<u>**Class Homework 6: 1D Unsteady convective diffusion case**</u>

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$$\begin{cases} u_t + a u_x - \nu u_{xx} = 0 & x \in (0,1),\ t \in (0, 0.6] \\ u(x,0) = u_0(x) & x \in (0,1) \\ u(0,t) = u(1,t) = 0 & t \in (0, 0.6] \end{cases}$$

$$u_0(x) = \frac{5}{7}\exp\{-(x-x_0)^2/L^2\}$$

$$a = 1, x_0 = 2/15, L = 7\sqrt{2}/300, \Delta x = 1/150$$

$$\nu = [3.3 \cdot 10^{-3}, 6.7 \cdot 10^{-4}, 3.3 \cdot 10^{-5}]^T$$

The Figure above shows the problem to be solved. It will be tested using different methods and different values of viscosity.

We first want to highlight the fact that linear finite elements in the standard Galerkin formulation do not ideally combine with the second order Crank-Nicolson time-stepping method in highly convective situations.

The results reported in Figure 1 for a Courant number C = 1 show that the second order time scheme performs well at low and moderate values of the Peclet number (Figure 1 left), but exhibits significant phase errors when the Peclet number is further increased (Figure 1 right). Moreover, the situation becomes worse when the time-step size corresponds to a Courant number larger than one. As shown by the next tests of the Gaussian hill, the situation improves very much when passing to third- and fourth-order accurate time-stepping algorithms like $R_{2,2}$ or $R_{3,3}$.

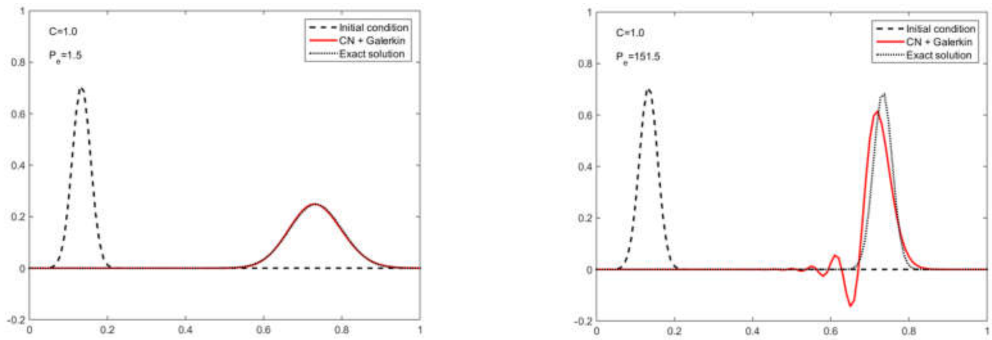

Figure 1: CN + G. Result with C = 1 and Pe = 1.5 (Left), C = 1 and Pe = 151.5 (right).

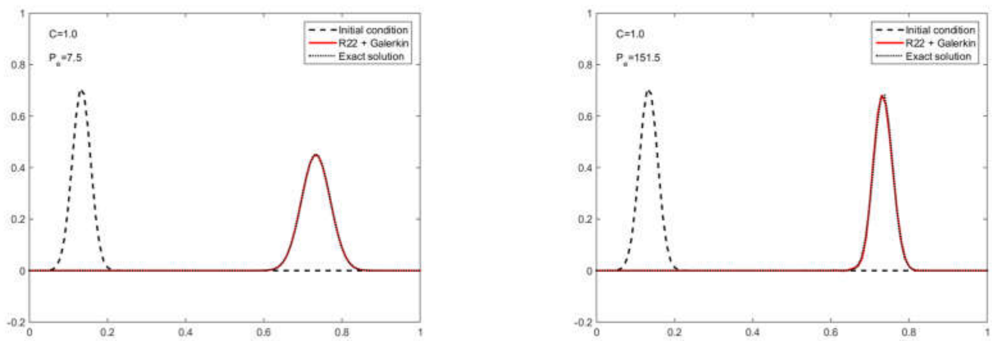

Figure 2: $R_{2,2}$ Result with C = 1 and Pe = 7.5 (Left), C = 1 and Pe = 151.5 (right).

Higher-order methods, such as $R_{2,2}$ or $R_{3,3}$, in time provide a gain in accuracy. Figure 2 shows, for a Courant number C = 1, that the fourth-order time scheme performs well for all values of the Peclet number. Obviously, results degrade when the time step is too large.