Report Finite Elements in Fluids

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Steady transport 1D

Using Galerkin method in different situations we get the following profiles

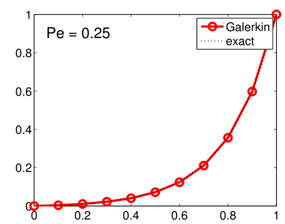

Figure 1: $a = 1$, $\nu = 0.2$ and 10 elements.

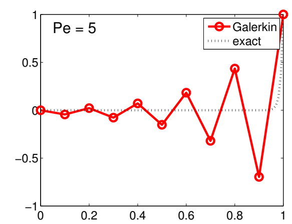

Figure 2: $a = 20$, $\nu = 0.2$ and 10 elements.

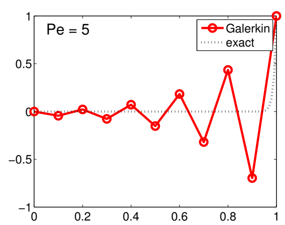

Figure 3: $a = 1$, $\nu = 0.01$ and 10 elements.

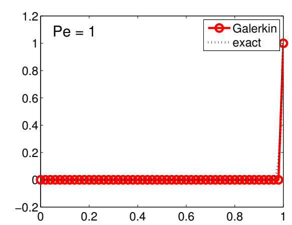

Figure 4: $a = 1$, $\nu = 0.01$ and 50 elements.

In the first example, as the Pclet number is lower than 1, there are no oscillations problems because is diffusion dominated and the numerical solution fits the exact one. The two following examples, despite having different values of a and ν, as they have the same proportion $\frac{a}{\nu}$ they have the same Pclet number, for this reason they draw the same profile u. As in both cases the Pclet number is higher than 1, there are oscillation point-to-point because is advection dominated. The last example, has the same values of a and ν compared to the third one, but changing the amount of elements in the simulation, obtains a different Pclet number that avoids oscillations point-to-point.

Changing the method to improve the results of the third case and avoid oscillations point-to-pint, the following result is obtained with all methods

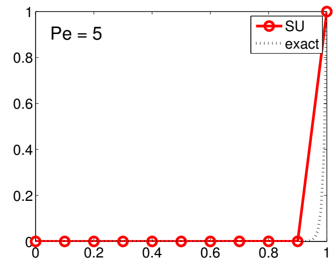

Figure 5: Third case using SU method.

that makes disappear oscillations without changing the number of elements as fourth case does. Seems that brings the exact solution for this mesh size. This solution has been obtained with the optimal τ coefficient that is 0.04005, but changing τ with SUPG method we get the following results

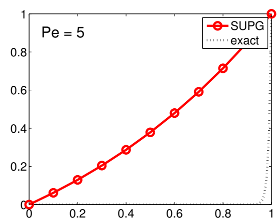

Figure 6: SUPG method with $\tau = 1$.

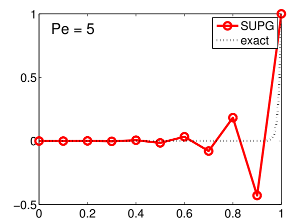

Figure 7: SUPG method with $\tau = 0.01$.

For large τ parameters the solution is stable but far from the exact one, while for little τ the solution is closer to the Galerkin method and oscillations appear again.

If instead of linear elements, quadratic elements are used, is not obtained nodal exact solution as the following figure shows

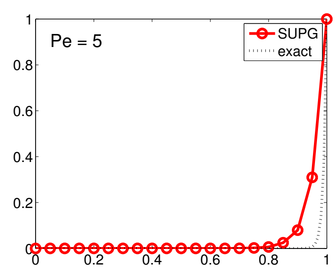

Figure 8: Quadratic elements.

Now adding reaction to the problem, we need to find the new exact solution

$$\frac{e^{\lambda_1 x} - e^{\lambda_2 x}}{e^{\lambda_1} - e^{\lambda_2}}$$

where λ_1 and λ_2 are the roots of $\nu\lambda^2 - a\lambda - \sigma = 0$. Using different methods we get the following results.

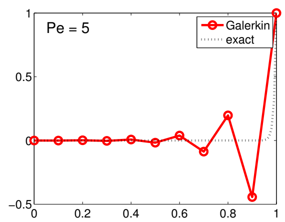

Figure 9: Reaction problem with Galerkin method.

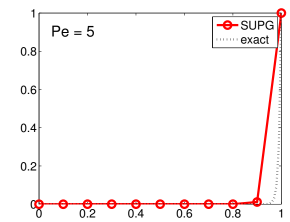

Figure 10: Reaction problem with SUPG method.

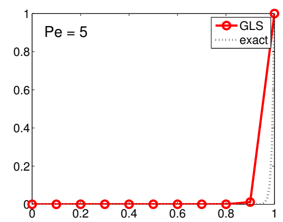

Figure 11: Reaction problem with GLS method.

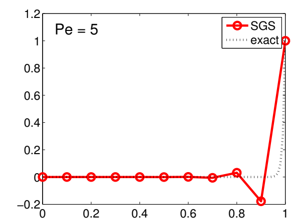

Figure 12: Reaction problem with SGS method.

The Galerkin method still has problems with oscillations and in the SGS method there oscillations to close to 1. But SUPG and GLS give good similar solutions close to the exact one. Refining the sharp front using a mesh with geometric ratio, the following results are obtained

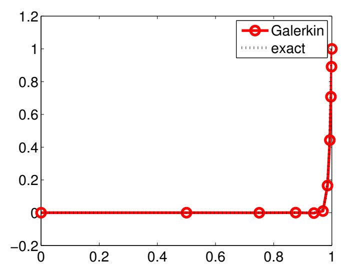

Figure 13: Geometric ratio mesh.

where the Galerkin solution fits really well the exact solution.

Steady transport 2D

The 2D convection-diffusion problem gives these solutions with Neumann boundary condition

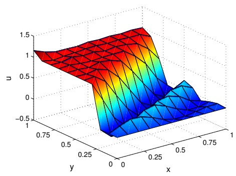

Figure 14: Galerkin method.

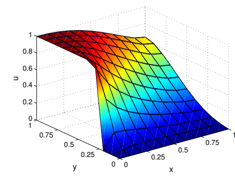

Figure 15: Art. diffusion method.

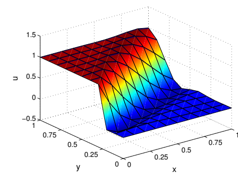

Figure 16: SUPG method.

In the Galerkin one is observed some oscillations and the artificial diffusion solution gives a smooth solution taking into account that the diffusion coefficient is so little. Otherwise, the SUPG solution seems that fits better the exact one. If instead, Dirichlet boundary conditions are imposed

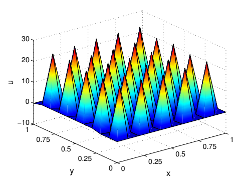

Figure 17: Galerkin method.

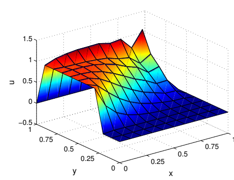

Figure 18: Art. diffusion method.

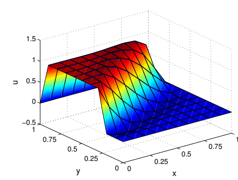

Figure 19: SUPG method.

using the Galerkin method is obtained a useless solution. The artificial diffusion method gives some oscillations and the SUPG method seems that gives a good solution. In these cases, where transport dominates, have no sense to impose boundary condition in the outflow, so the case with Neumann boundary conditions is more real. Now, if reaction is added, the convection-reaction dominated case behaves

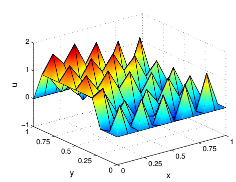

Figure 20: Galerkin method.

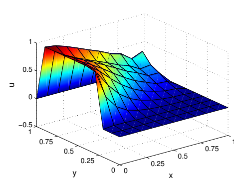

Figure 21: Art. diffusion method.

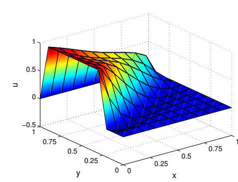

Figure 22: SUPG method.

where methods give a solution similar to the previous figures where SUPG gave the better solution.

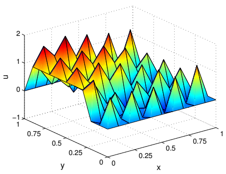

Figure 23: Galerkin method.

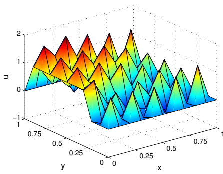

Figure 24: Art. diffusion method.

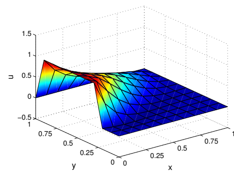

Figure 25: SUPG method.

In reaction dominated problems, SUPG method is also the better one.