Lab Report - 6

Incompressible flow Matlab Exercises

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1.1

The L2 error norm of the pressure, the H1 error semi norm of the velocity and the L2 error norm of velocity were computed for N×N elements where N = 2, 4, 8, 16, 24, 32.

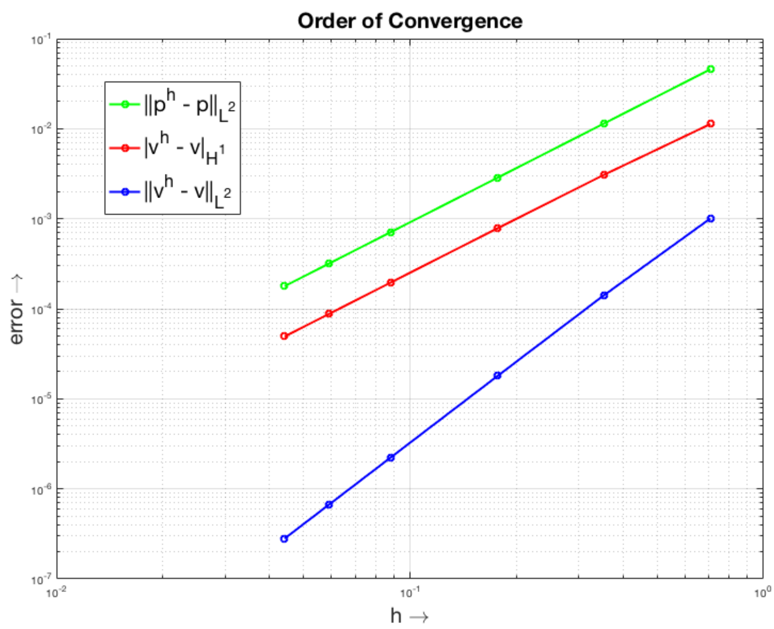

Fig 1. Order of Convergence - Q_2Q_1 element

Expected order of 3 was observed for L^2 error of the velocity because of the use of Q_2 elements to interpolate velocity. Expected order of 2 was observed for H^1 semi-norm error of the velocity and expected order of 2 was observed for L^2 error of the pressure beacause of the use of Q_1 elements to interpolate pressure.

1.999635493524672	1.873302176674072	2.832325805975174
2.000330546619946	1.973753558430197	2.978879222764457
2.000030956597092	1.993369649775693	2.996589698011148
2.000003845615942	1.997883037101279	2.999112251694083
2.000001108133545	1.998952202313913	2.999587524766460
L^2 norm Pressure	H^1 semi norm Velocity	L^2 norm Velocity

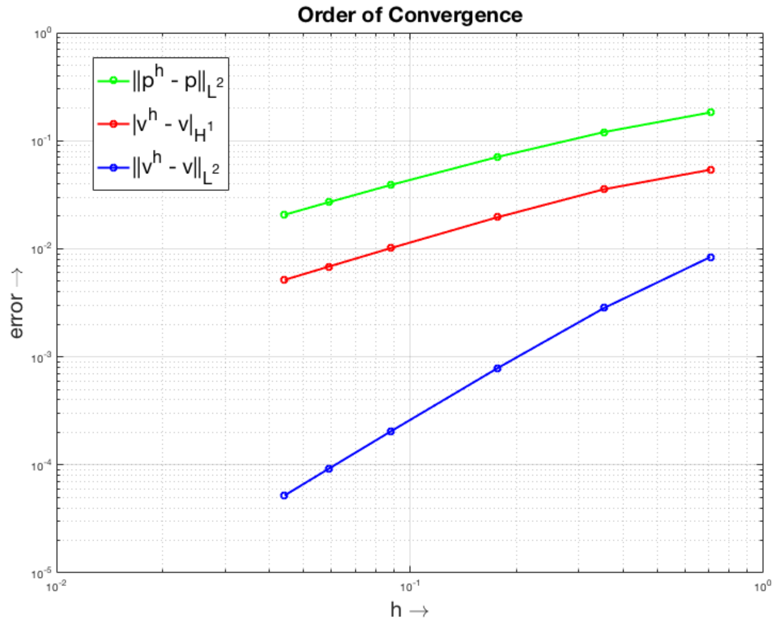

Fig 2. Order of Convergence - Q_2Q_0 element

Sub optimal order of 2 was observed for L^2 error of the velocity because of the use of Q_2 elements to interpolate velocity. **Sub optimal order** of 1 was observed for H^1 semi-norm error of the velocity and expected order of 1 was observed for L^2 error of the pressure because of the use of Q_0 elements to interpolate pressure.

0.603207639658958	0.599564594480739	1.570261149805783
0.764928669723566	0.861079350149105	1.847611293290563
0.859539857030234	0.946183972006160	1.938299914637524
0.908531624097754	0.974407462367428	1.969355957741998
0.930408655436988	0.983754077155205	1.979955365670362
L^2 norm Pressure	H^1 semi norm Velocity	L^2 norm Velocity

1.2

GLS stabilization was implemented in order to use non LBB stable elements of first order namely Q_1Q_1 and P_1P_1 elements.

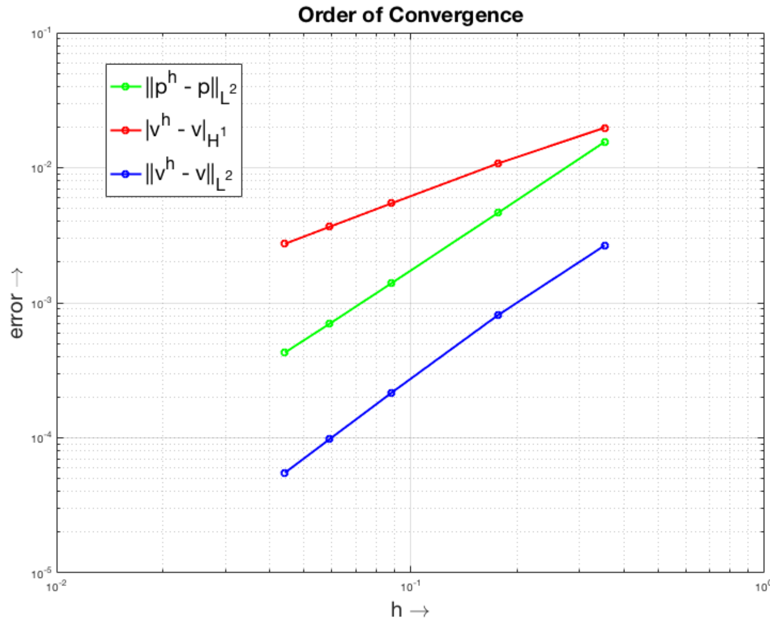

Fig 3. Order of Convergence - P_1P_1 element

Expected order of 2 was observed for L^2 error of the velocity because of the use of P_1 elements to interpolate velocity. Expected order of 1 was observed for H^1 semi-norm error of the velocity and expected order of 2 was observed for L^2 error of the pressure beacause of the use of P_1 elements to interpolate pressure.

1.743087048803921	0.877747449506963	1.712039935988664
1.731865301828069	0.982035276250644	1.910869797988448
1.716936612119631	0.994044887871000	1.962705036840820
1.686190434792798	0.997873006922521	1.977760564254682
L^2 norm Pressure	H^1 semi norm Velocity	L^2 norm Velocity

2

2.1

The stokes equations are solved on an uniform Cartesian mesh with 20 elements in each direction for a lid driven cavity problem where the flow is driven by the constant velocity of the top wall.

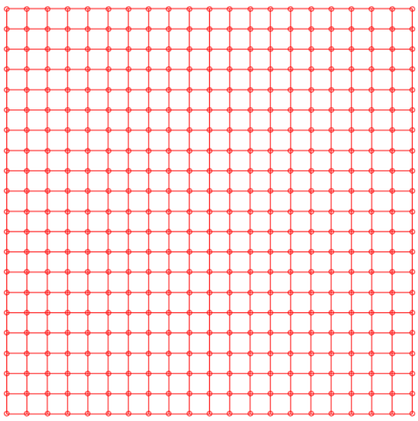

Fig 4a: Uniform Mesh

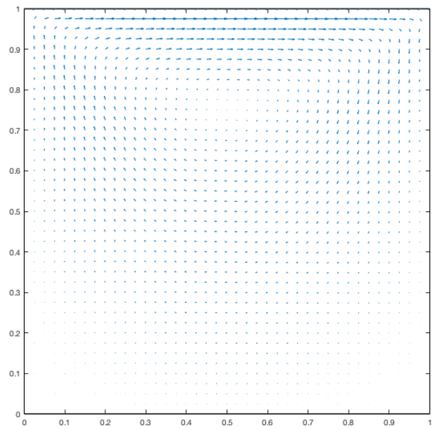

Fig 4b: Velocity Profile

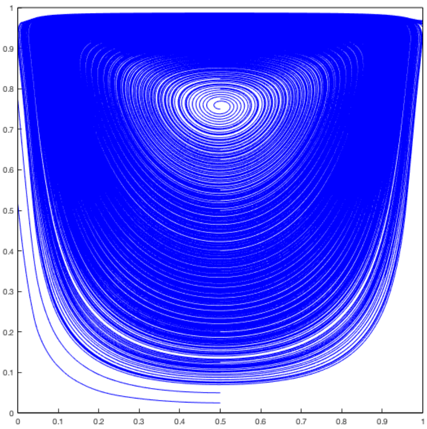

Fig 4c: Velocity streamlines

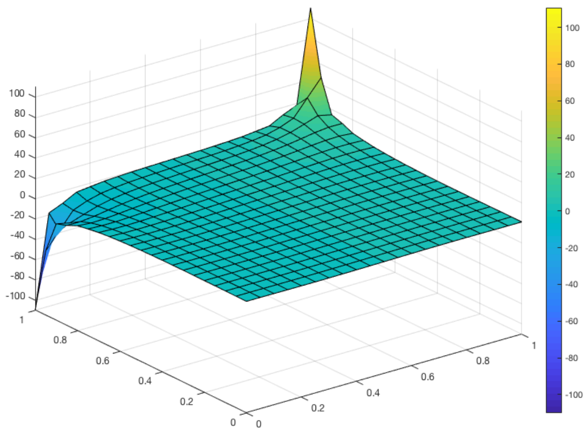

Fig 4d: Pressure Profile

The velocity profile obtained show that the velocity near the upper wall is almost horizontal because of the no slip condition and it causes a vortex structure in the domain. The velocity near the other boundaries are almost zero in accordance with the boundary conditions. The pressure at the top corners tend to go to infinity and because of the coarseness of the mess the behavior is not well captured. There is need to refine the mesh closer to the walls in order to capture the physics better. Hence we go for an adapted mesh with 20 elements on each direction and solve the lid driven cavity problem.

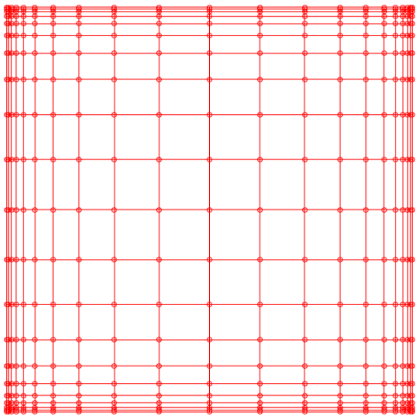

Fig 5a: Adapted Mesh

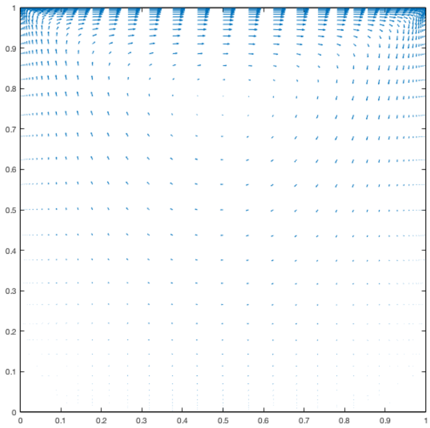

Fig 5b: Velocity Profile

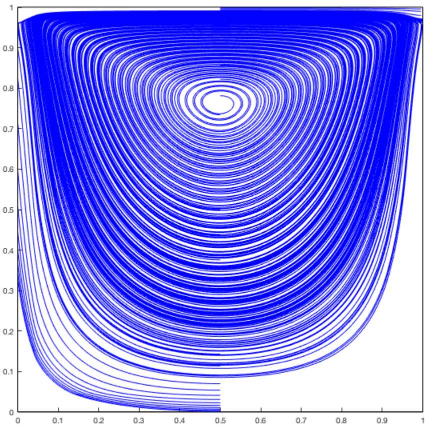

Fig 5c: Velocity streamlines

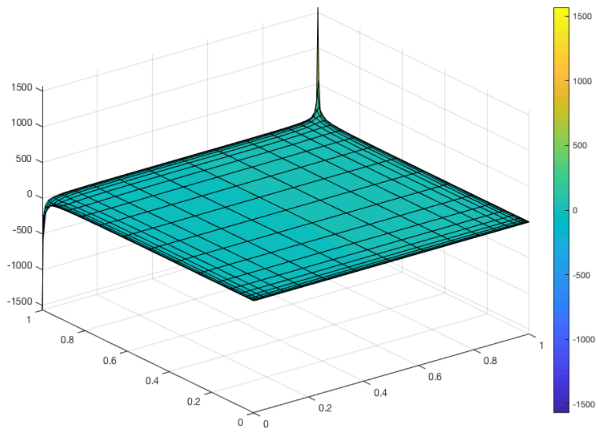

Fig 5d: Pressure Profile

As expected, it can be observed that the adapted mesh captures the physics of the problem at hand better.

2.2

The Navier stokes equation was solved using Picard's method for the lid driven cavity problem for Re = 100, 500, 1000, 2000 on an adapted mesh shown previously.

It was observed that for low Reynold's number the stokes solution can be recovered. As the Reynold's number is increased the position of the eye of the vortex moves closer to the center of the domain. It can also be observed that the size of the vortex became bigger with increase in Re. The peak pressure at the top corners of the domain decreased with increase in Re.

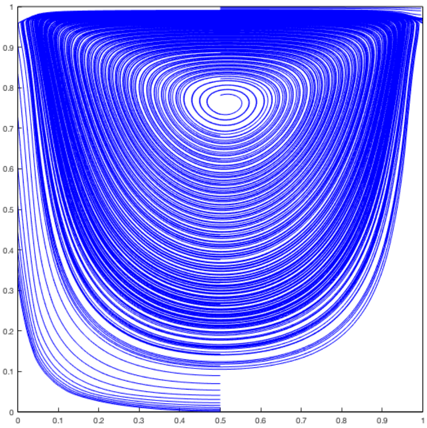

Fig 6a: Re = 10

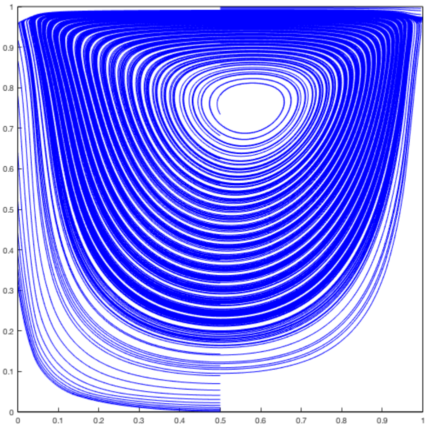

Fig 6b: Re = 50

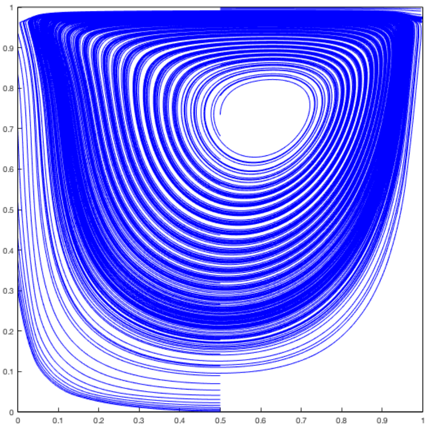

Fig 6c: Re = 100

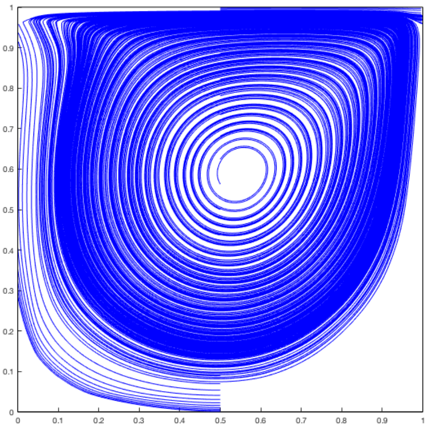

Fig 6d: Re = 500

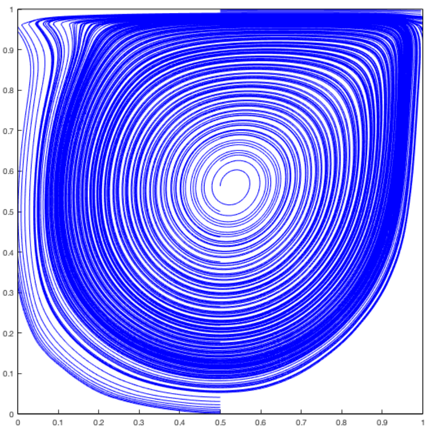

Fig 6e: Re = 1000

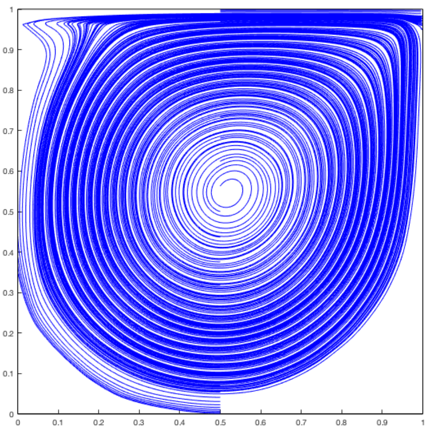

Fig 6f: Re = 2000

It was also observed that the residual decreased in a linear way because of the Picard method. This can prove costly when handling large scale time dependent problems.

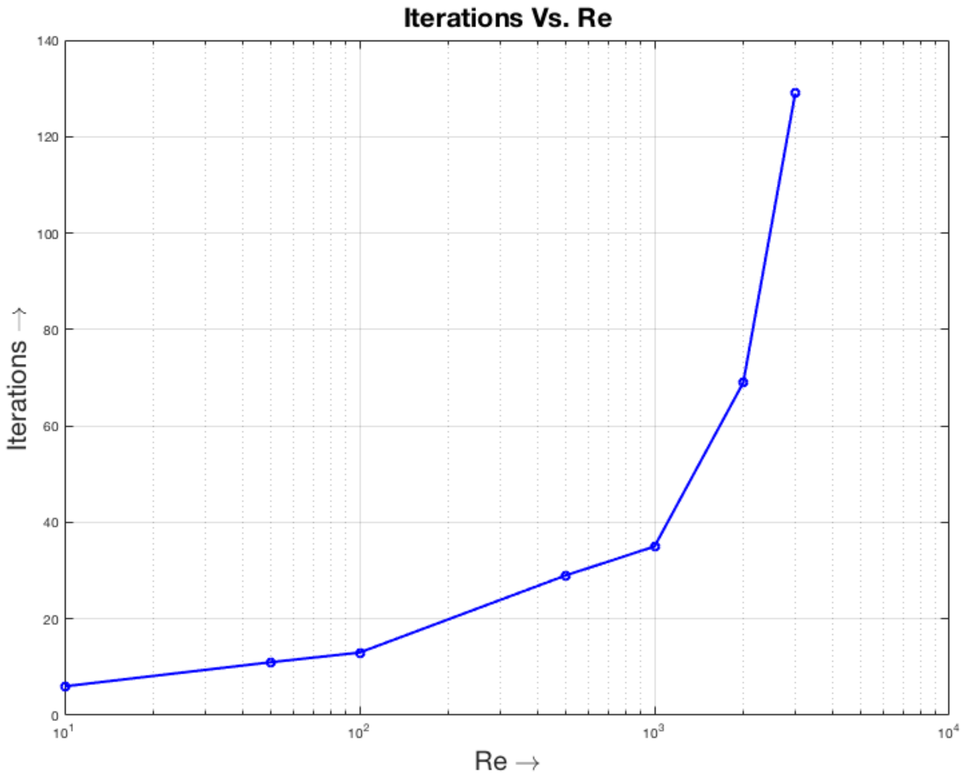

Fig 7: Computational cost Vs Reynold's number

It was also observed that with increase in Re the number of iterations needed to converge grew. This can be attributed towards micro scale instabilities as the Re tends towards the laminar limit into transition and finally the turbulent phase. The Newton Raphson algorithm has to be implemented in order to decrease the number of iterations as it has a rate of convergence of 2. In order to solve problem for higher Re, it is necessary to solve the RANS equations along with a suitable turbulence model for the problem