Solution:

Leap frog:-

Scheme:- $\dfrac{u^{n+1}-u^{n-1}}{2\Delta t} = u_t^n = s^n - \boldsymbol{a} * \boldsymbol{\nabla} u^n$

For the given 1D problem it is reduced to: (Galerkin form)

$(w,\dfrac{u^{n+1}}{2\Delta t}) = (w,\dfrac{u^{n-1}}{2\Delta t}) + (aw_x\,,u^n)$

The first iteration is performed using the Lax-Wendroff method. At intermediate and short wavelengths the phase error of the LF scheme becomes positive as the time step is increased and deteriorates. The relative phase errors of the leap-frog schemes using a consistent mass representation are reported in Figures 1for the C=0.5. It can be observe in the Figure 2.

TG3:

TG3 Scheme:-

$\dfrac{u^{n+1}-u^n}{\Delta t} = u_t^n + \dfrac{\Delta t}{2}u_{tt}^n + \dfrac{\Delta t^2}{6}u_{ttt}^n + o(\Delta t^3);$

Galerkin Formulation:-

TheGalerkin formulation of the given 1D problem for this scheme becomes:

$(w,\dfrac{\Delta u}{\Delta t}) + \dfrac{\Delta t^2}{6}(aw_x\,,\boldsymbol{a}.\boldsymbol{\nabla}\dfrac{\Delta u}{\Delta t}) = (aw_x\,,u^n - \dfrac{\Delta t}{2}\boldsymbol{a}.\boldsymbol{\nabla}u^n)$

The advantage here is that, with similar computational cost as TG2, we obtain third-order accuracy. The necessary condition for the numerical stability in 1D is C < 1. The obtained solution has been represented in Figure 1.

Tg3-2s

Scheme:

$\tilde{u}^n = u^n + \dfrac{\Delta t}{3}u_t^n + \alpha\,\Delta t^2 u_{tt}^n$

$u^{n+1} = u^n + \Delta t\,u_t^n + \dfrac{\Delta t^2}{2}\tilde{u}_{tt}^n$

Galerkin Formulation for the given problem:-

$(w,\dfrac{\tilde{u}^n - u^n}{\Delta t}) = \dfrac{1}{3}(\boldsymbol{a}.\boldsymbol{\nabla}w,u^n) - \alpha * \Delta t * (\boldsymbol{a}.\boldsymbol{\nabla}w,\boldsymbol{a}.\boldsymbol{\nabla}u^n)$

$(w,\dfrac{u^{n+1} - u^n}{\Delta t}) = (\boldsymbol{a}.\boldsymbol{\nabla}w,u^n) - \dfrac{\Delta t}{2}(\boldsymbol{a}.\boldsymbol{\nabla}w,\boldsymbol{a}.\boldsymbol{\nabla}\tilde{u}^n)$

The two-step versions of the explicit Taylor—Galerkin method TG3 which include second time derivatives only and are thus easier to implement than the one-step method TG3. For the choice α = 1/9 the two-step procedure reproduces exactly the phase-speed characteristics of the single-step TG3 scheme. It can be observed from the Figure 1 and Figures 3.
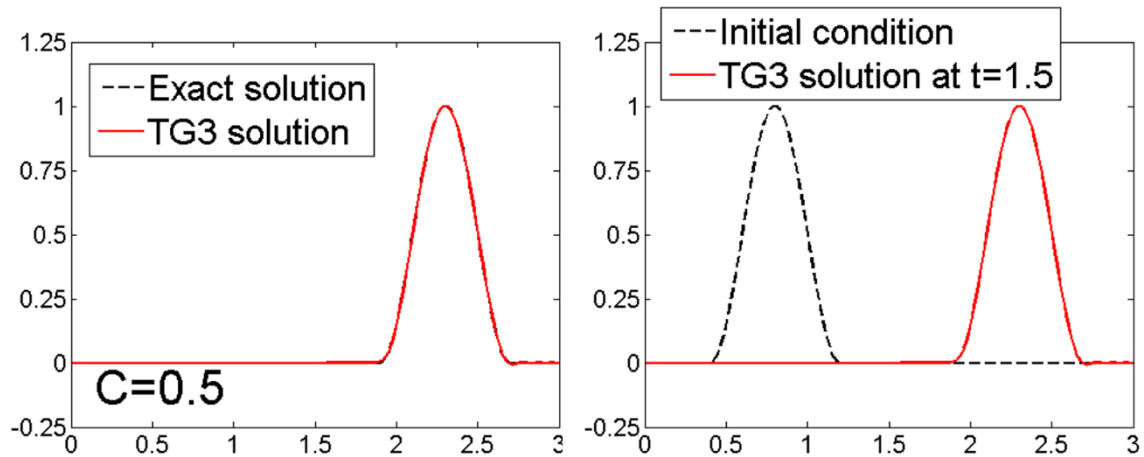
TG3:-

Figure1: Solutions obtained using TG3

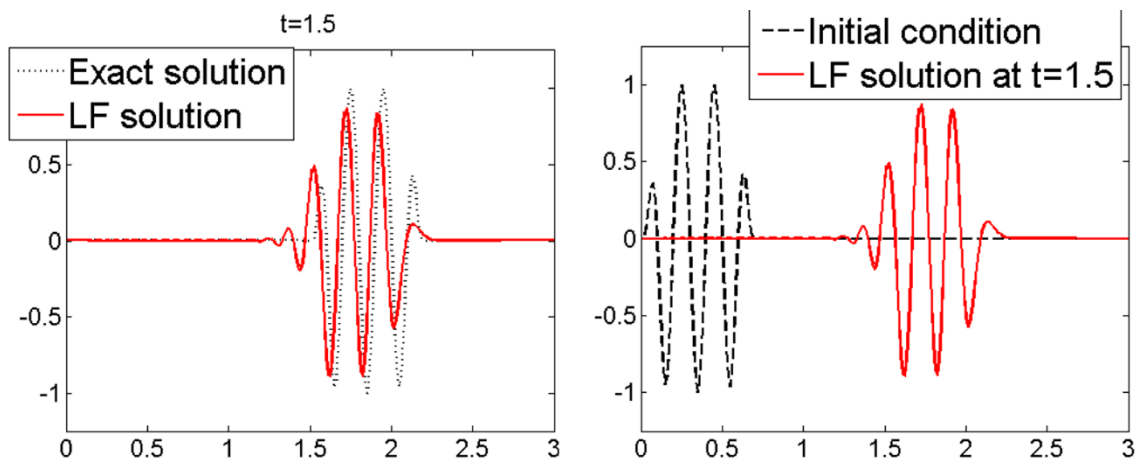**Problem 2 initial condition:- (Leap Frog)**

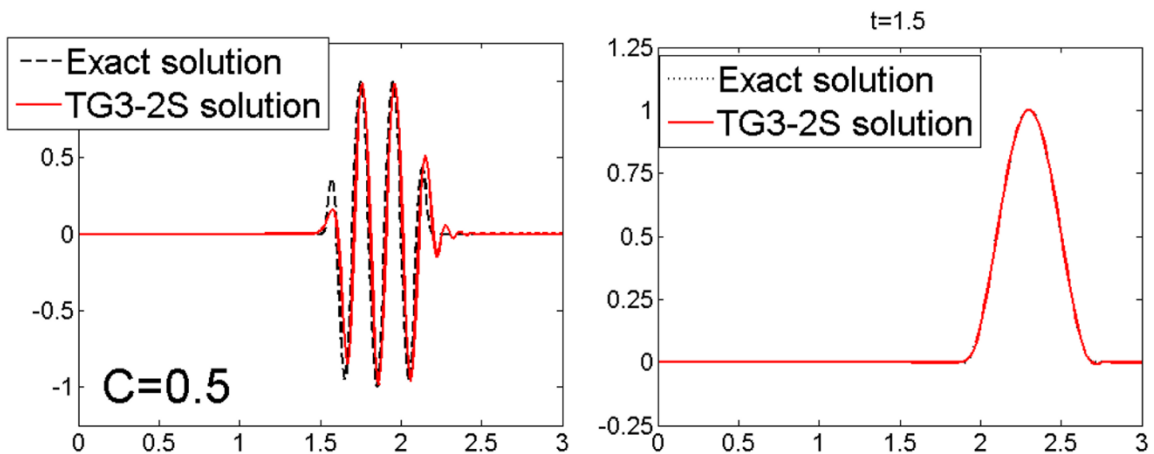**Figure 2:- Solutions obtained from Leap Frog**

TG3-2S

Figure 3:- Solutions from TG3-2S (A: Problem=2; B: problem=1)

CODE:

TG3:-

```
case 5 % Third order Taylor-Galerkin + Galerkin
A=M+dt^3*a^2/6*K;
B=-a*dt*C-dt^2/2*a^2*K;
methodName = 'TG3';
```

LEAP FROG:-

Code:

```
case 3 % LEAP-FROG
        A=M;
B=-dt*2*a*C;
methodName = 'LF';
```

Solution:-

```
for n = 1:nStep
if method ==3
tum=1;
    [A1,B1,methodName] = System(tum,M,K,C,a,dt);

% size(B*u(ind_unk,n))
if n==1
                Du = A1\(B1*u(:,n) + f);
u(1:nPt,n+1) = u(1:nPt,n) + Du;

else
    [A,B,methodName] = System(3,M,K,C,a,dt);
    Du = A\(B*u(1:nPt,n) + f);
u(1:nPt,n+1) = u(1:nPt,n-1) + Du;
end

else
```

```
    Du = A\(B*u(ind_unk,n) + f);
u(ind_unk,n+1) = u(ind_unk,n) + Du;
end
end
```

TG3:2S;

A1=M;

B1=-dt/3*C-dt/2*a^2*K

A2=M;

B2=-dt*C

C2=-dt/2*a^2*K;