Question: The 1D convection diffusion equation with constant coefficients and Diriclet boundary conditions:

$$au_x - \gamma u_{xx} = s \quad x \in [0,1]$$

$$u(0) = u_0;\ u(1) = u_1$$

aim :-

* To solve the given problem using Galerkin, SU, SUPG, GLS and SGS methods with 10 linear elements a

>> To solve the problem, changes are made in the codes and added to the original program given in the class assignment.

The changes are made in SUPG, GLS and SGS which is shown as below:

```
for ig = 1:ngaus
    N_ig = N(ig,:);
    Nx_ig = Nxi(ig,:)*2/h;
    N2x_ig = N2xi(ig,:)*2/h;
    w_ig = wgp(ig)*h/2;
    x = N_ig*Xe; % x-coordinate of the gauss point
    s = SourceTerm(x,example);
    Ke = Ke + w_ig*(N_ig'*a*Nx_ig + Nx_ig'*nu*Nx_ig) + ...
    + w_ig*((a*Nx_ig')*tau*(a*Nx_ig + nu*N2x_ig));
    fe = fe + (w_ig*(N_ig)'*s) +(w_ig*(a*Nx_ig)'*tau*s);
end
```

SUPG Codes

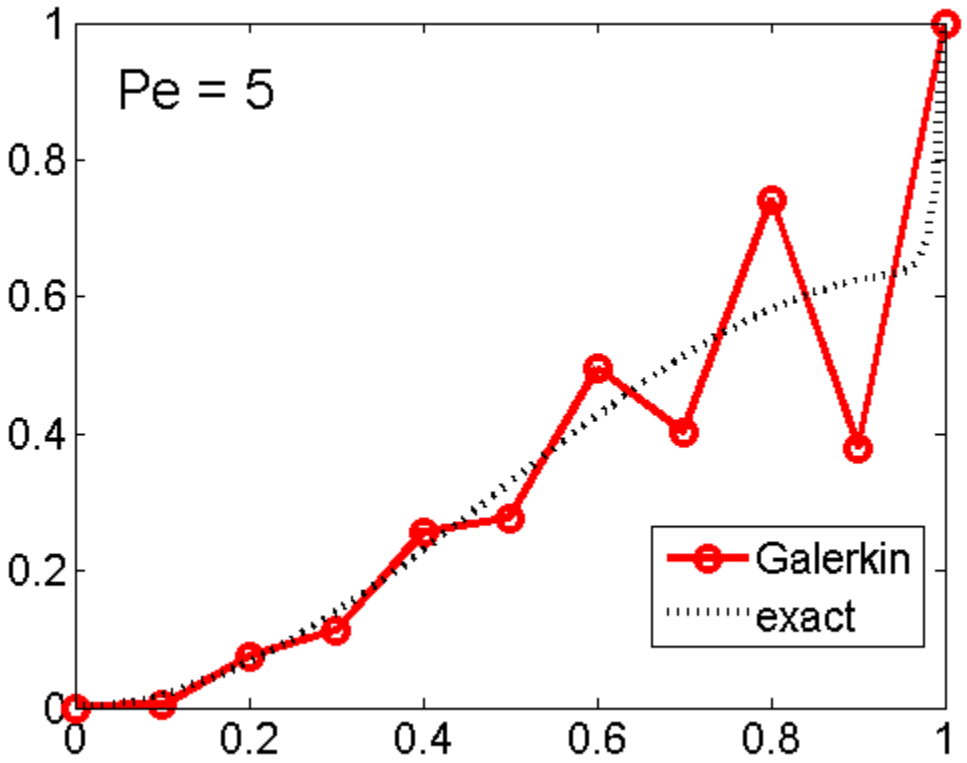
```
for ig = 1:ngaus
    N_ig = N(ig,:);
    Nx_ig = Nxi(ig,:)*2/h;
    w_ig = wgp(ig)*h/2;
    Ke = Ke + w_ig*(N_ig'*(a*Nx_ig) + Nx_ig'*(nu*Nx_ig));
    x = N_ig*Xe; % x-coordinate of the gauss point
    s = SourceTerm(x,example);
    fe = fe + w_ig*(N_ig')*s;
end
```

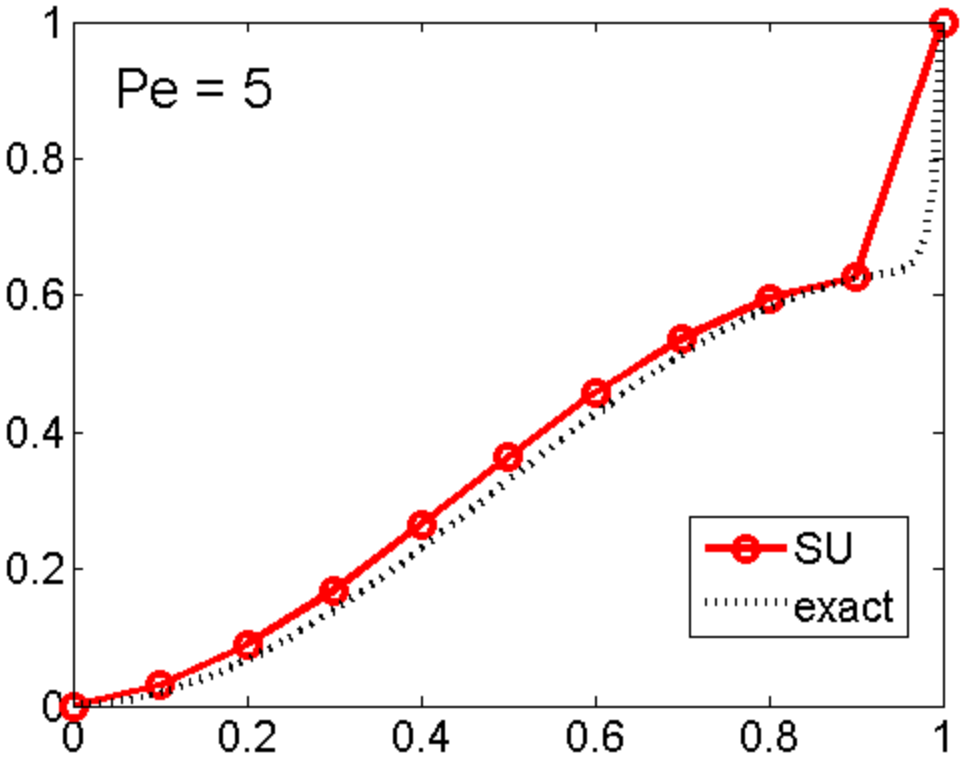
GLS Codes

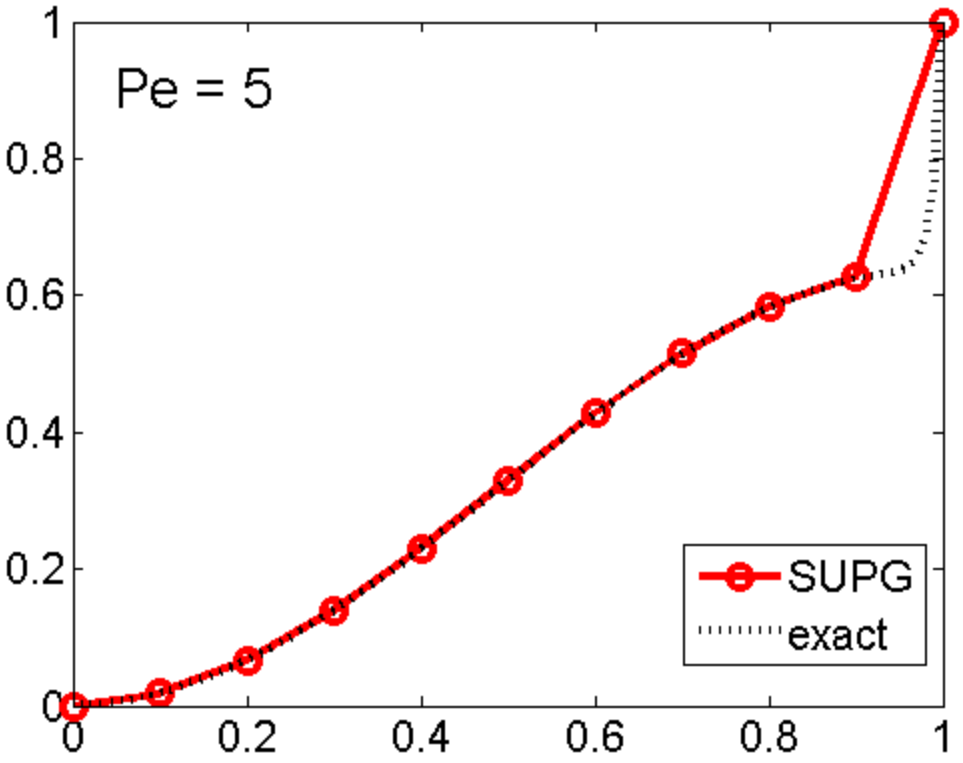
```
for ig = 1:ngaus
    N_ig = N(ig,:);
    Nx_ig = Nxi(ig,:)*2/h;
    Nx_2ig=N2xi(ig,:)*2/h;
    w_ig = wgp(ig)*h/2;
    x = N_ig*Xe; % x-coordinate of the gauss point
    s = SourceTerm(x,example);
    Ke = Ke + w_ig*(N_ig'*a*Nx_ig + Nx_ig'*nu*Nx_ig) ...
        + w_ig*(a*Nx_ig'- nu*Nx_2ig'*tau*a*Nx_ig+nu*Nx_2ig);
    fe = fe + w_ig*(N_ig)'*s + w_ig*(a*Nx_ig)'*tau*s;
end
```

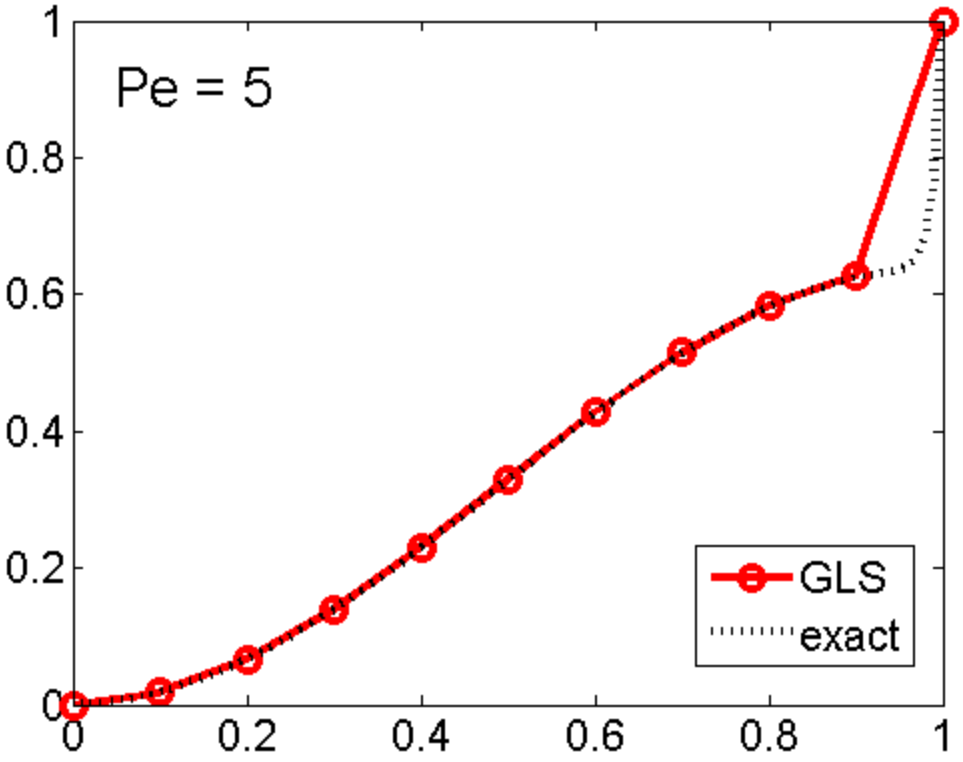
SGS Codes

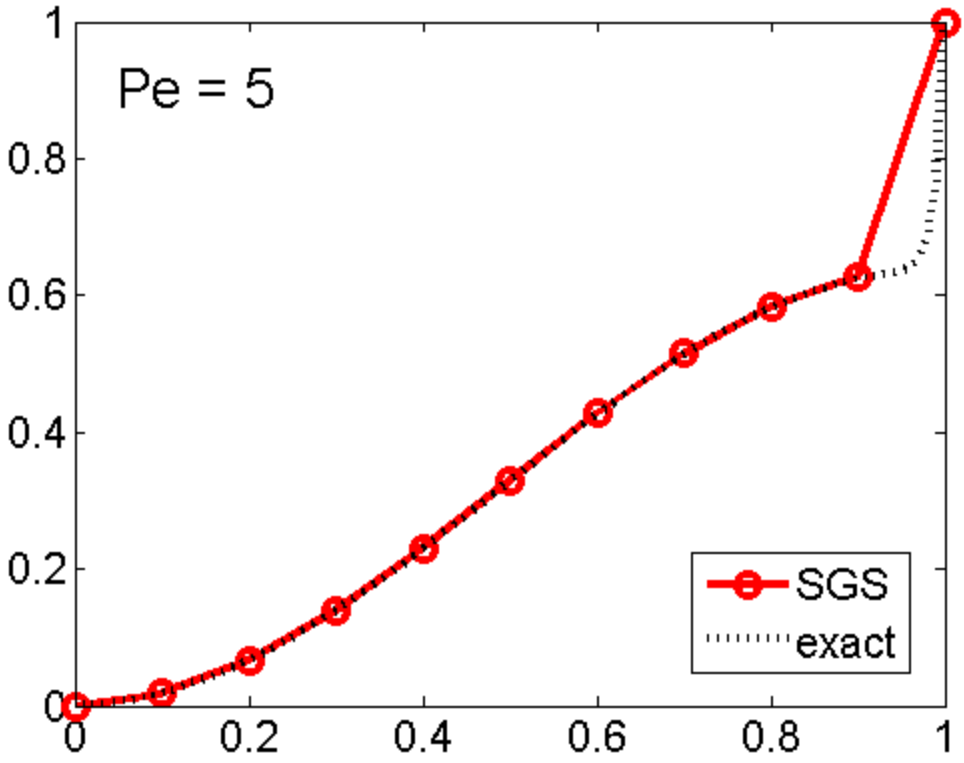
The Graphs




Pe = 5
SU
exact
0
0.2
0.4
0.6
0.8
1


Pe = 5
SUPG
exact
0
0.2
0.4
0.6
0.8
1


Pe = 5
GLS
exact
0
0.2
0.4
0.6
0.8
1



Conclusion

As expected Galerkin method is unstable for Pe=5, that is Pe >=1.