Finite element in fluid

Assignment 4

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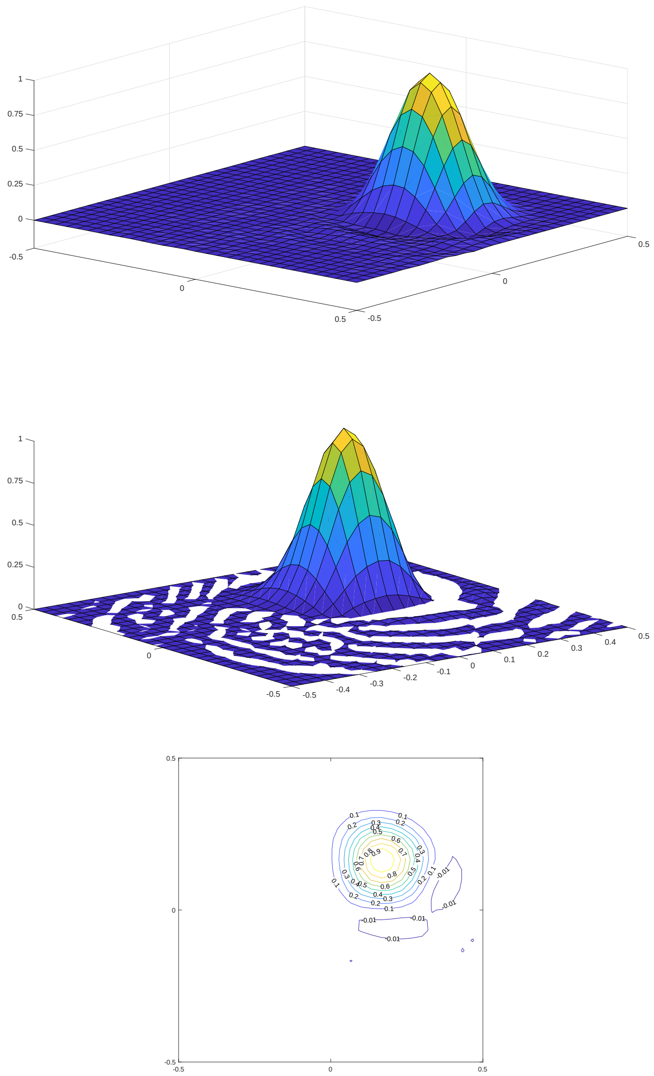
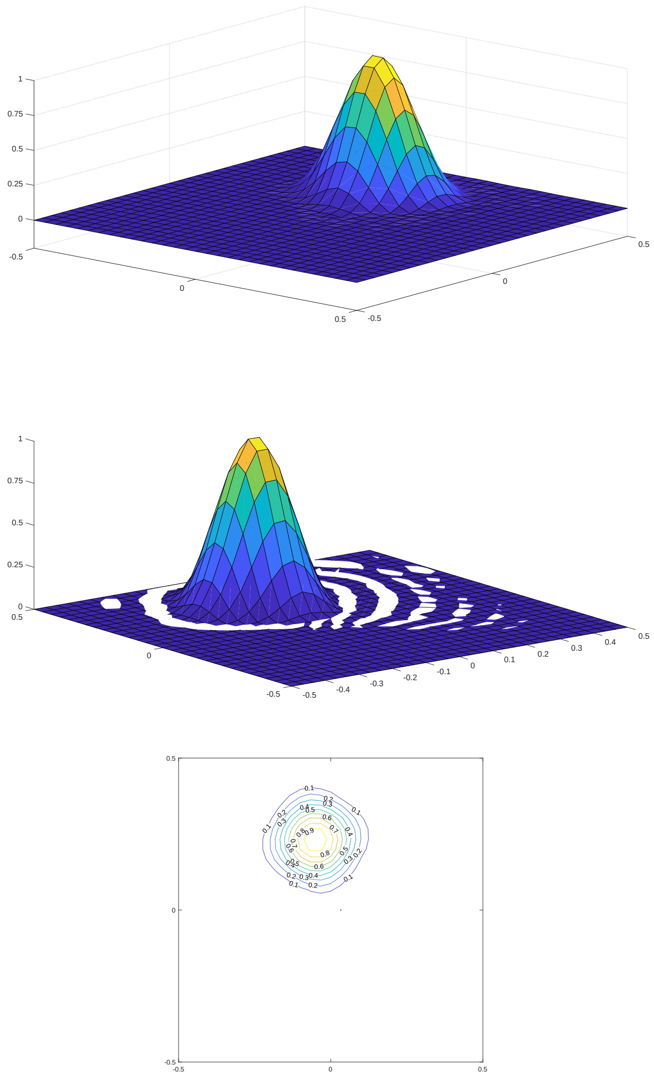


Figure 1: Crank Nicolson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0.5
0.25
0
-0.5
0
0.5
-0.5
0
0.5
1
0.75
0.5
0.25
0
0.5
0
-0.5
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.5
0
-0.5
-0.5
0
0.5

Figure 2: TG2

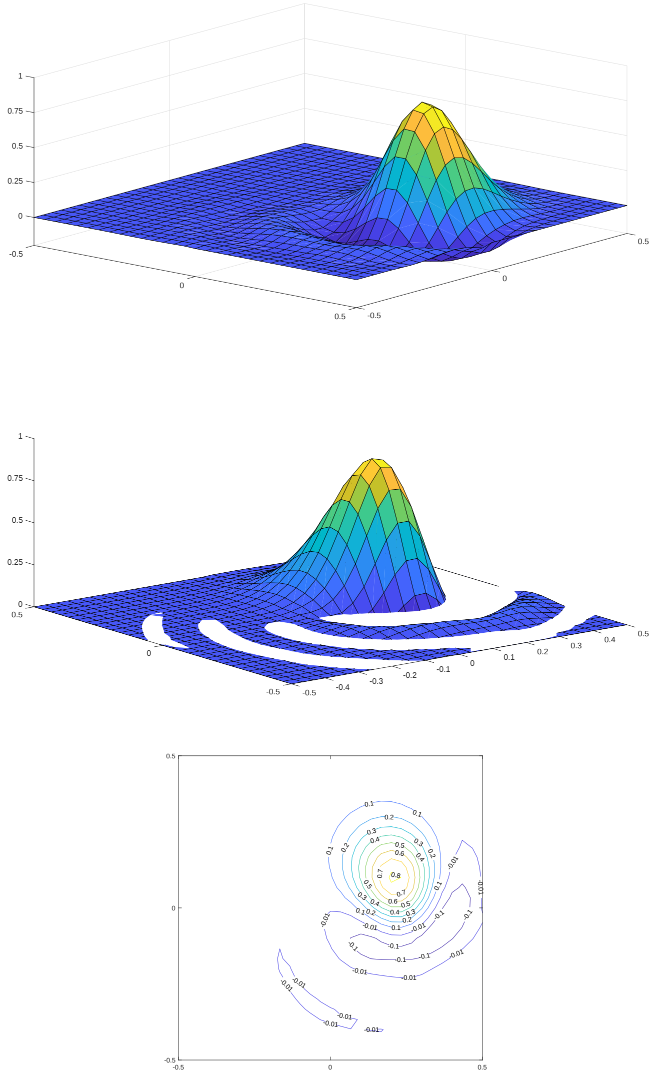


Figure 3: Lax wandroff with lumped mass matrix

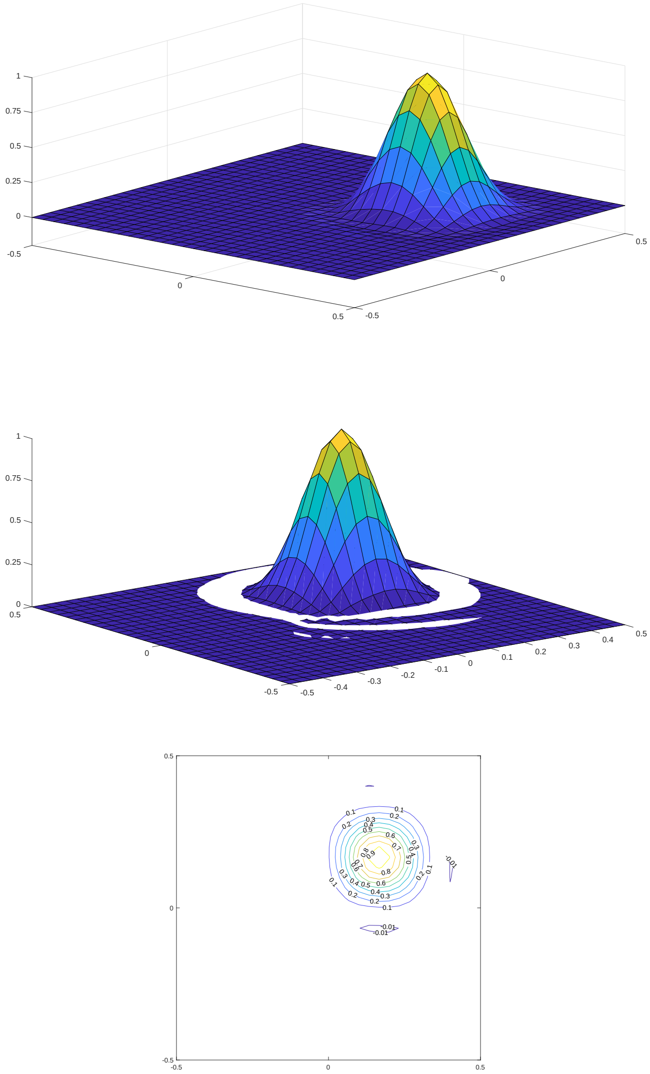


Figure 4: TG3

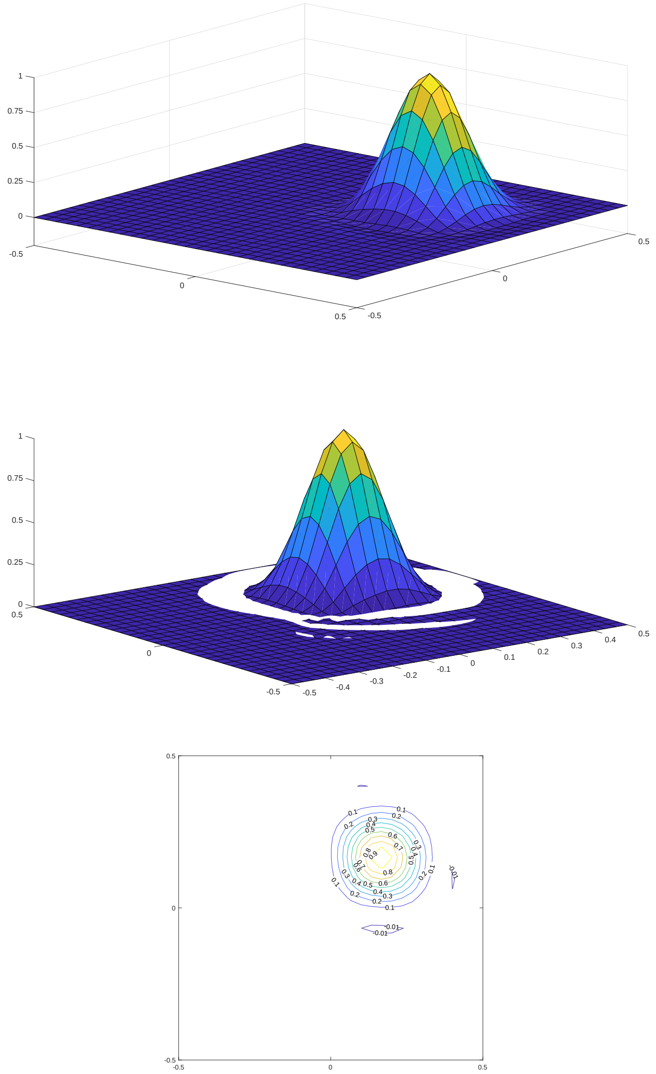
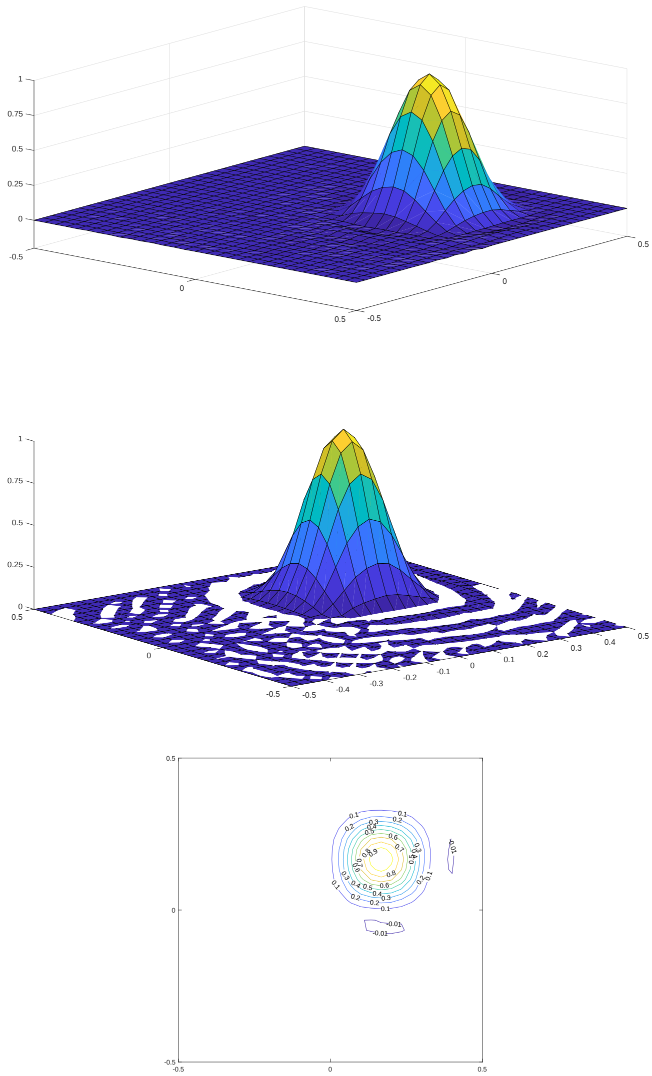


Figure 5: TG3-2step


1
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0.5
0.25
0
-0.5
0
0.5
-0.5
0
0.5
1
0.75
0.5
0.25
0
0.5
0
-0.5
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.5
0
-0.5
-0.5
0
0.5

Figure 6: TG4

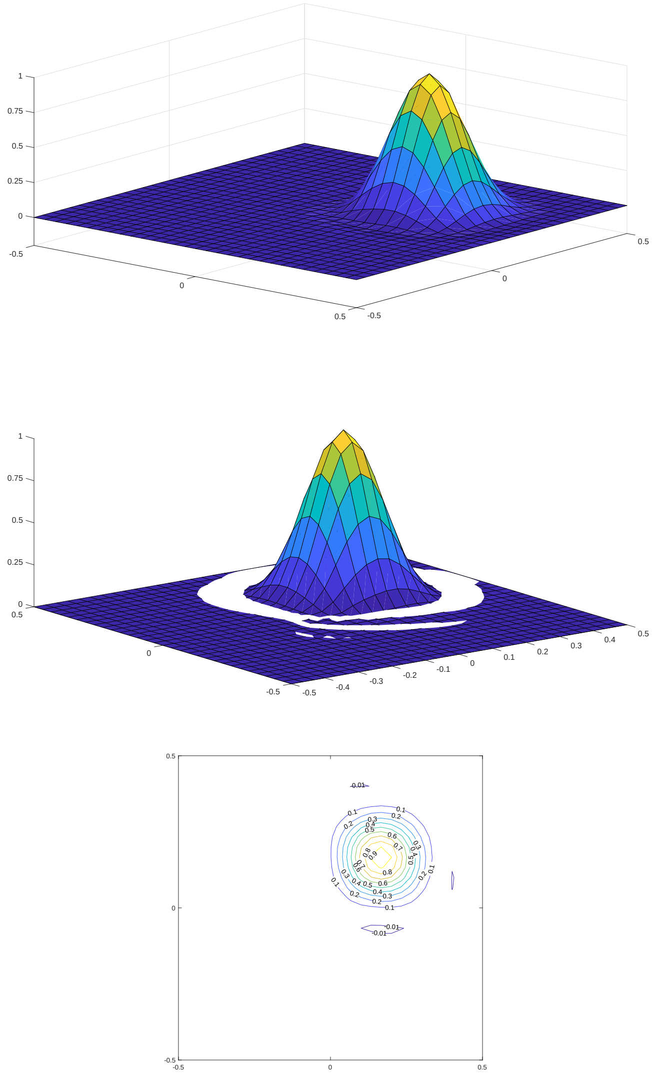

Figure 7: TG4-2step

	u_{max}	u_{min}
TG2	0.9833	-0.0104
Lax wendroff	0.9971	-0.0572
Crank Nicolson	0.9848	-0.0107
Crank Nicolson with lumped matrix	0.999	-0.0590
TG3	0.98318	-0.0104
TG3-2Step	0.9831	-0.0104
TG4	0.9849	-0.0107
TG4-2Step	0.9831	-0.0104

2D convection schemes considers the convection of a product cosine hill in a pure rotation velocity field.
Comparing the accuracy of various explicit method.
Showing maximum and minimum values for various explicit method.

The Lax wendroff with consistent mass matrix and Taylor Galerkin have good accuracy and Lax wendroff with diagonal mass matrix is computationally less expensive than Lax wendroff with consistent mass matrix.

For Crank Nicolson phase accuracy of the method decrease when the time step start to increase.
We have non physic osculations when we increase the time step.

TG3

$$\tilde{u}^n = u^n + \frac{1}{3}\Delta t u_t^n + \alpha \Delta t^2 u_{tt}^n$$

$$\tilde{u}^n = u^n + \frac{1}{3}\Delta t(s^n - a.\nabla u^n) + \alpha \Delta t^2 (s_t^n - a.\nabla s^n + (a.\nabla)^2 u^n)$$

$$\tilde{u}^n = u^n + \frac{1}{3}\Delta t s^n - \frac{1}{3}\Delta t a.\nabla u^n + \alpha \Delta t^2 s_t^n - \alpha \Delta t^2 a.\nabla s^n + \alpha \Delta t^2 (a.\nabla^2) u^n$$

$$(w, \tilde{u}^n) = (w, u^n) + \frac{1}{3}\Delta t(w, s^n) - \frac{1}{3}\Delta t(w, a.\nabla u^n) - \alpha \Delta t^2 (w, a.\nabla s^n) + \alpha \Delta t^2 (w, (a.\nabla)^2 u^n)$$

$$[1 - \frac{\Delta t^2}{6}(a.\nabla)^2]\frac{u^{n+1} - u^n}{\Delta t} = -(a.\nabla)u^n + \frac{\Delta t}{2}(a.\nabla)^2 u^n$$

$$(w, \frac{\Delta u}{\Delta t} + \frac{\Delta t^2}{6}(w, a.\nabla \frac{\Delta u}{\Delta t}) = -(w, a.\nabla u^n) - \frac{\Delta t}{2}(w, \nabla)^2 u^n)$$

$$(w, \frac{\Delta u}{\Delta t} - \frac{\Delta t^2}{6}(w, (a.\nabla)^2 \frac{\Delta u}{\Delta t}) = -(w, a.\nabla u^n) + \frac{\Delta t}{2}(w, a.\nabla u^n)$$

$$u^{n+1} = u^n + \Delta t u_t^n + \frac{1}{2}\Delta t^2 \tilde{u_{tt}}^n$$

$$u^{n+1} = u^n + \Delta t(s^n - a.\Delta t(s^n - a.\nabla u^n) + \frac{1}{2}\Delta t^2 (s_t^n - a.\nabla s^n + (a.\nabla)^2 u^n)$$

$$u^{n+1} = u^n + \Delta t s^n - \Delta t a.\nabla u^n - \frac{1}{2}\Delta t^2 a.\nabla s^n + \frac{1}{2}\Delta t^2 (a.\nabla)^2 \tilde{u}$$

$$(w, u^{n+1}) = (w, u^n) + \Delta t(w, s^n) - \Delta t(w, a.\nabla u^n) - \frac{1}{2}\Delta t^2 (w, a.\nabla s^n) + \frac{1}{2}\Delta t^2 (w, (a.\nabla)^2 \tilde{u_n})$$

TG4

$$\tilde{\mathrm{u}} = \mathrm{u}^{\mathrm{n}} + \frac{1}{3}\Delta \mathrm{t} \mathrm{u}_{\mathrm{t}}^{\mathrm{n}} + \frac{1}{12}\Delta \mathrm{t}^2 \mathrm{u}_{\mathrm{tt}}^{\mathrm{n}}$$

$$\tilde{\mathrm{u}} = \mathrm{u}^{\mathrm{n}} + \frac{1}{3}\Delta \mathrm{t}(\mathrm{s}^{\mathrm{n}} - \mathrm{a}.\nabla \mathrm{u}^{\mathrm{n}}) + \frac{1}{12}\Delta \mathrm{t}^2(\mathrm{s}_{\mathrm{t}}^{\mathrm{n}} - \mathrm{a}.\nabla \mathrm{s}^{\mathrm{n}} + (\mathrm{a}.\nabla)^2 \mathrm{u}^{\mathrm{n}})$$

$$\tilde{\mathrm{u}} = \mathrm{u}^{\mathrm{n}} + \frac{1}{3}\Delta \mathrm{t} \mathrm{s}^{\mathrm{n}} - \frac{1}{3}\Delta \mathrm{t} \mathrm{a}.\nabla \mathrm{u}^{\mathrm{n}} + \frac{1}{12}\Delta \mathrm{t}^2 \mathrm{s}_{\mathrm{t}}^{\mathrm{n}} - \frac{1}{12}\Delta \mathrm{t}^2 \mathrm{a}.\nabla \mathrm{s}^{\mathrm{n}} + \frac{1}{12}\Delta \mathrm{t}^2(\mathrm{a}.\nabla)^2 \mathrm{u}^{\mathrm{n}}$$

$$(\mathrm{w}, \tilde{\mathrm{u}}) = (\mathrm{w}, \mathrm{u}^{\mathrm{n}}) + \frac{1}{3}\Delta \mathrm{t}(\mathrm{w}, \mathrm{s}^{\mathrm{n}}) - \frac{1}{3}\Delta \mathrm{t}(\mathrm{w}, \mathrm{a}.\nabla \mathrm{u}^{\mathrm{n}}) - \frac{1}{12}\Delta \mathrm{t}^2(\mathrm{w}, \mathrm{a}.\nabla \mathrm{s}^{\mathrm{n}}) + \frac{1}{12}\Delta \mathrm{t}^2(\mathrm{w}, (\mathrm{a}.\nabla)^2 \mathrm{u}^{\mathrm{n}})$$

$$\frac{\Delta \mathrm{u}}{\Delta \mathrm{t}} = \frac{1}{2}(\mathrm{u}_{\mathrm{t}}^{\mathrm{n+1}} + \mathrm{u}_{\mathrm{t}}^{\mathrm{n}}) - \Delta \mathrm{t}\frac{1}{12}(\mathrm{u}_{\mathrm{tt}}^{\mathrm{n+1}} - \mathrm{u}_{\mathrm{tt}}^{\mathrm{n}})$$

$$\frac{\Delta \mathrm{u}}{\Delta \mathrm{t}} = \frac{1}{2}(\mathrm{s}^{\mathrm{n+1}} - \mathrm{a}.\nabla \mathrm{u}^{\mathrm{n+1}} + \mathrm{s}^{\mathrm{n}} - \mathrm{a}.\nabla \mathrm{u}^{\mathrm{n}}) - \frac{\Delta \mathrm{t}}{12}\mathrm{s}_{\mathrm{t}}^{\mathrm{n+1}} - \mathrm{a}.\nabla \mathrm{s}^{\mathrm{n+1}} + (\mathrm{a}.\nabla)^2 \mathrm{u}^{\mathrm{n+1}} - \mathrm{s}_{\mathrm{t}}^{\mathrm{n}} + \mathrm{a}.\nabla \mathrm{s}^{\mathrm{n}} - (\mathrm{a}.\nabla)^2 \mathrm{u}^{\mathrm{n}}$$

$$\frac{\Delta \mathrm{u}}{\Delta \mathrm{t}} = \mathrm{s} - \frac{1}{2}\mathrm{a}.\nabla \mathrm{u}^{\mathrm{n+1}} - \frac{1}{2}\mathrm{a}.\nabla \mathrm{u}^{\mathrm{n}} + \frac{\Delta \mathrm{t}}{12}(\mathrm{a}.\nabla)^2 \Delta \mathrm{u}$$

$$\mathrm{u}(\mathrm{t}^{\mathrm{n+1}}) = \mathrm{u}(\mathrm{t}^{\mathrm{n}}) + \Delta \mathrm{t} \mathrm{u}_{\mathrm{t}}^{\mathrm{n}}(\mathrm{t}^{\mathrm{n}}) + \frac{\Delta \mathrm{t}^2}{2}\mathrm{u}_{\mathrm{tt}}^{\mathrm{n}}(\mathrm{t}^{\mathrm{n}}) + \frac{\Delta \mathrm{t}^3}{6}\mathrm{u}_{\mathrm{ttt}}^{\mathrm{n}}(\mathrm{t}^{\mathrm{n}}) + \frac{\Delta \mathrm{t}^4}{24}\mathrm{u}_{\mathrm{tttt}}^{\mathrm{n}}(\mathrm{t}^{\mathrm{n}}) + \mathrm{o}(\Delta \mathrm{t}^5)$$

Crank Nicolson

$$(\mathrm{w}, \frac{\Delta \mathrm{u}}{\Delta \mathrm{t}}) - \frac{1}{2}(\nabla \mathrm{w}, \mathrm{a}\Delta \mathrm{u}) + \frac{1}{2}((\mathrm{a.n})\mathrm{w}, \Delta \mathrm{u}) = (\nabla \mathrm{w}, \mathrm{a}\mathrm{u}^{\mathrm{n}}) - ((\mathrm{a.n})\mathrm{w}, \mathrm{u}^{\mathrm{n}})$$

$$(\mathrm{w}, \Delta \mathrm{u}) - \Delta \mathrm{t}\frac{1}{2}(\nabla \mathrm{w}, \mathrm{a}\Delta \mathrm{u}) + \Delta \mathrm{t}\frac{1}{2}((\mathrm{a.n})\mathrm{w}, \Delta \mathrm{u}) = \Delta \mathrm{t}(\nabla \mathrm{w}, \mathrm{a}\mathrm{u}^{\mathrm{n}}) - \Delta \mathrm{t}((\mathrm{a.n})\mathrm{w}, \mathrm{u}^{\mathrm{n}})$$

Lax wendroff

$$(\mathrm{w}, \frac{\Delta \mathrm{u}}{\Delta \mathrm{t}}) = (\mathrm{a}.\nabla \mathrm{w}, \mathrm{u}^{\mathrm{n}} - \Delta \mathrm{t}\frac{1}{2}(\mathrm{a}.\nabla)\mathrm{u}^{\mathrm{n}}) - ((\mathrm{a.n})\mathrm{w}, \mathrm{u}^{\mathrm{n}} - \Delta \mathrm{t}\frac{1}{2}(\mathrm{a}.\nabla)\mathrm{u}^{\mathrm{n}})$$

$$\mathrm{u}(\mathrm{t}^{\mathrm{n}+1}) = \mathrm{u}(\mathrm{t}^{\mathrm{n}}) + \Delta \mathrm{t}\mathrm{u}_{\mathrm{t}}(\mathrm{t}^{\mathrm{n}}) + \frac{1}{2}\Delta \mathrm{t}^2\mathrm{u}_{\mathrm{tt}}(\mathrm{t}^{\mathrm{n}}) + \mathrm{o}(\Delta \mathrm{t}^3)$$

$$\mathrm{u}_{\mathrm{t}}^{\mathrm{n}} = \mathrm{s}^{\mathrm{n}} - \mathrm{a}.\nabla \mathrm{u}^{\mathrm{n}}$$

$$\mathrm{u}_{\mathrm{tt}}^{\mathrm{n}} = \mathrm{s}_{\mathrm{t}}^{\mathrm{n}} - \mathrm{a}.\nabla \mathrm{u}_{\mathrm{t}}^{\mathrm{n}} = \mathrm{s}_{\mathrm{t}}^{\mathrm{n}} - \mathrm{a}.\nabla \mathrm{s}^{\mathrm{n}} + (\mathrm{a}.\nabla \mathrm{s}^{\mathrm{n}} + (\mathrm{a}.\nabla)^2\mathrm{u}^{\mathrm{n}}$$

$$\frac{\Delta \mathrm{u}}{\Delta \mathrm{t}} = -\mathrm{a}.\nabla \mathrm{u}^{\mathrm{n}} + \Delta \mathrm{t}\frac{1}{2}(\mathrm{a}.\nabla)^2\mathrm{u}^{\mathrm{n}} + \mathrm{s}^{\mathrm{n}} + \Delta \mathrm{t}\frac{1}{2}(\mathrm{s}_{\mathrm{t}}^{\mathrm{n}} - \mathrm{a}.\nabla \mathrm{s}^{\mathrm{n}})$$

$$(\mathrm{w}, \frac{\Delta \mathrm{u}}{\Delta \mathrm{t}}) = -(\mathrm{w}, \mathrm{a}.\nabla \mathrm{u}^{\mathrm{n}} - \Delta \mathrm{t}\frac{1}{2}(\mathrm{a}.\nabla)^2\mathrm{u}^{\mathrm{n}}) + (\mathrm{w}, \mathrm{s}^{\mathrm{n}} + \Delta \mathrm{t}\frac{1}{2}(\mathrm{s}_{\mathrm{t}}^{\mathrm{n}} - \mathrm{a}.\nabla \mathrm{s}^{\mathrm{n}}))$$

$$(\mathrm{w}, \frac{\Delta \mathrm{u}}{\Delta \mathrm{t}}) = (\mathrm{a}.\nabla \mathrm{w}, \mathrm{u}^{\mathrm{n}} + \frac{\Delta \mathrm{t}}{2}[\mathrm{s}^{\mathrm{n}} - (\mathrm{a}.\nabla)\mathrm{u}^{\mathrm{n}}]) - ((\mathrm{a.n})\mathrm{w}, \mathrm{u}^{\mathrm{n}} + \frac{\Delta \mathrm{t}}{2}[\mathrm{s}^{\mathrm{n}} - (\mathrm{a}.\nabla)\mathrm{u}^{\mathrm{n}}])$$

$$+(\mathrm{w}, \mathrm{h}^{\mathrm{n}+\frac{1}{2}}) + (\mathrm{w}, \mathrm{s}^{\mathrm{n}} + \Delta \mathrm{t}\mathrm{s}_{\mathrm{t}}^{\mathrm{n}})$$

Leap frog

$$\mathrm{u(t^{n+1}) = u(t^{n-1}) + 2\Delta t u_t(t^n) + o(\Delta t^3)}$$

$$\mathrm{\frac{u^{n+1} - u^{n-1}}{2\Delta t} = u_t^n = s^n - a.\nabla u^n}$$

$$\mathrm{(w, \frac{u^{n+1}}{2\Delta t}) = (w, \frac{u^{n-1}}{2\Delta t}) + (w, s^n - a.\nabla u^n)}$$

$$\mathrm{(w, \frac{u^{n+1}}{2\Delta t}) = (w, \frac{u^{n-1}}{2\Delta t} + s^n) + (a.\nabla w, u^n) - (w, u^n(a.n)) + (w, h^n)}$$

Annex

$ifmeth == 1A = M;$
$B = dt * (C - (dt/2) * K - Mo + (dt/2) * Co);$
$f = dt * (v1 + (dt/2) * (v2 - vo));$

$elseifmeth == 2$
$Md = diag(M * ones(numnp, 1));$
$Mod = diag(Mo * ones(numnp, 1));$
$A = Md;$
$B = dt * (C - (dt/2) * K - Mod + (dt/2) * Co);$
$f = dt * (v1 + (dt/2) * (v2 - vo));$

$elseifmeth == 3$
$A = M + (dt^2/6) * (K - Co);$
$B = dt * (C - (dt/2) * K - Mo + (dt/2) * Co);$
$f = dt * ((dt/2) * (v2 - vo) + v1);$

$elseifmeth == 4A = M - (dt/2) * C + (dt/2) * Mo;$
$B = dt * C - dt * Mo;$
$f = dt * v1;$

$elseifmeth == 5$
$Md = diag(M * ones(numnp, 1));$
$Mod = diag(Mo * ones(numnp, 1));$
$A = Md - (dt/2) * C + (dt/2) * Mod;$
$B = dt * C - dt * Mod;$
$f = dt * v1;$

$elseifmeth == 6$
$A = M + (dt/2) * (C + C') + (dt^2/4) * K;$
$B = -dt * (C' + (dt/2) * K);$
$f = dt * v1 + (dt^2/2) * v2;$

$elseifmeth == 7$
$alpha = 1/9;$
$A1 = M;$
$B1 = (dt/3) * (C - Mo) - alpha * dt^2 * (K - Co);$
$f1 = (dt/3) * v1 + alpha * dt^2 * (v2 - vo);$
$A2 = M;$
$B2 = dt * (C - Mo);$
$C2 = -(dt^2/2) * (K - Co);$
$f2 = dt * v1 - (dt^2/2) * (v2 - vo);$

$elseifmeth == 8A = M + (dt/2) * C' - (dt^2/12) * (K - Co); B = -dt * C';$
$f = dt * v1;$

$elseifmeth == 9$
$alpha = 1/12;$
$A1 = M;$
$B1 = (dt/3) * (C - Mo) - alpha * dt^2 * (K - Co);$
$f1 = (dt/3) * v1 + alpha * dt^2 * (v2 - vo);$
$A2 = M;$
$B2 = dt * (C - Mo);$
$C2 = -(dt^2/2) * (K - Co);$
$f2 = dt * v1 - (dt^2/2) * (v2 - vo);$