HOMEWORK 3: BURGERS EQUATION

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1 PROBLEM

In this assignment we solve the following equation and boundary conditions

$$u_t + uu_x = 0$$
$$u(x,0) = u_0(x)$$
$$x \in (0,4)$$

Two initial conditions we will test. The first problem has the following initial condition

$$u_0(x) = \begin{cases} 0 & \text{if } x < 1, \\ 1 & \text{if } x \geqslant 1. \end{cases}$$

For the second problem

$$u_0(x) = \begin{cases} 1-x & \text{if } x < 3, \\ 0 & \text{if } x \geqslant 1. \end{cases}$$

2 CODE MODIFICATION

To incorporate Newton-Raphson we create a new function in the script `burgers_imNR.m` based on the file `burgers_im.m`. The following lines were added to the code

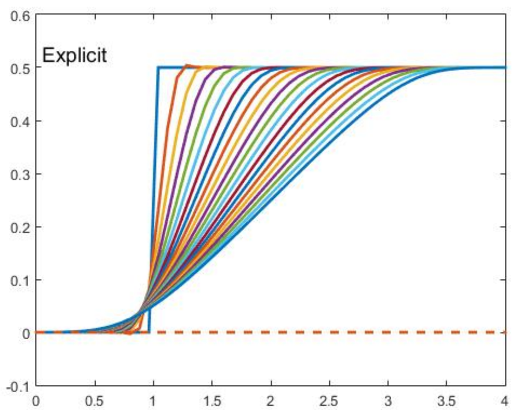
```
 J = M + E*At*K + At*C1 + At*C2;
f = (M + At*C1 + E*At*K)*U1 - M*U0;
s=J\f;
U1 = U1 - s;
```

3 RESULTS

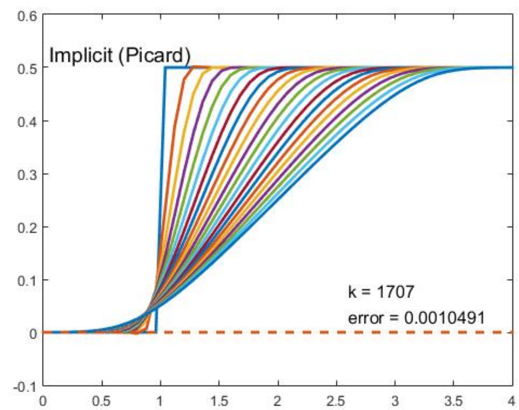
3.1 Problem 1

In this problem the implicit methods are stable for different time discretization and values of ε. However the explicit method is not stable when the time discretization is not small enough as seen figure 3a.

Between the implicit Picard and Newton-Raphson methods, we compare the summation of iteration per time step (k) and the norm of the error per time step. Picard and Newton-Raphson method have the same number of iterations.



(a)



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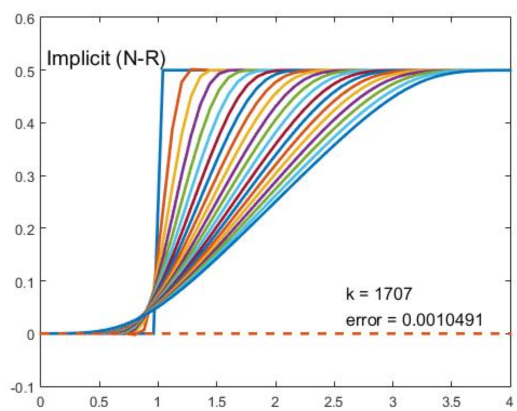
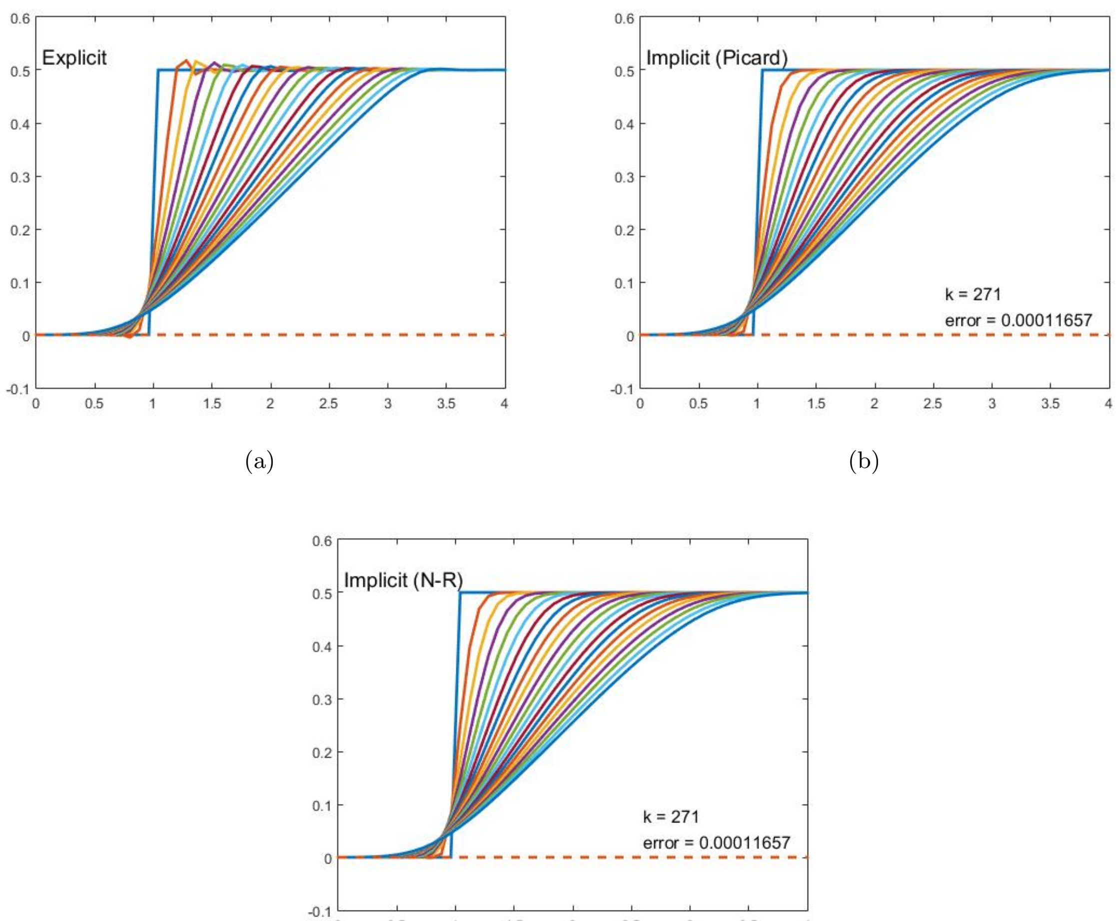


(c)

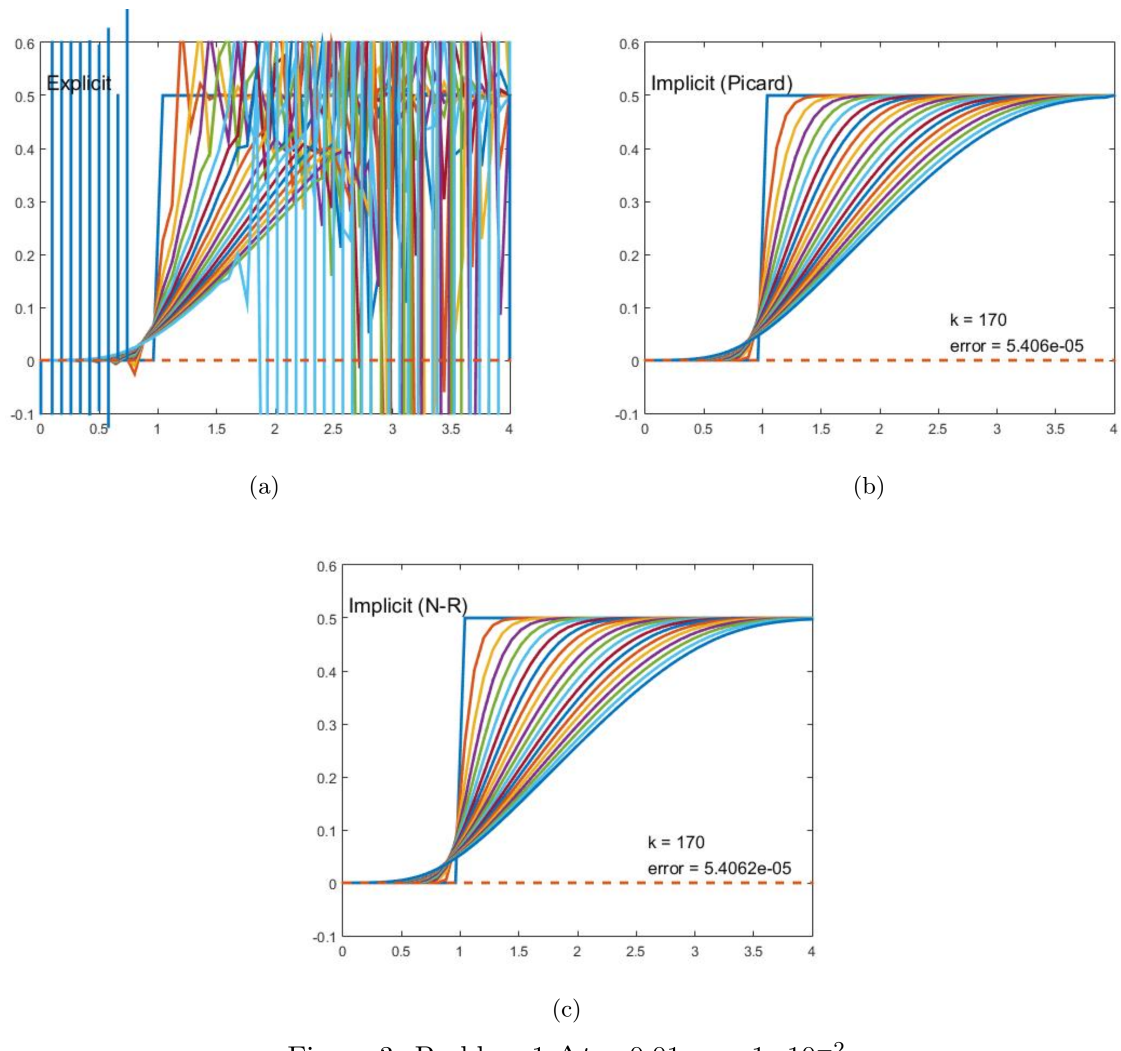
Figure 1: Problem 1 $\Delta t = 0.005$, $\varepsilon = 1 \cdot 10^{-2}$



(a) (b)

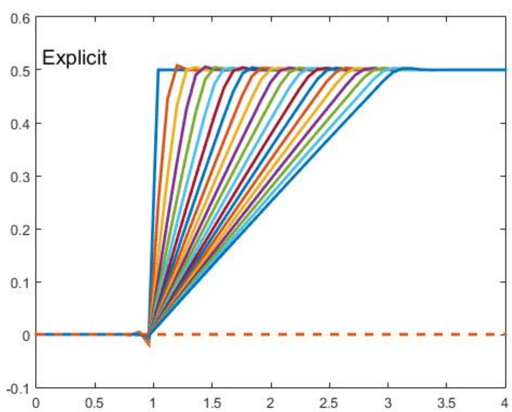
(c)

Figure 2: Problem 1 $\Delta t = 0.05$, $\varepsilon = 1 \cdot 10^{-2}$

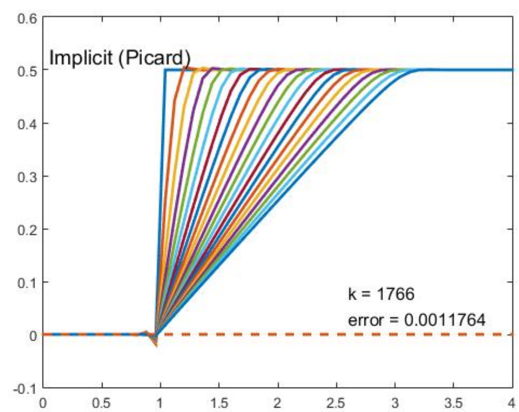


(a) (b) (c)

Figure 3: Problem 1 $\Delta t = 0.01$, $\varepsilon = 1 \cdot 10^{-2}$



(a)



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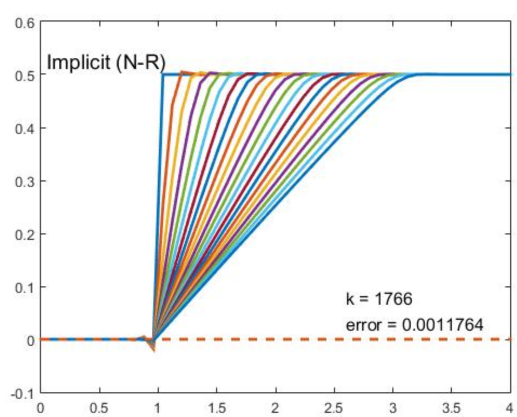


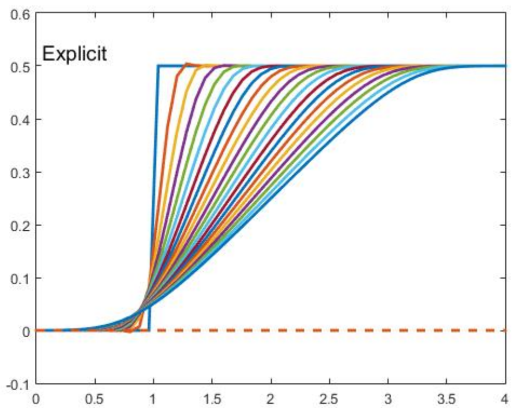
(c)

Figure 4: Problem 1 $\Delta t = 0.005$, $\varepsilon = 1 \cdot 10^{-4}$

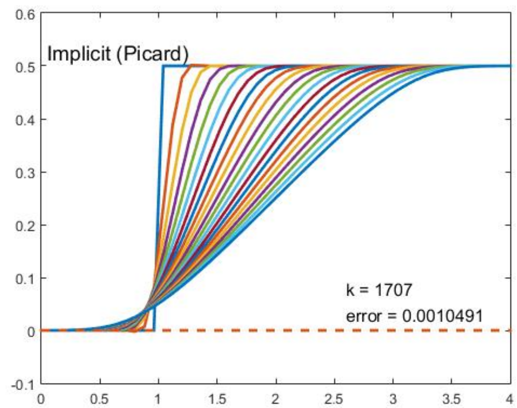
3.2 Problem 2

The number of iterations per time step in Picard and Newton Raphson methods are the same. The reason may be that the rate convergence is the same for the two problems or that Newton-Raphson method is not well implemented in the code.

In the second problem we observe that high values of ε could affect the solutions of explicit and implicit methods as shown figure 6.



(a)



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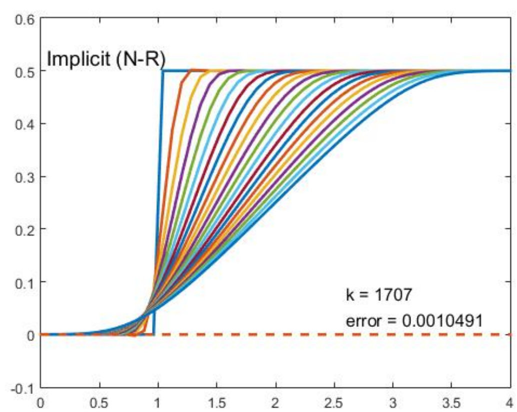
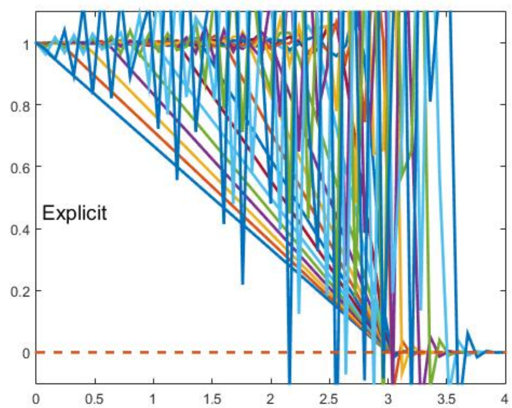
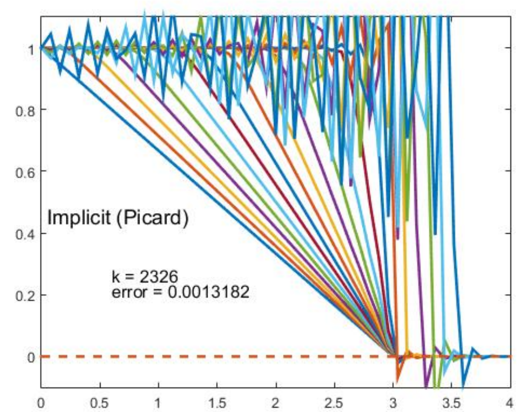


(c)

Figure 5: Problem 2 $\Delta t = 0.005$, $\varepsilon = 1 \cdot 10^{-2}$



(a)



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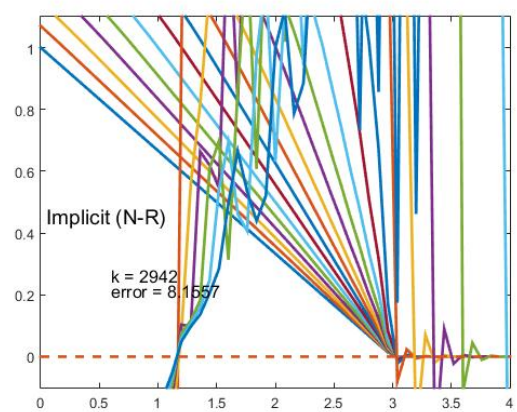

(c)

Figure 6: Problem 2 $\Delta t = 0.005$, $\varepsilon = 1 \cdot 10^{-4}$