Lab Report - 2

1D Transport problems

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We consider the pure transport equation in 1D.

$$\begin{aligned} u_t + (a.\nabla)u &= s & in\ \Omega \times [0,T] \\ u(x,0) &= u_0(x) & on\ \Omega\ at\ t=0 \\ u = &= u_D & on\ \Gamma_D^{in} \\ -a\ u.n &= h & \Gamma_N^{in} \end{aligned}$$

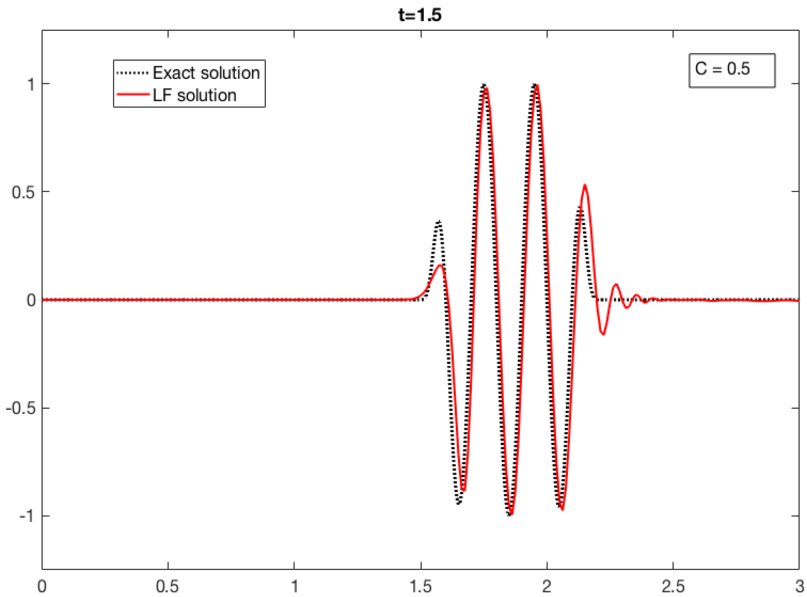
Zero source term is considered for all the examples with homogeneous Dirichlet boundary conditions on the inflow boundary. The leap frog method is given by

$$\begin{aligned} \frac{u^{n+1} - u^{n-1}}{2\Delta t} &= u_t^n = s^n - a.\nabla u^n \\ \Delta u &= u^{n+1} - u^{n-1} \\ \Delta u &= -2\Delta t\ a.\nabla u^n \\ (\Delta u, w) &= -2\Delta t\ a(\nabla u^n, w) \end{aligned}$$

The leap frog scheme is not a self starting scheme. Hence the first time step, the Lax wendroff method is used.

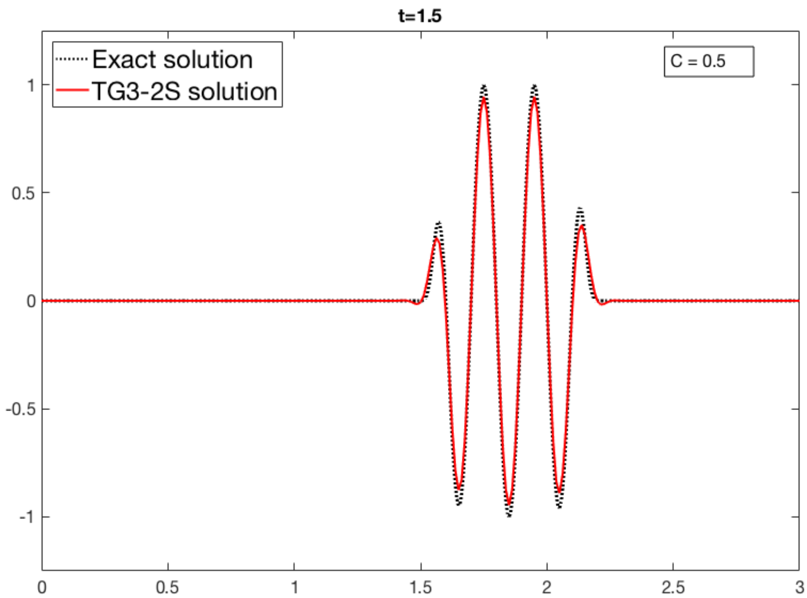
```
if method == 8 % Leap frog
        [At,Bt,methodNamet] = System(1,M,K,C,a,dt);
        At = At(ind_unk,ind_unk);
        Bt = Bt(ind_unk,ind_unk);
        Du = At\(Bt*u(ind_unk,1) + f);
        u(ind_unk,2) = u(ind_unk,1) + Du;
        for n = 2:nStep
                Du = A\(B*u(ind_unk,n) + f);
                u(ind_unk,n+1) = u(ind_unk,n-1) + Du;
        end
else ...
case 8 % Leap Frog + Galerkin
        A = M;
        B = -2*a*dt*C;
        methodName = 'LF';
```

The same stability limits of the lax wendroff scheme of $C^2 < 1/3$ is observed.



The two step Taylor galerkin method is given by

$$\begin{aligned}
\tilde{u}^n &= u^n + \frac{1}{3}\Delta t u_t^n + \frac{1}{9}\Delta t^2 u_{tt}^n \\
u^{n+1} &= u^n + \Delta t u_t^n + \frac{1}{2}\Delta t^2 \tilde{u}_{tt}^n
\end{aligned}$$




```
if method ==7
        Du = At\ (Bt*u(ind_unk,n) + f);
        utilde(ind_unk) = u(ind_unk,n) + Du;
```

```
            Du = A\ (B*u(ind_unk,n) + D*utilde(ind_unk) + f);
            u(ind_unk,n+1) = u(ind_unk,n) + Du;
        else ...
```

The A and B matrices for the 2 steps are codes separately and solved together as above. The stability limits of $C^2 < 3/4$ is observed. The solutions obtained with 2 step TG3 were slightly more diffusive than TG3.

In addition, The TG4 and TG3 were also implemented for comparisons.