Lab Report - 1

Steady Convection Diffusion Equation

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1 Quadratic Finite Elements

$$
\begin{aligned}
N_1(\xi) &= \frac{\xi^2 - \xi}{2} \\
N_2(\xi) &= -\xi^2 + 1 \\
N_3(\xi) &= \frac{\xi^2 + \xi}{2} \\
x &= x_1(\frac{\xi^2 - \xi}{2}) + x_2(-\xi^2 + 1) + x_3(\frac{\xi^2 + \xi}{2}) \\
\frac{dx}{d\xi} &= \frac{x_3 - x_1}{2} \\
\frac{d\xi}{dx} &= \frac{2}{h} \\
\frac{d^2\xi}{dx^2} &= 0 \\
\frac{\partial N_i}{\partial x} &= \frac{\partial N_i}{\partial \xi}\frac{d\xi}{dx} \\
\frac{\partial^2 N_i}{\partial x^2} &= \frac{\partial N_i}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 N_i}{\partial \xi^2}(\frac{d\xi}{dx})^2 = \frac{4}{h^2}\frac{\partial^2 N_i}{\partial \xi^2}
\end{aligned}
$$

The cordinates of the nodes and the mesh connectivity matrix were changed as follows.

```
X = (dom(1):h/2:dom(2))';
T = [1:2:nPt-2; 2:2:nPt-1; 3:2:nPt]';
```

For SUPG stabilization, the second derivative of the shapefunction was used and the Element Stiffness matrix was modified as follows.

```
N2xi = referenceElement.N2xi;
N2x_ig = N2xi(ig,:)*4/h^2;
Ke = Ke + w_ig*(N_ig'*a*Nx_ig + Nx_ig'*nu*Nx_ig) ...
+ w_ig*(tau*a*Nx_ig)'*((a*Nx_ig) -nu*N2x_ig - s);
```

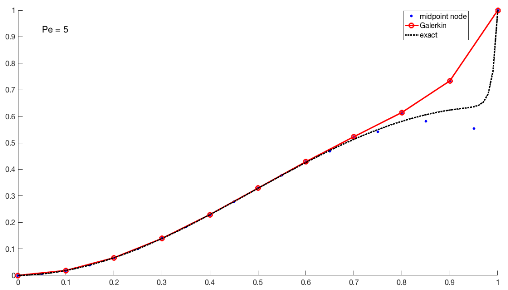
For GLS stabilization, the second derivative of the shapefunction was used and the Element Stiffness matrix was modified as follows.

```
N2xi = referenceElement.N2xi;
N2x_ig = N2xi(ig,:)*4/h^2;
Ke = Ke + w_ig*(N_ig'*a*Nx_ig ...
+ Nx_ig'*nu*Nx_ig) + w_ig*tau*...
(a*Nx_ig -nu*N2x_ig)'*...
((a*Nx_ig) -nu*N2x_ig - s);
```

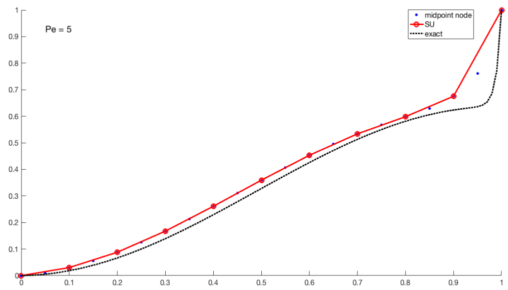
1.1 Numerical Results - Quadratic Elements

$$
\begin{aligned}
au_x - \nu u_{xx} &= f(x) \qquad in\ \Omega \in [0,1] \\
u(x=0) &= 0 \\
u(x=1) &= 1 \\
f(x) &= sin(\pi x)
\end{aligned}
$$

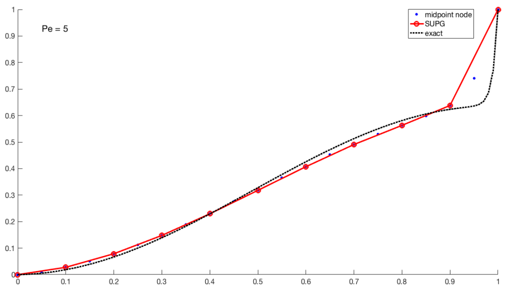
All cases presented are for $a = 1$, $\nu = 0.01$, Peclet number 5 and 10 quadratic elements.



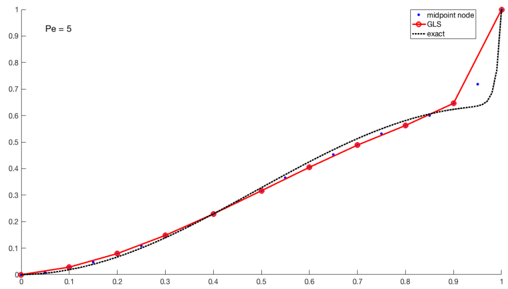
(a) Galerkin Method



(b) SU



(c) SUPG



(d) GLS

The Galerkin case provides a nodally oscillating solution (observe the midpoint node values) and the rest of the stabilization methods provide a stable solution.

2 Linear Finite Elements

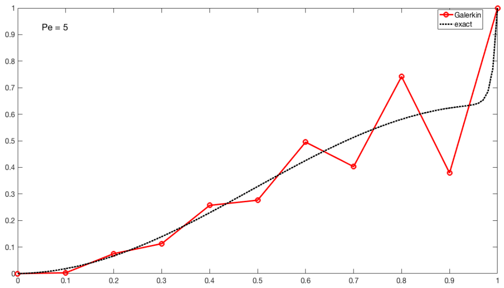
For linear Finite elements, SUPG and GLS are the same because the diffusive part of the stabilization term is 0. The following change was made to the element stiffness matrix for SUPG and GLS stabilization.

```
Ke = Ke + w_ig*(N_ig'*a*Nx_ig + Nx_ig'*nu*Nx_ig) ...
+ w_ig*(tau*a*Nx_ig)'*(a*Nx_ig - s);
```

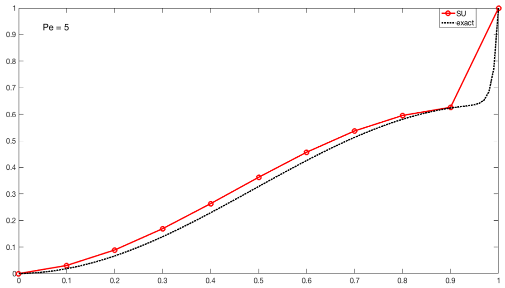
2.1 Numerical Results - Linear Elements

$$\begin{aligned} au_x - \nu u_{xx} &= f(x) \qquad in\ \Omega \in [0,1] \\ u(x=0) &= 0 \\ u(x=1) &= 1 \\ f(x) &= sin(\pi x) \end{aligned}$$

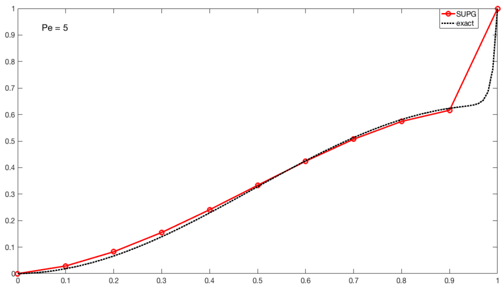
All cases presented are for $a = 1$, $\nu = 0.01$, Peclet number 5 and 10 linear elements.



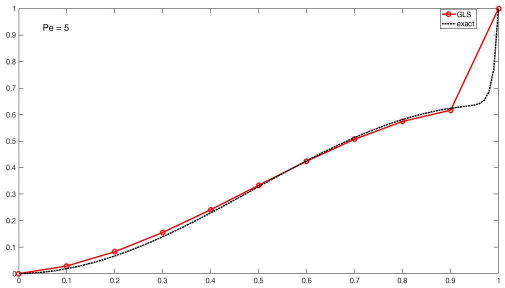
(a) Galerkin Method



(b) SU



(c) SUPG



(d) GLS