The methods used in this report are all discretized in space using the classical Galerkin method. However, this is an unsteady problem thus time discretization is required. This is conducted in this report using the Crank-Nicholson, Lax-Wendroff and the 3rd order Taylor-Galerkin (TG3) methods. Two versions where applied for the Crank-Nicholson and the Lax-Wendroff methods: one with a normal mass matrix and another with a lumped mass matrix.

Crank-Nicholson

The first method to be analyzed is the Crank-Nicholson time discretization scheme. The following results are obtained for the normal mass matrix and the lumped mass matrix for different courant numbers.

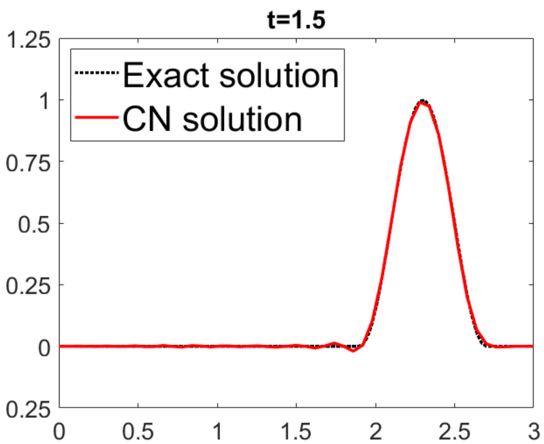

Figure 1 Crank Nicholson solution for C = 0.1389

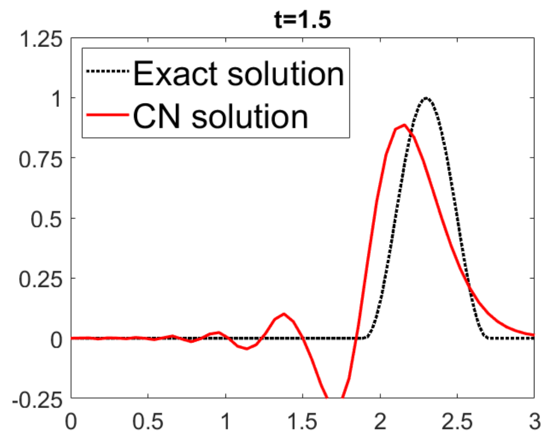

Figure 2 Crank Nicholson solution for C = 2.5

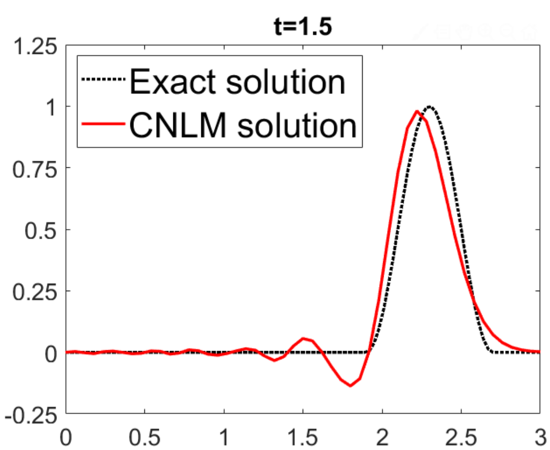

Figure 3 Crank-Nicholson lumped mass solution for C=0.1389

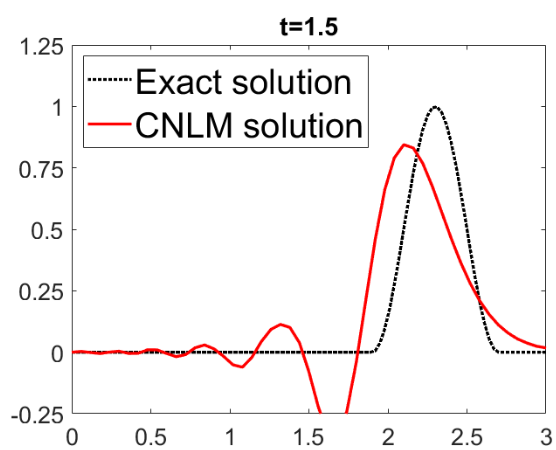

Figure 4 Crank-Nicholson lumped mass solution for C=2.5

The above graphs show the Crank-Nicholson solution with the normal mass matrix (Figure 1 and Figure 12) and with lumped mass matrix (Figure 13 and Figure 14) for Courant numbers of 0.1389 and 2.5. All the solutions presented are considered to be stable. This due to the fact that the Crank-Nicholson method is unconditionally stable. However, using the normal mass matrix provides a more accurate solution compared to the lumped mass matrix. This is due to the fact that the normal formulation provides fourth order accuracy, while using the lumped mass matrix provides only second order accuracy.

Lax-Wendroff method

The second method to be analyzed is the Lax-Wendroff time discretization scheme. The following results are obtained for the normal mass matrix and the lumped mass matrix for different courant numbers.

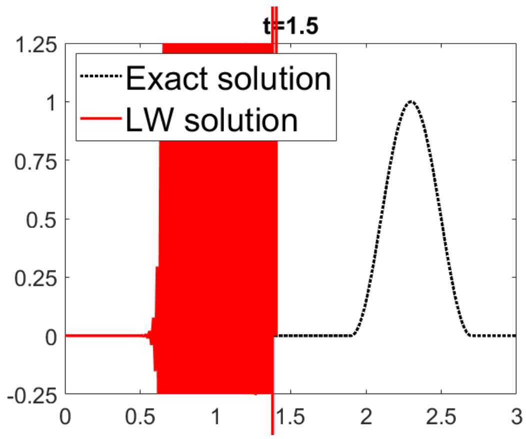

Figure 5 Lax-Wendroff solution for C=0.667

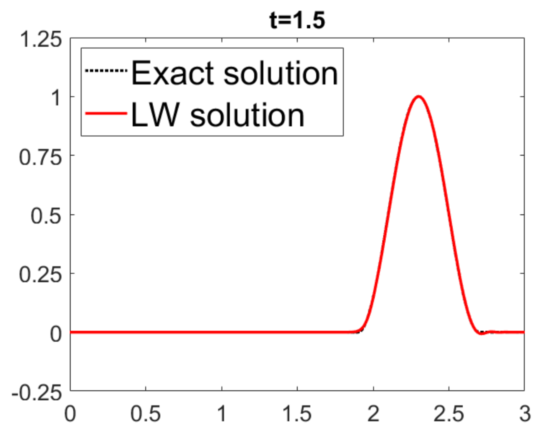

Figure 6 Lax-Wendroff solution for C=0.5

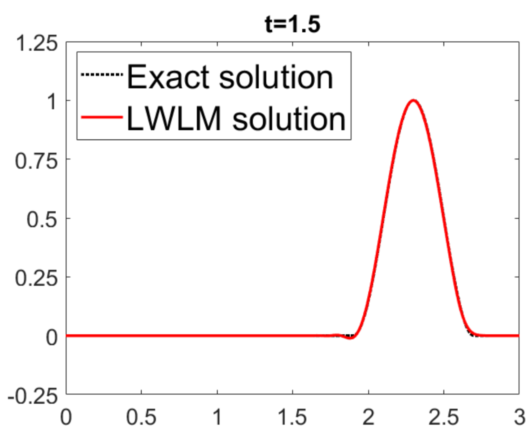

Figure 7 Lax-Wendroff with Lumped mass solution for C=0.667

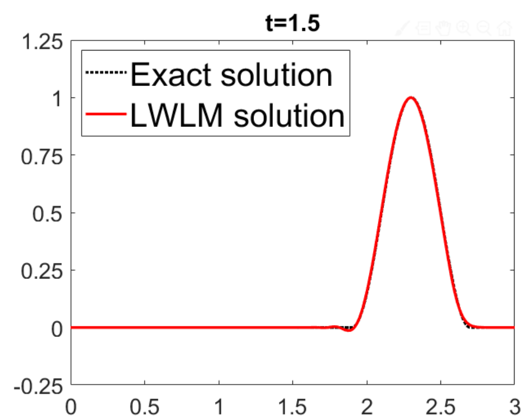

Figure 8 Lax-Wendroff with lumped mass solution for C=0.5

The above graphs show the Lax-Wendroff solution with normal mass matrix (Figure 55 and Figure 56) and with lumped mass matrix (Figure 57 and Figure 58). The courant numbers 0.667 and 0.5 respectively. It is shown that the utilization of a normal mass matrix results in an unstable solution. This is due to the unconditional nature of this method where the solution is only stable for C < 0.577. However, utilizing the lumped mass matrix results in a stable solution for a Courant number up to 1. A selection of a Courant number beyond the mentioned value (C=2.4) yields an unstable solution as shown in Figure 9.

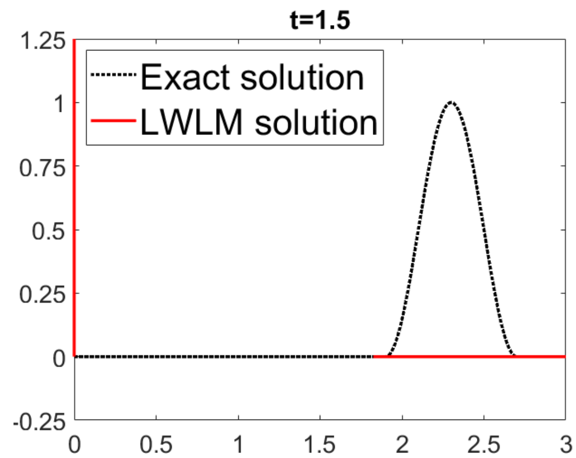

Figure 9 Lax-Wendroff with lumped mass solution for C=2.4

Third-order explicit Taylor-Galerkin (TG3)

The third method to be analyzed is the Third-order explicit Taylor-Galerkin (TG3) time discretization scheme. The following results are obtained for different courant numbers.

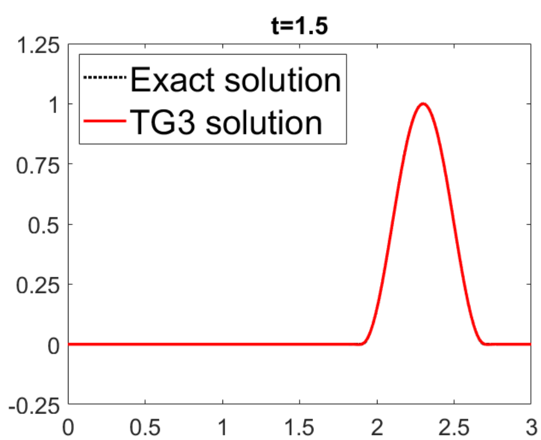

Figure 10 Taylor-Gakerkin (TG3) solution for C=0.5

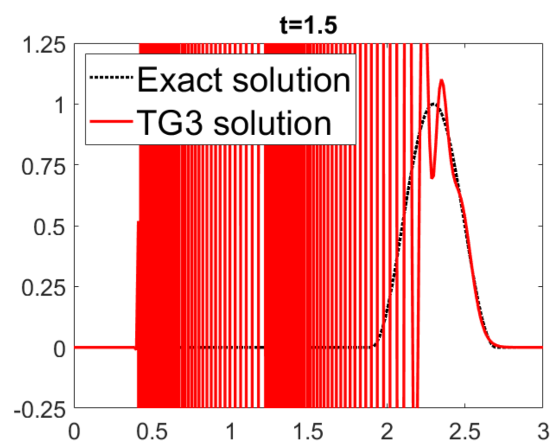

Figure 11 Taylor-Gakerkin (TG3) solution for C=2.4

The above figures 10 and 11 show the solution of the application of the third order Taylor-Galerkin (TG3) method using Courant numbers of 0.5 and 2.4 respectively. The TG3 provides third order accuracy; however, it's conditionally stable for values of $C < 1$. This justifies the results shown in the figures above where a Courant number of 2.4 yields an unstable solution while a Courant number of 0.5 yields a stable solution.

Thus, it could be concluded that Crank-Nicholson method is unconditionally stable thus to achieve a better solution it is better to use the normal mass matrix instead of a lumped one. The Lax-Wendroff method is conditionally stable thus however using a lumped mass matrix yields more stable results for higher Courant numbers. The third order Taylor Galerkin method is a conditionally stable method; however, it has a higher range of stability compared to the Lax-Wendroff method.