2D Steady Unsteady Transport

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Goal: a) 2D Steady Transport
b) 2D Unsteady Transport

SECTION A:- 2D Steady Transport

The GLS method has been implemented in the following way.

```
elseif method == 3
    % GLS
    Ke = Ke + (nu*(Nx'*Nx+Ny'*Ny) + N_ig'*(ax*Nx+ay*Ny) + N_ig'*sgma*N_ig ...
        + tau*(nu*(Nx+Ny) +(ax*Nx+ay*Ny))'*(ax*Nx+ay*Ny))*dvolu;
    aux = N_ig*Xe;
    f_ig = SourceTerm(aux);
    fe = fe + (N_ig+tau*(nu*(Nx+Ny) +(ax*Nx+ay*Ny)))'*(f_ig*dvolu + sgma);
end
```

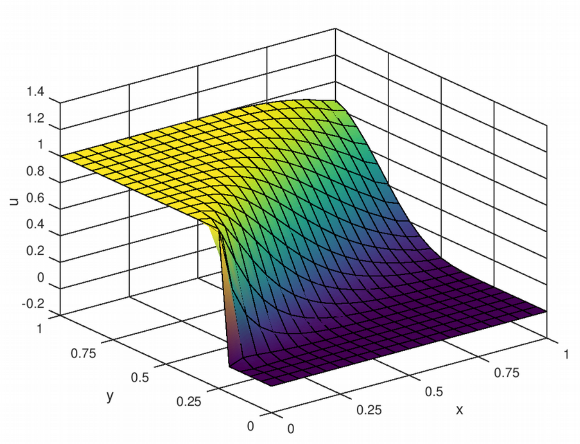
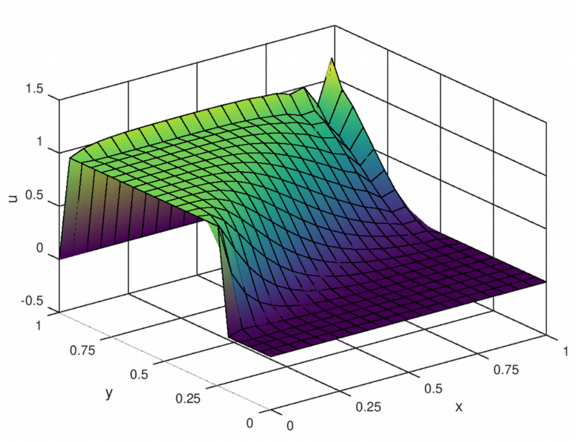
Implementation of GLS Method

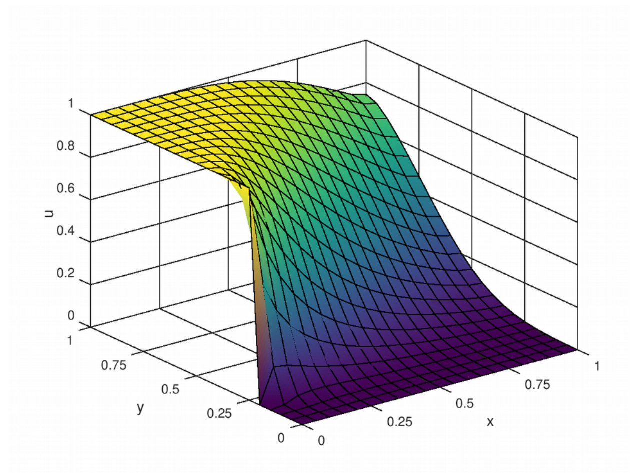
The zero Dirichlet boundary condition is applied at the outlet boundary has been implemented in the following way.

```
if BC==1
% nodes on which solution is u=1
nodesDir1 = nodes_x0( X(nodes_x0,2) > 0.2 );
% nodes on which solution is u=0
nodesDir0 = [nodes_x0( X(nodes_x0,2) <= 0.2 ); nodes_y0];
% Boundary condition matrix
C = [nodesDir1, ones(length(nodesDir1),1);
     nodesDir0, zeros(length(nodesDir0),1)];
elseif BC==2
% nodes on which solution is u=1
nodesDir1 = nodes_x0( X(nodes_x0,2) > 0.2 );
% nodes on which solution is u=0
nodesDir0 = [nodes_x0( X(nodes_x0,2) <= 0.2 ); nodes_y0; nodes_x1; nodes_y1];
% Boundary condition matrix
C = [nodesDir1, ones(length(nodesDir1),1);
     nodesDir0, zeros(length(nodesDir0),1)];
```

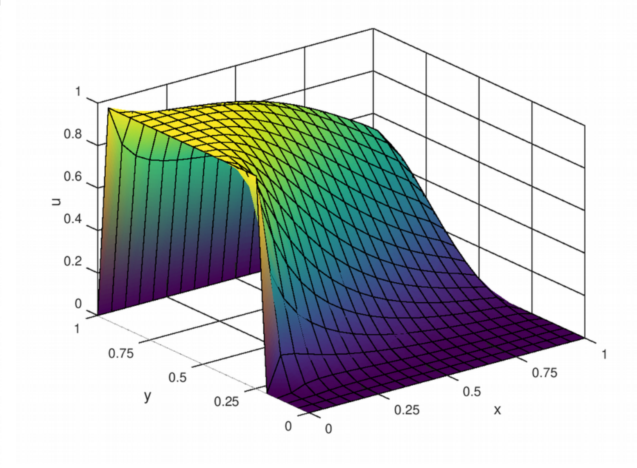
Implementation of Diriclet Boundary Condition

Comparison of Implementation of Neumann and Dirichlet Conditions

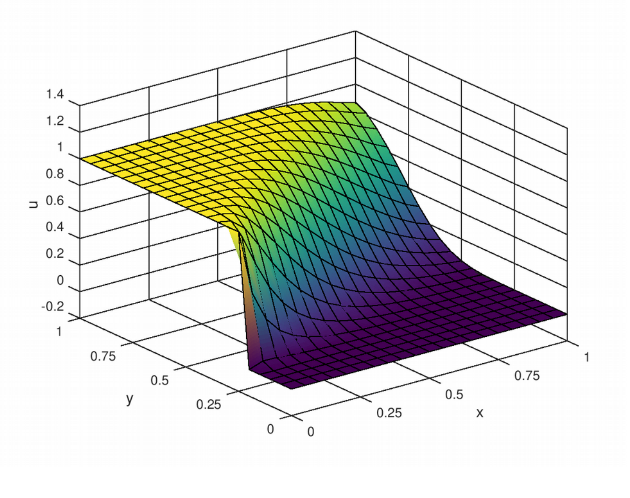
	
Galerkin Neumann	Galerkin Dirichlet



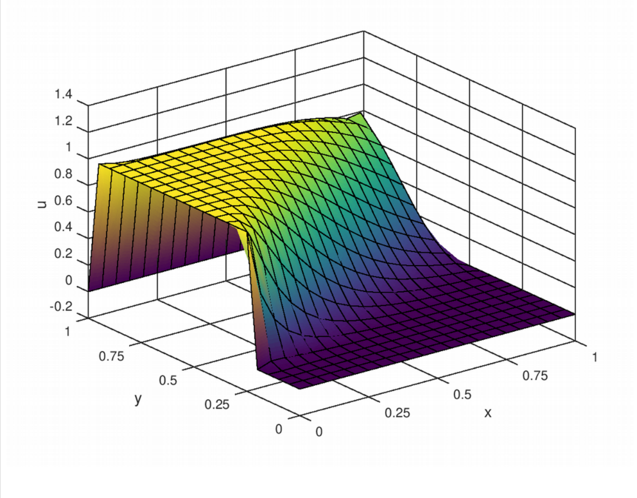
Artificial Neumann



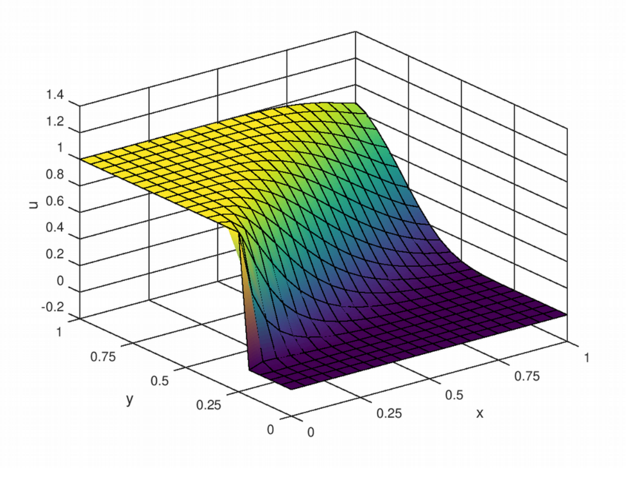
Artificial Dirichlet



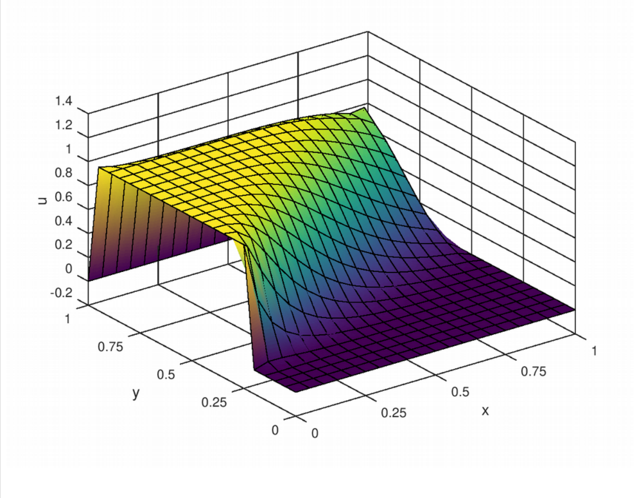
SUPG Neumann



SUPG Dirichlet

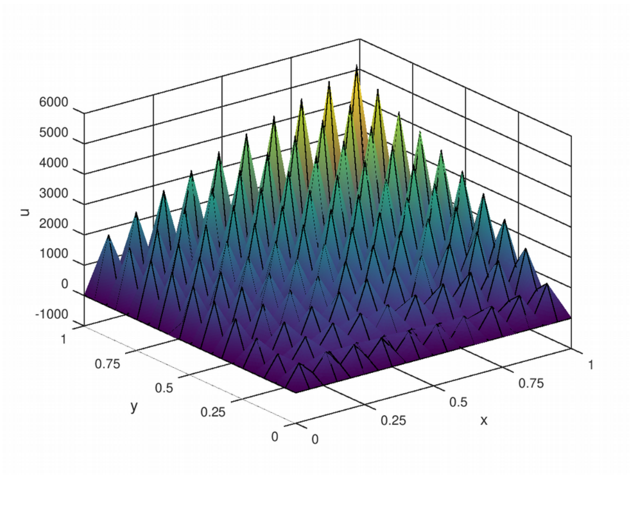


GLS Neumann

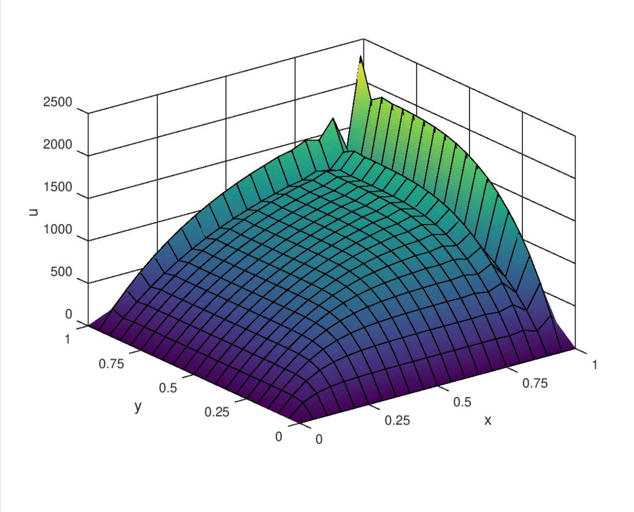


GLS Neumann

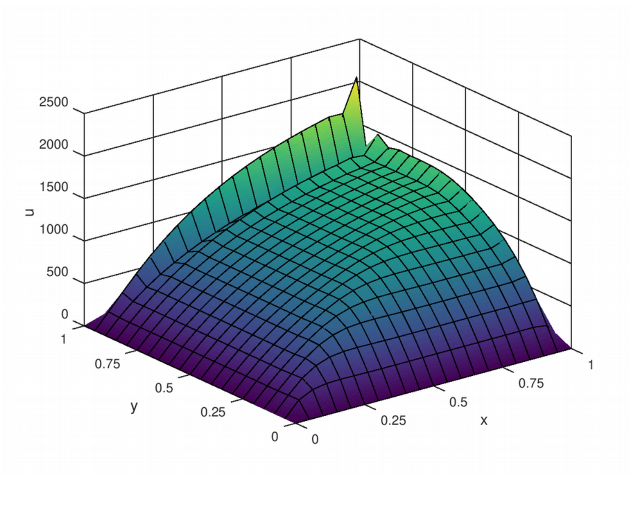
Implementation of Case 1 with $\|a\|=\frac{1}{2};\ \nu=10^{-4};\ \sigma=1$



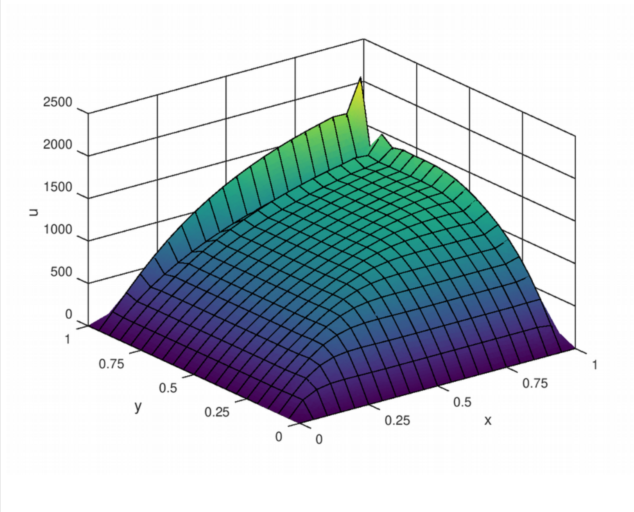
Galerkin Case 1



Artificial Diffusion Case 1

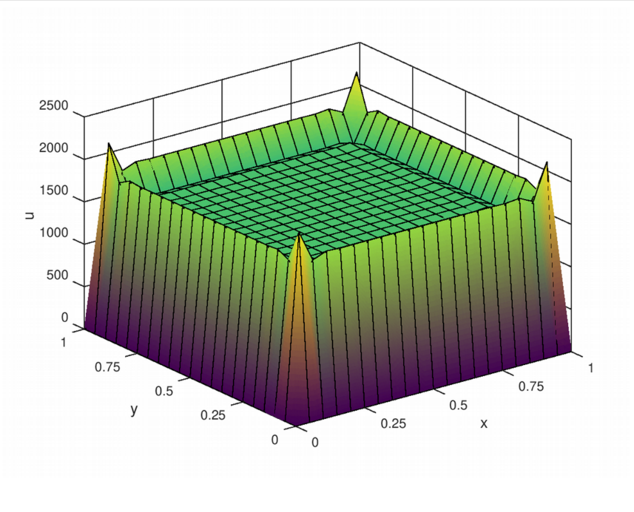


SUPG Case 1

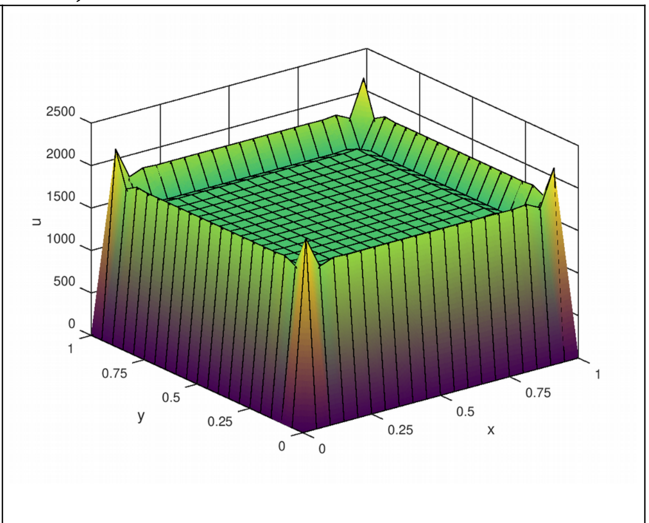


GLS Case 1

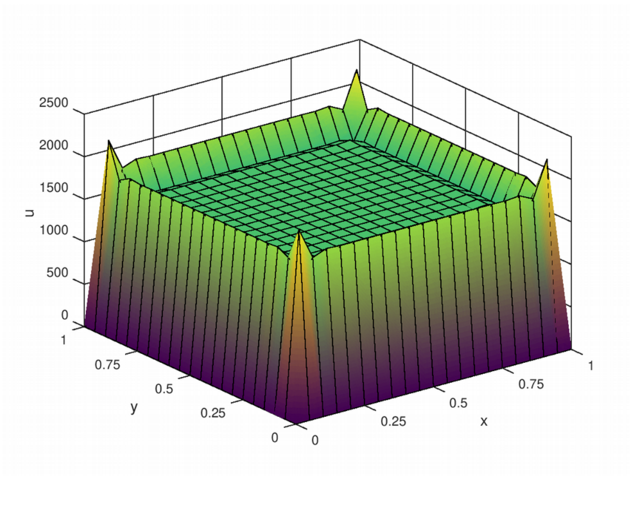
Implementation of Case 2 with $\|a\|=10^{-3};\ \nu=10^{-4};\ \sigma=1$



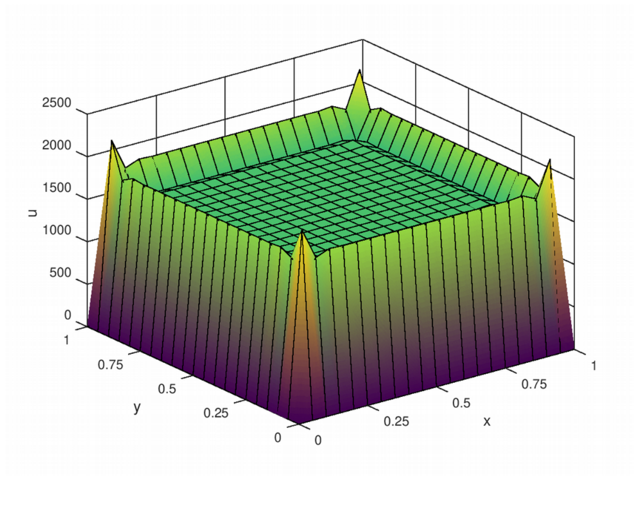
Galerkin Case 2



Artificial Diffusion Case 2



SUPG Case 2



GLS Case 2

Conclusion (A): In case 1, the problem is reaction dominated, hence we see instability in Pure Galerkin Formulation and stabilization is achieved by using methods like Artificial Diffusion, SUPG and GLS method.

In case 2, the problem is diffusion dominated, hence we see stability in Pure Galerkin Formulation and stabilization methods like Artificial Diffusion, SUPG and GLS method generate similar results.

SECTION B:- 2D Unsteady Transport

Goal: To solve the problem 2D homogeneous convection equation with initial condition and homogeneous Dirichlet conditions on the inlet boundary

$$U(x,0)=\frac{1}{4}\times(1+\cos\pi X_1)(1+\cos\pi X_2) \text{ If } X_1^2+X_2^2\leq 1$$

otherwise,

$$U(x,0)=0$$

The numerical solution has been computed using following finite element schemes:

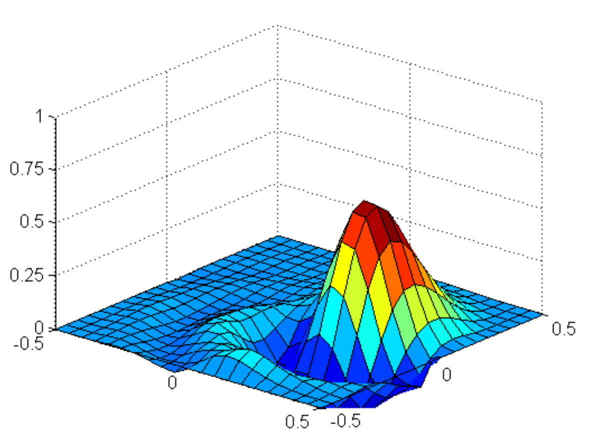
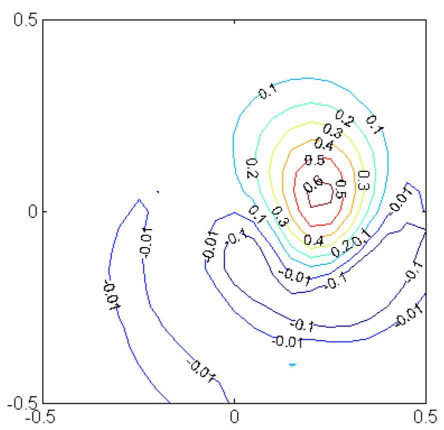
i. Lax-Wendroff + Galerkin (and with lumped mass matrix);
ii. Crank-Nicolson +Galerkin (and lumped mass matrix);
iii. The third-order explicit Taylor-Galerkin scheme (TG3);
iv. Two-step third order Taylor-Galerkin-2S method (TG3-2S);

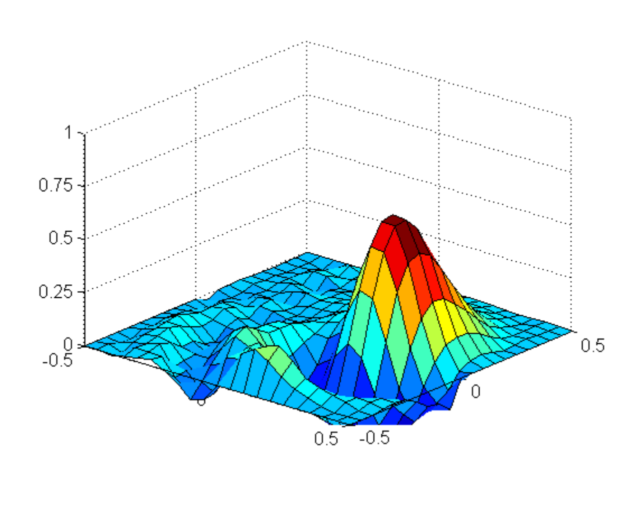
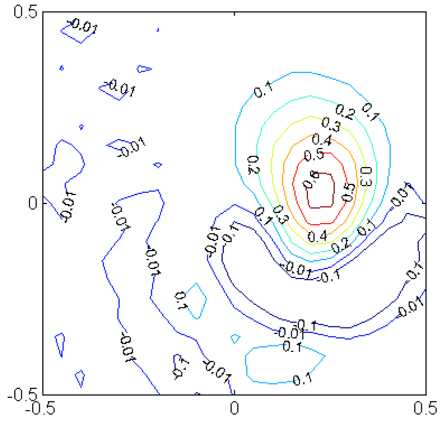
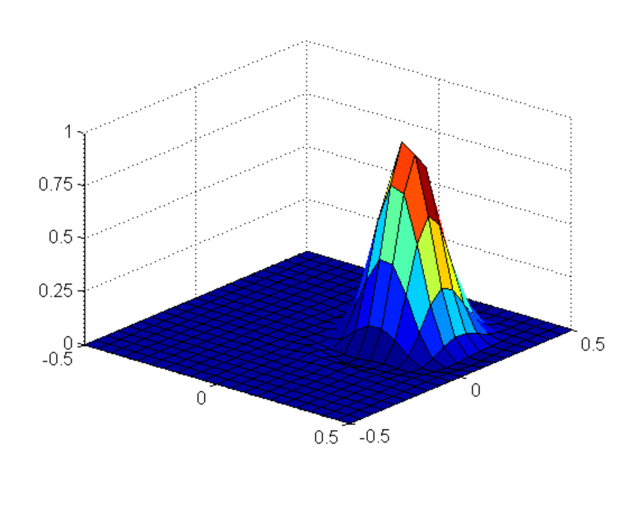
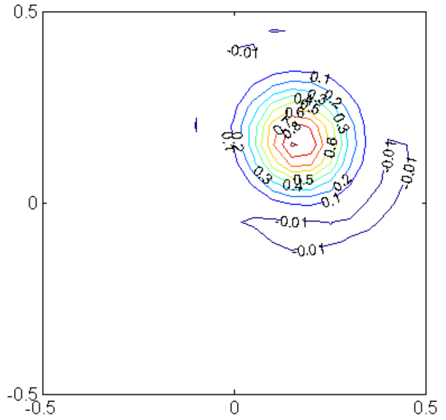
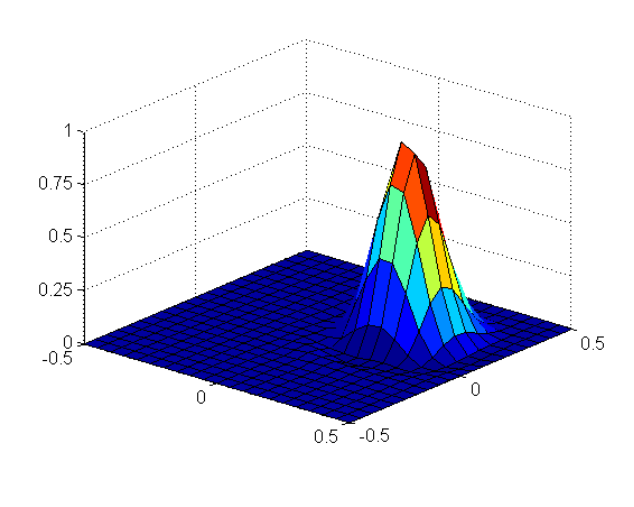
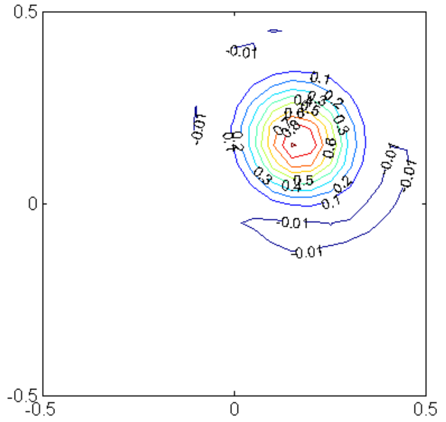
Two-step fourth order Taylor-Galerkin method (TG4-2S).

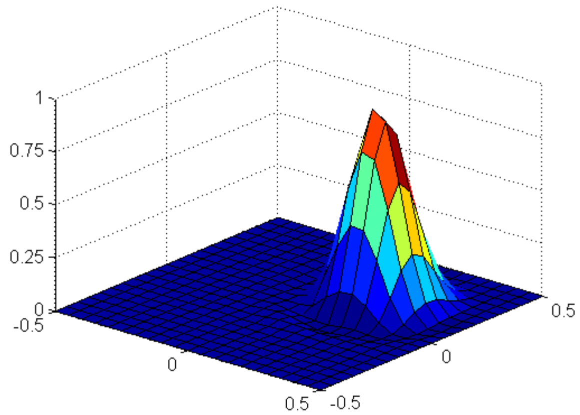
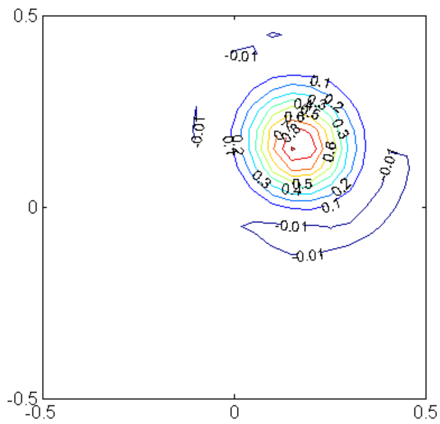
```
elseif meth == 8
 %TG4-2S
    alpha = 1/12;
    A1 = M;
    B1 = -(dt/3)*C'- alpha*dt^2*(K - Co);
    f1 = (dt/3)*v1 + alpha*dt^2*(v2 - vo);
    A2 = M;
    B2 = -dt*C';
    C2 = - (dt^2/2)*(K-Co);
    f2 = dt*v1 - (dt^2/2)*(v2 - vo);
else
    error('Unavailable method')
end
```

Implementation of TG4 -2S Method

Graphs: Convection of Cosine hill in pure rotation velocity field

	
Lax Wendroff	Lax Wendroff

	
Crank Nicolson	Crank Nicolson
	
TG3	TG3
	
TG3-2S	TG3-2S

	
TG4-2S	TG4-2S

Terms of Equations:
Lax Wendroff

In the code

$$(w,\Delta u/\Delta t)=(a.\nabla w,u^n-\Delta t/2(a.n)u^n)+((a.n)w,u^n-\Delta t/2(a.\nabla)u^n)_{\Gamma^{out}}$$

where,

M represents $(w,\nabla u)$

B represents $(a.\nabla w,u^n)$

K represents $(a.\nabla w)(a.\nabla u^n)$

Mo represents $((a.n)w,u^n)_{\Gamma^{out}}$

Co represents $((a.n)w,(a.\nabla)u^n)_{\Gamma^{out}}$

v1 represents (w,s^n)

v2 represents $(a.\nabla w,s^n)$

v0 represents $((a.n)w,s^n)_{\Gamma^{out}}$ but, s=0;

TG3

$$(W,\frac{\Delta u}{\Delta t})+\Delta t^2/6(a.\nabla w,(a.\nabla\frac{\Delta u}{\Delta t})-\frac{\Delta t^2}{6}(a.n)w,(a.\nabla\frac{\Delta u}{\Delta t}))_{\Gamma^{out}}=(a.\nabla w,u^n-\frac{\Delta t}{2}(a.\nabla)u^n)$$
$$-((a.n)w,u^n-\frac{\Delta t}{2}(a.\nabla)U^n)_{\Gamma^{out}}$$

M represents $(w,\nabla u)$

B represents $(a.\nabla w,u^n)$

K represents $(a.\nabla w)(a.\nabla u^n)$

Mo represents $((a.n)w,u^n)_{\Gamma^{out}}$

Co represents $((a.n)w,(a.\nabla)u^n)_{\Gamma^{out}}$

Crank Nicolson

$$(w,\frac{\Delta u}{\Delta t})-\frac{1}{2}(\nabla w,a\Delta u)+\frac{1}{2}((a.n)w,\Delta u)_{\Gamma^{out}}=(\Delta w,au^n)-((a.n)w,u^n)_{\Gamma^{out}}$$

M represents $(w,\Delta u)$

B represents $(a.\nabla w,u^n)$

C represents $(a.\nabla w,u^n)$

Mo represents $((a.n)w, u^n)_{\Gamma^{out}}$

Co represents $((a.n)w, (a.\nabla)u^n)_{\Gamma^{out}}$

TG3-2S (Two – step Method)

$$(w, \frac{\bar{u}^n - u^n}{\Delta t}) = \frac{1}{3}(a.\nabla w, u^n) - \alpha(\Delta t)(a.\nabla w, a.\nabla u^n) + \alpha(\Delta t)((a.n)w, (a.\nabla)u^n)_{\Gamma^{out}}$$

K represents $(a.\nabla w)(a.\nabla u^n)$

Co represents $((a.n)w, (a.\nabla)u^n)_{\Gamma^{out}}$

A2 represents $(w, u^{n+1} - u^n)$

B2 represents $(a.\nabla w)(a.\nabla \bar{u}^n)$

C2 represents $(a.\nabla w)(a.\nabla \bar{u}^n)_{\Gamma^{out}}$