Lab Report - 5

1D Unsteady Transport problems

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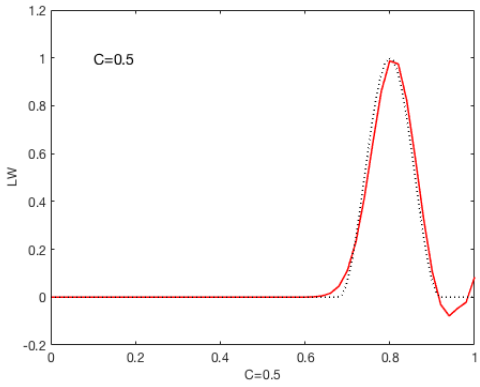
1 Propagation of cosine profile

$$\begin{cases} u_t + a u_x = 0 & x \in (0,1),\ t \in (0, 0.6] \\ u(x,0) = u_0(x) & x \in (0,1) \\ u(0,t) = 0 & t \in (0, 0.6] \end{cases}$$

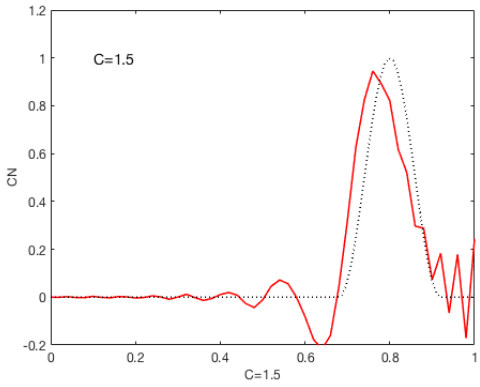
$$u_0(x) = \begin{cases} \frac{1}{2}(1 + \cos(\pi(x - x_0)/\sigma)) & \text{if } |x - x_0| \leq \sigma, \\ 0 & \text{otherwise} \end{cases}$$

$$a = 1, x_0 = 0.2, \sigma = 0.12, \Delta x = 2 \cdot 10^{-2}$$

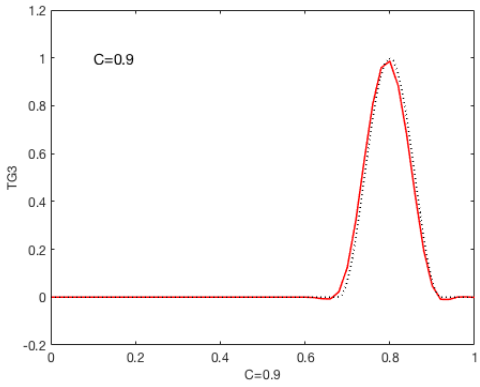
Problem statement



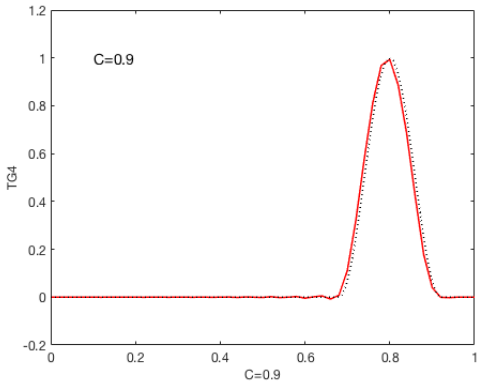
Lax wendroff + galerkin C=0.5



Crank nicolson + galerkin C = 1.5



TG3 + galerkin C=0.9



TG4 + galerkin C =0.9

It can be observed that the lax wendroff is stable when $C^2 < 1/3$ and crank nicolson is unconditionally stable while the TG3 and TG4 are stable for $C < 1$. However the crank nicolson is unconditionally stable, the numerical solution is inaccurate and prone to spurious oscillations for higher courant numbers.

2 Propagation of Steep front

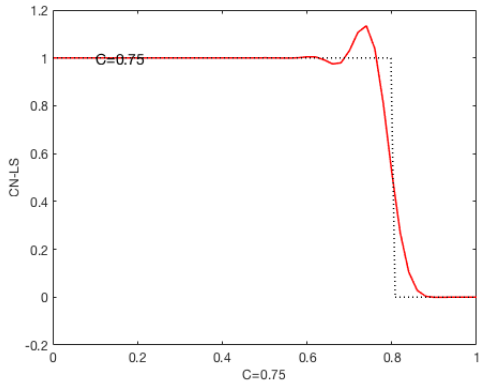
$$\begin{cases} u_t + a u_x = 0 & x \in (0,1),\ t \in (0, 0.6] \\ u(x,0) = u_0(x) & x \in (0,1) \\ u(0,t) = 1 & t \in (0, 0.6] \end{cases}$$

$$u_0(x) = \begin{cases} 1 & \text{if } x \leq 0.2, \\ 0 & \text{otherwise} \end{cases}$$

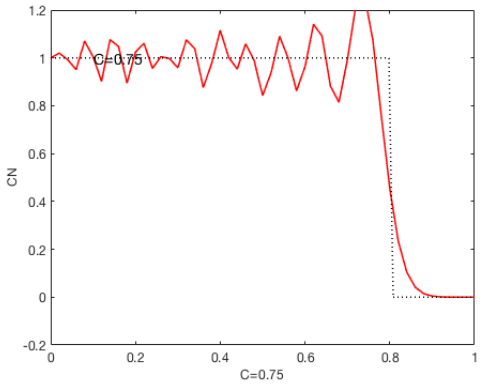
$$a = 1, \Delta x = 2 \cdot 10^{-2}, \Delta t = 1.5 \cdot 10^{-2}$$

Problem statement

The courant number is given by $C = a\Delta t/h = 0.75$.

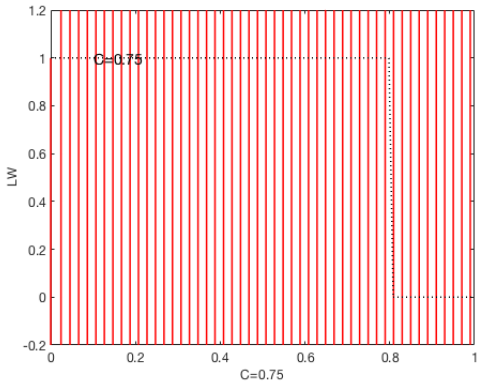


Crank nicolson + Least squares C = 0.75



Crank nicolson + galerkin C = 0.75

The Crank nicolson with least squares method exhibited much lesser spurious oscillations compared to the galerkin space formulation even at higher courant numbers.



Lax wendroff + galerkin C = 0.75

The lax wendroff method provides an unstable solution for $C^2 < 1/3$, hence the solution obtained is not accurate.