Master Of Science in Computational Mechanics

Finite Elements in Fluid

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Assignment 4

Burgers' equation (nonlinear hyperbolic equation)

The problem studied consist of Burgers' equation which is a nonlinear hyperbolic equation represented in convective or non convective form as follows:

$$\begin{cases} u_t + (\frac{u^2}{2})_x = 0 & x \in (a,b), t \in (0,\infty) \\ u(x,0) = u_0(x) & x \in (a,b) \end{cases} \qquad \begin{cases} u_t + uu_x = 0 & x \in (a,b), t \in (0,\infty) \\ u(x,0) = u_0(x) & x \in (a,b) \end{cases}$$

As per given data time interval of (0, 4) is considered for solving the equations, Figure 1 depicts initial conditions implemented.

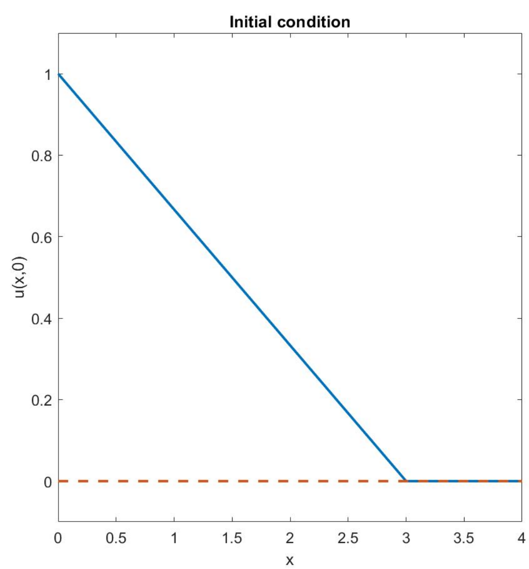


Figure 1: initial conditions.

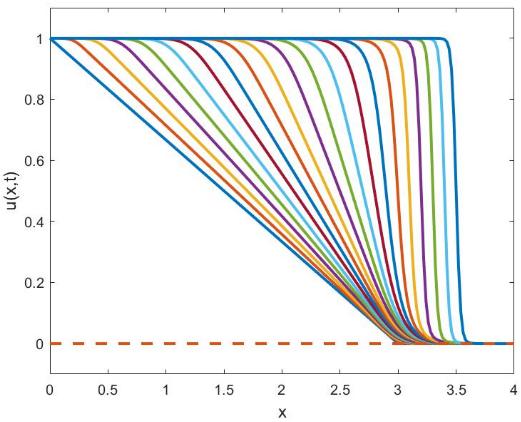
The provided files already consist of the complete code to solve Burgers' equation using an Forward Euler and Backward Euler scheme with Picard method.The main challenge is to implement Newton Rapson method for the same equation. for this new function file $burgersimNR.m$ was created.

The code implemented to calculate value for each iteration is as follows:

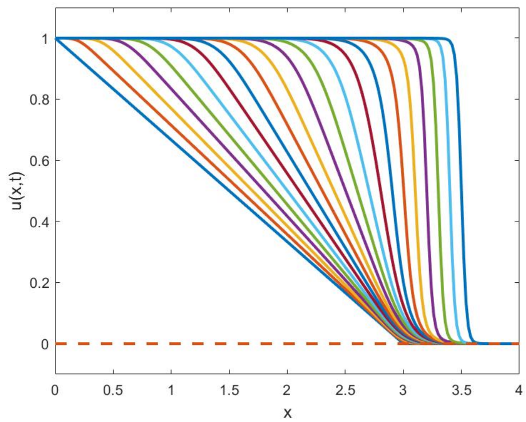
```
for n = 1:nTimeSteps
    U0 = U(:,n);
    error_U = 1; k = 0;
    while (error_U > 0.5e-5) && k < 20
        C = computeConvectionMatrix(X,T,U0);
        F = (M + At*C+At*E*K)*U0-M*U(:,n);
        J = M+2*At*C+At*E*K;
        U1 = U0-J\F;
        error_U = norm(U1-U0)/norm(U1);
        U0 = U1; k = k+1;
    end
    U(:,n+1) = U1;
end
```

The solution of Burgers' equation for different method are shown in figure 2. Time t_f is taken to be 4, space discretization factor $h = 0.02$ and time discretization factor $\Delta t = 0.05s$.

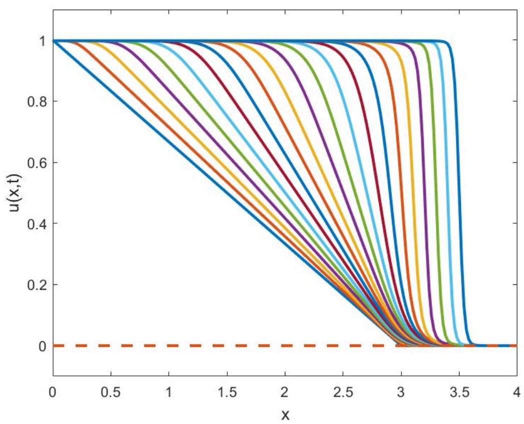
As seen from the results all three method behaves very similarly for the given domain with given conditions. Newton Rapson compute in quadratic manner so it is computationally expensive then Picard method but, the convergence is much faster.



(a) Explicit scheme.



(b) Picard's Method.



(c) Newton Rapson method.

Figure 2: Solution for Burgers' equation