<u>**Class Homework 3: 2D Steady Convection-Diffusion**</u>

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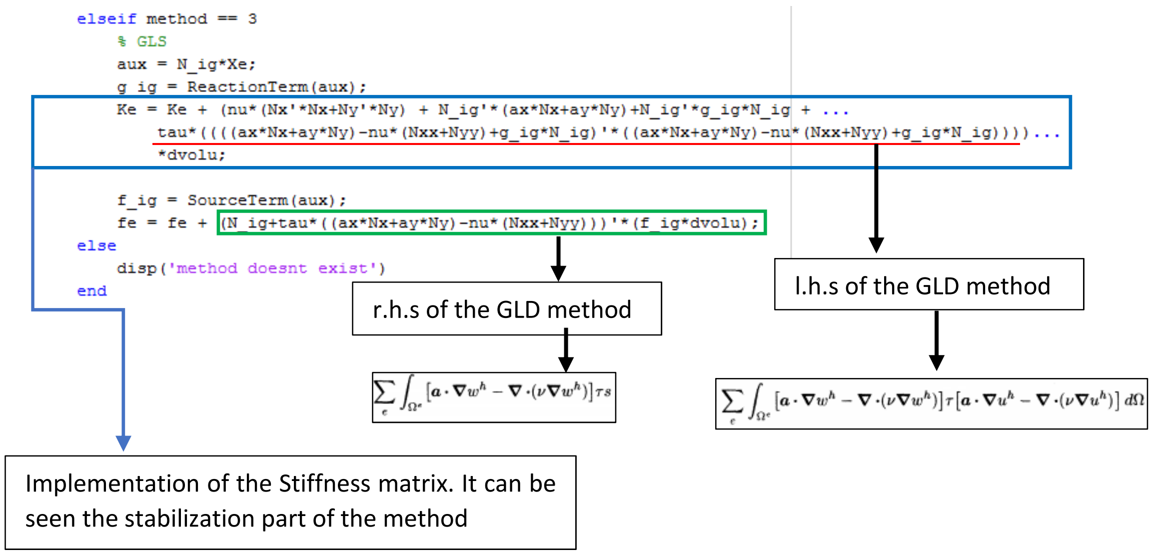
<u>**GLS code implementation:**</u>

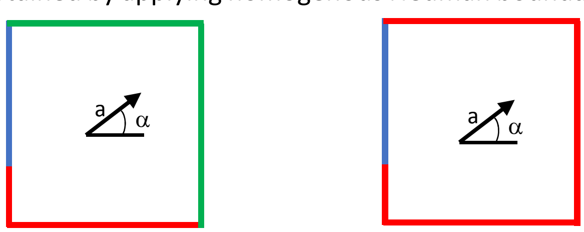
The next figure shows the line of the function code called “FEM_system” where GLS method was implemented.




The reaction term was implemented not only for GLS, but also for Galerkin, Artificial diffusion and SUPG, in order to compare different results when the reaction part of the equation is present.

<u>**Result:**</u>

first, it will be compared the results obtained by applying zero Dirichlet boundary condition on the outlet and the results obtained by applying homogenous Neuman boundary condition on the outlet.




<u>**Boundary Conditions:**</u>

Green line: Homogeneous Neumann bc

Red line: u = 0 Dirichlet bc

Blue line: u = 1 Dirichlet bc

<u>**Parameters of the problem:**</u>

a = 1 (convective velocity)

$\alpha = 30°$

nu = 0.001 (diffusion coefficient)

h = 0.05 (mesh size)

Pe = 25 (Pecle number)

It will be used bilinear quadrilateral elements (Quad4), 20 elements per side.
Galerking, Artificial diffusion, SUPG and GLS will be the fem techniques used to solve the problem.

## Homogeneous Neuman bc case

Results obtained using different techniques and taking into account homogeneous Neumann bc on the outlet of the domain.

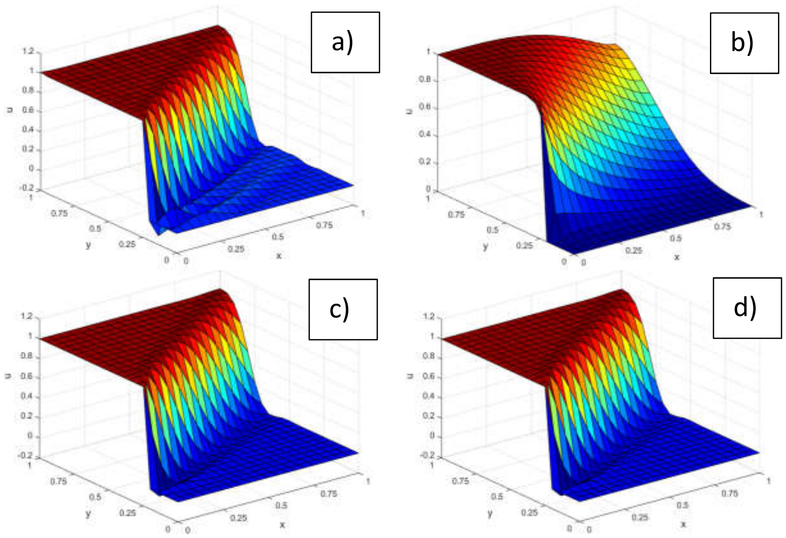



**Figure 1:** Solution obtained by a) Galerkin, b) Artificial diffusion, c) SUPG, d) GLS.

It can be seen as the Peclé number is very high, because of this, it can be observed the solution obtained by Galerkin showed some instability. As is expected the artificial diffusion method introduced too much crosswind diffusion on the solution. On the other hand, SUPG and GLS, gave a stability solution with less crosswind diffusion.

## Zero Dirichlet bc case

Results obtained using different techniques and taking into account zero Dirichlet bc on the outlet of the domain.

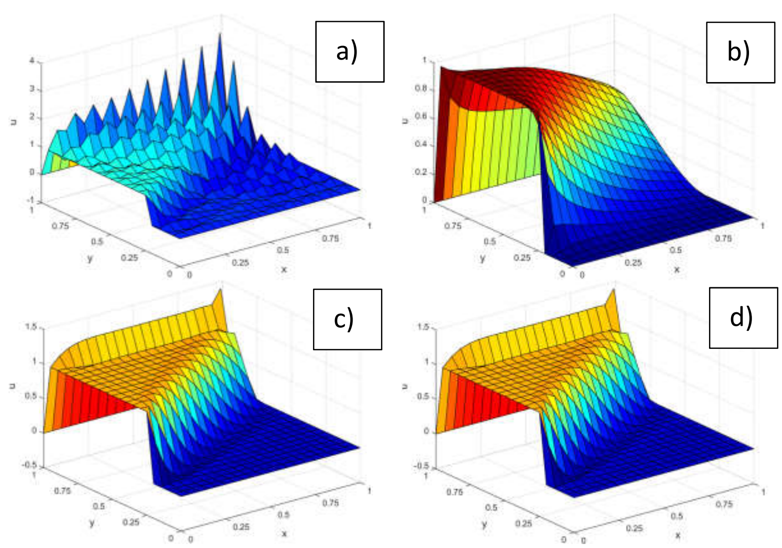



**Figure 2:** Solution obtained by a) Galerkin, b) Artificial diffusion, c) SUPG, d) GLS.

In term of stability of each method, there is no big difference between this problem and the problem described below. The Galerkin showed a bad solution with a very high instability. The big difference lays on the outlet boundary. For Neumann bc case can be seen how the velocity produces a drag effect of the solution. This means, the solution on the inlet almost remain constant until reach the outlet boundary. On the other hand, for Dirichlet bc case it can not be used the same argument written before, since the solution on the outlet boundary is already known. It can be noticed that GLP and SUPG gave the same results, this is due to it has used linear elements and there is no reaction term, hence:

$$\mathrm{GLS} = \mathrm{P(w)} = \mathrm{L(w)} = \mathrm{a} \cdot \nabla \mathrm{w} - \cancel{\nabla \cdot (\nabla \mathrm{w})} + \cancel{\sigma \mathrm{w}} = \mathrm{a} \cdot \nabla \mathrm{w} = \mathrm{SUPG}$$

**<u>Convection reaction dominate and reaction cases</u>**

In these two cases will be used the zero Dirichlet bc described above. It will be used different type of elements and orders.

**<u>Parameters for convection-reaction case:</u>**

a = 0.5 (convective velocity)
$\alpha = 30^{\circ}$
$\mu = 1x10^{-4}$ (diffusion coef.)
$\sigma = 1$ (reaction coef.)
h = 0.05 (mesh size)
Pe = 12.5 (Pecle number)

<u>Results for bilinear quadrilateral elements QUAD4</u>

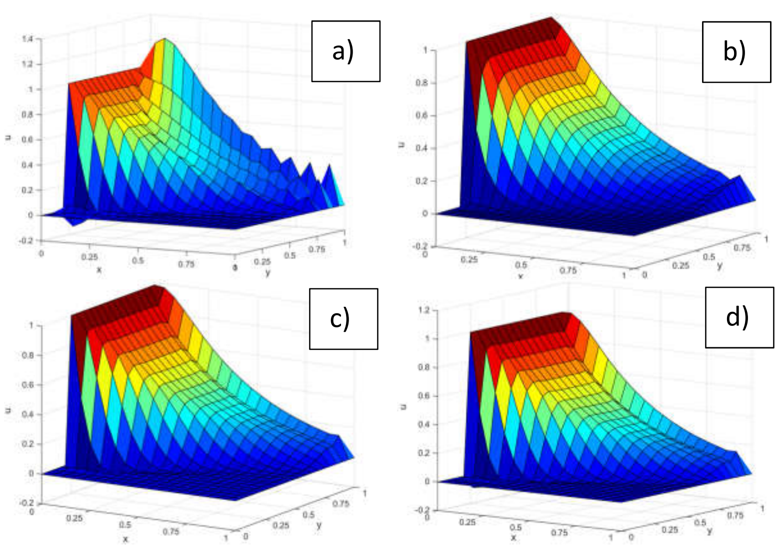



**Figure 3:** Solution obtained by a) Galerkin, b) Artificial diffusion, c) SUPG, d) GLS, using QUAD4 elements.

Results for quadratic quadrilateral elements QUAD9

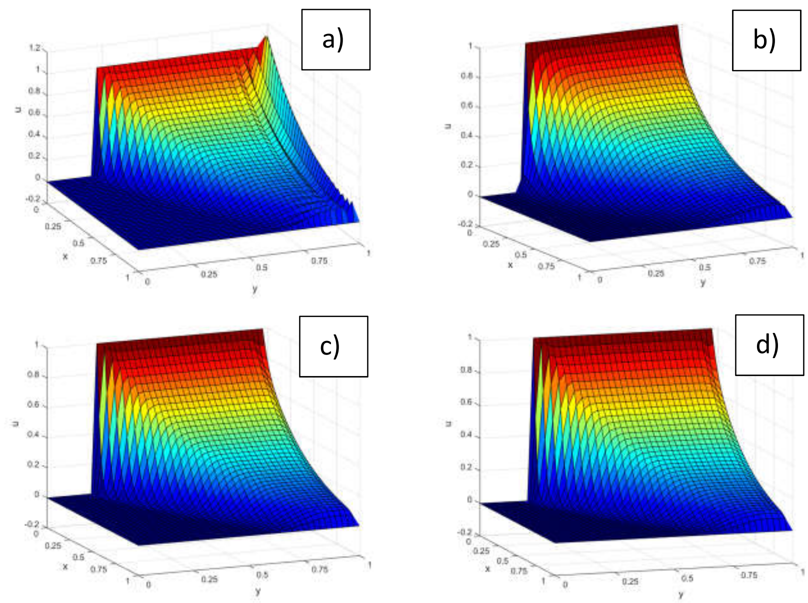



**Figure 4:** Solution obtained by a) Galerkin, b) Artificial diffusion, c) SUPG, d) GLS, using QUAD9 elements.

Results for bilinear triangular elements TRI3

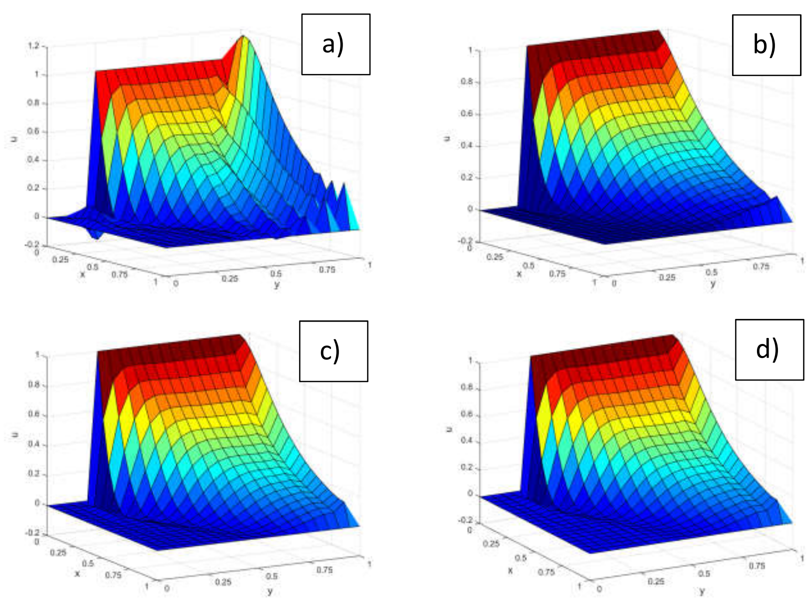



**Figure 5:** Solution obtained by a) Galerkin, b) Artificial diffusion, c) SUPG, d) GLS, using TRI3 elements.

Results for quadratic triangular elements TRI6

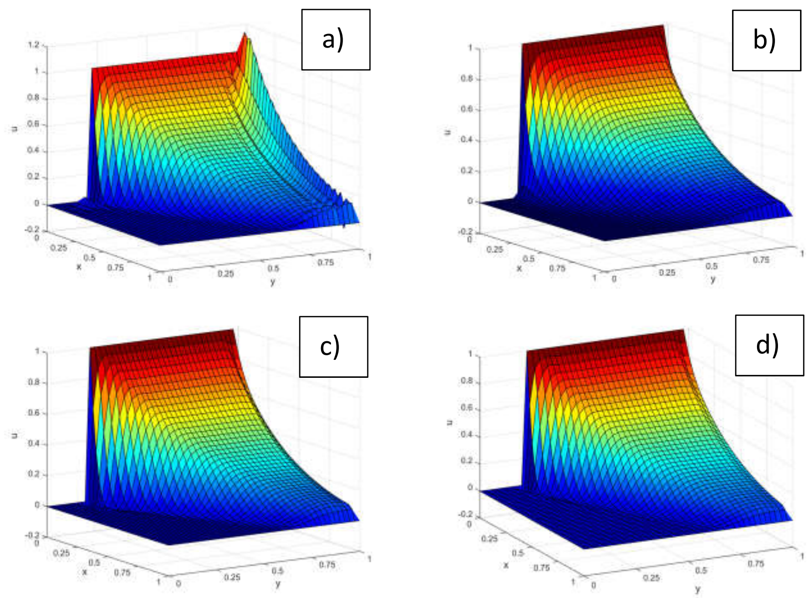


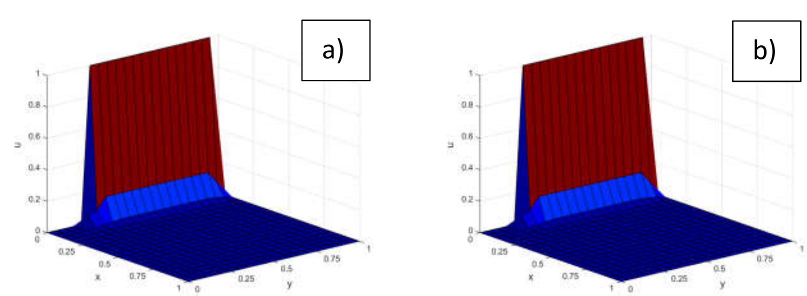

**Figure 6:** Solution obtained by a) Galerkin, b) Artificial diffusion, c) SUPG, d) GLS, using TRI6 elements.

It has obtained good results using high order elements instead of bilinear elements. Using linear elements GLS is SUPG with Galerkin term weighted $1+\sigma\tau$ times more. This implies that the instabilities introduced by Galerkin are little more amplified in GLS compared with SUPG [Donea and Huerta book].

**Parameters for reaction dominated case:**

a = $1x10^{-3}$ (convective velocity)
$\alpha = 30^{\circ}$
$\upsilon = 1x10^{-4}$ (diffusion coef.)
$\sigma = 1$ (reaction coef.)
h = 0.05 (mesh size)
Pe = 0.025 (Pecle number)

Results for bilinear quadrilateral elements QUAD4



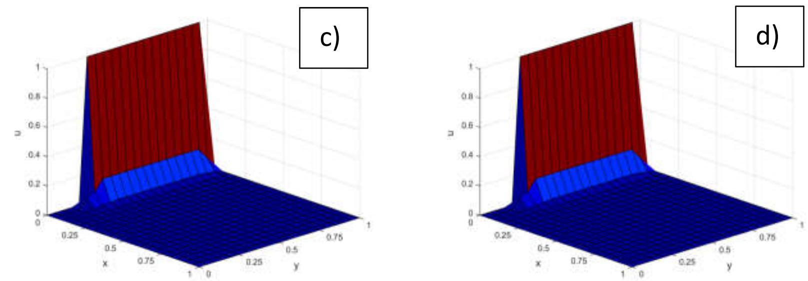



**Figure 7:** Solution obtained by a) Galerkin, b) Artificial diffusion, c) SUPG, d) GLS, using QUAD4 elements.

Results for quadratic quadrilateral elements QUAD9

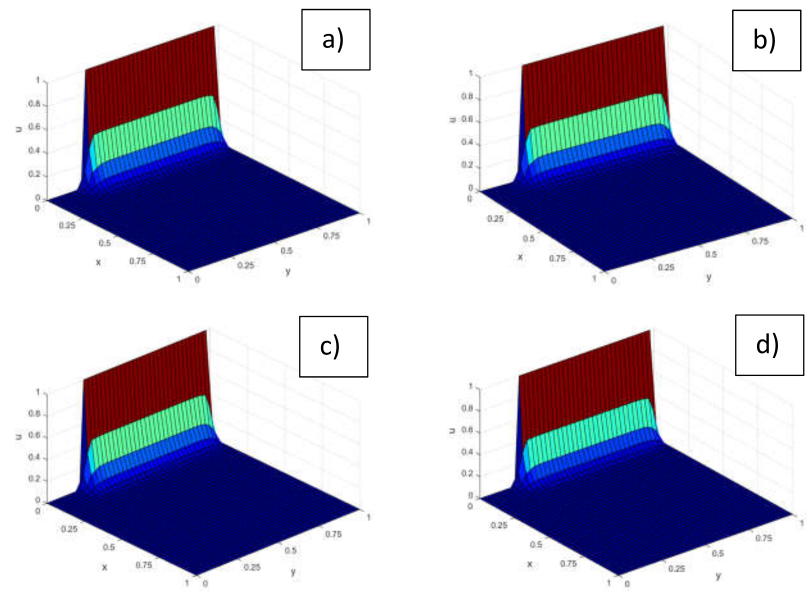



**Figure 8:** Solution obtained by a) Galerkin, b) Artificial diffusion, c) SUPG, d) GLS, using QUAD9 elements.

Results for bilinear triangular elements TRI3
It has obtained the same result than bilinear quadrilateral elements QUAD4.

Results for quadratic triangular elements TRI6
It has obtained the same result than quadratic quadrilateral elements QUAD9.

It can be seen that high order elements have obtained smooth solutions than bilinear elements. Using linear elements GLS is SUPG with Galerkin term weighted $1+\sigma\tau$ times more. This implies that the instabilities introduced by Galerkin are little more amplified in GLS compared with SUPG [Donea and Huerta book]. It is pretty obvious to see that in reaction dominant problems the variable u does not varies too much in the domain. This is due to both the convective and the diffusion terms do not have any effect on the variable u. There is neither transport nor diffusion.