Finite Element in Fluids

Compressible Flow

Home Work -5

Adnan Ali Imran

MS-Computational Mechanics

Problem

Problem data

$$u_t + uu_x = 0$$

$$u(x,0) = u_o(x)$$

The physical correct weak solution of the inviscid Burger equation corresponds to the solution of Burger's equation when viscosity goes to zero.

$$u^{\epsilon}_t + u^{\epsilon}u^{\epsilon}_x = \epsilon u^{\epsilon}_{xx}$$

Use the mesh of **200** elements, with two different cases with respect to total time.

a) Total Time t = 2

Let's consider the time step of $dt = 0.005$ and $\epsilon = 0$. And compare the results for Explicit, Implicit Picard and Implicit Newton Raphson schemes.

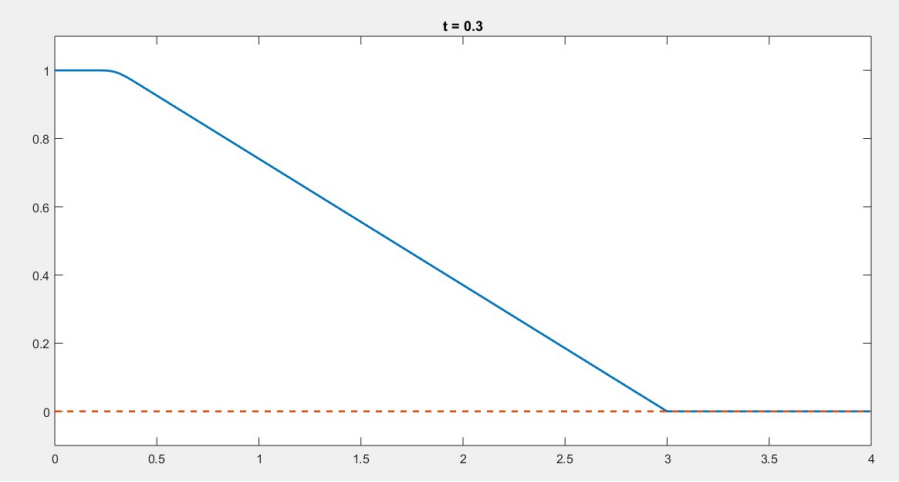

Figure-1: Initial conditions

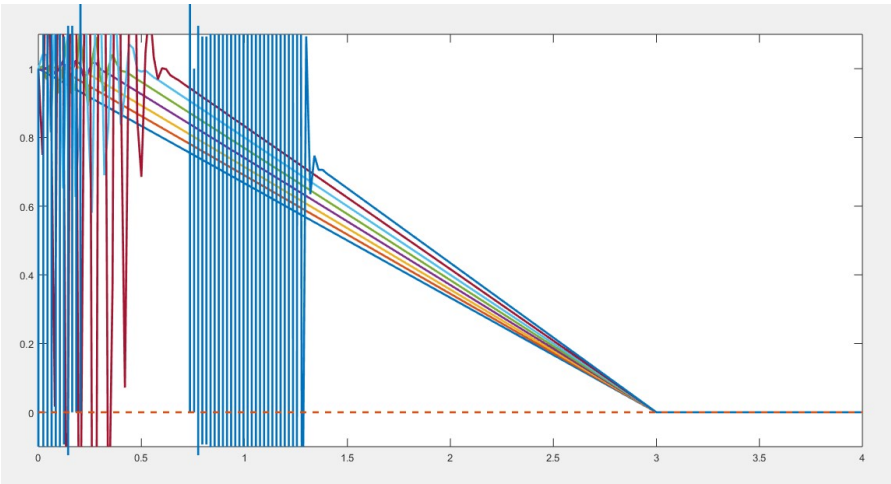

Figure-2: Response by Explicit Scheme

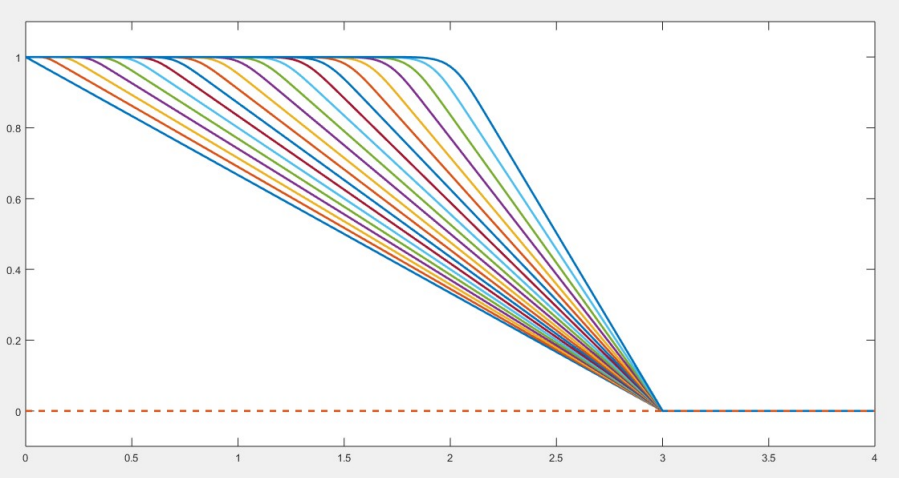

Figure-3: Response by Implicit Scheme (Picard)

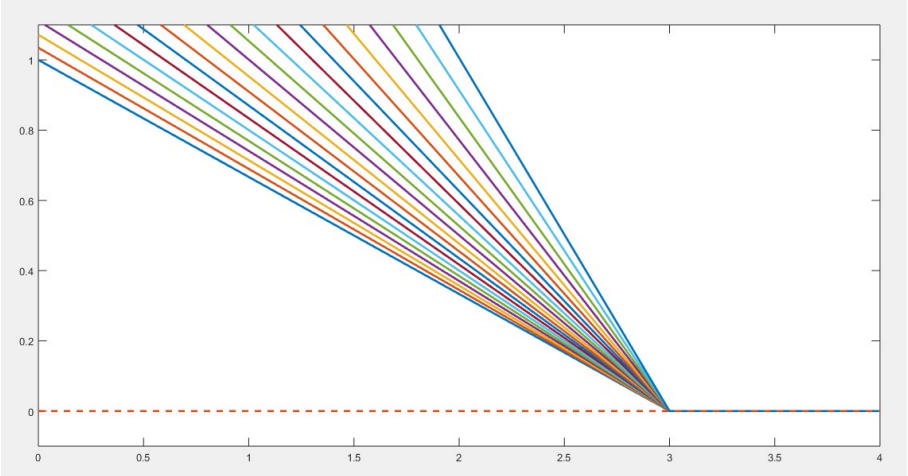

Figure-4: Response by Implicit Scheme (Newton Raphson)

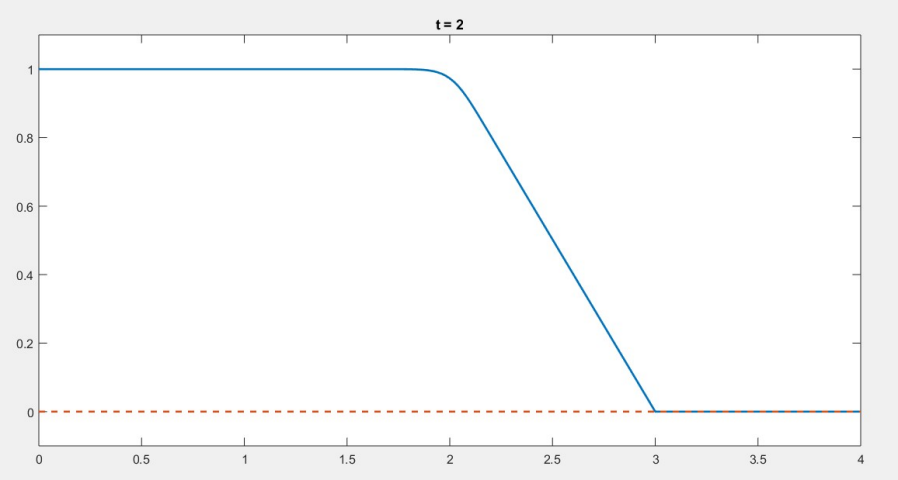

Figure-5: Total Response time

Let's consider the time step of *dt = 0.005* and *ϵ = 1e-2.* And compare the results for Explicit, Implicit Picard and Implicit Newton Raphson schemes.

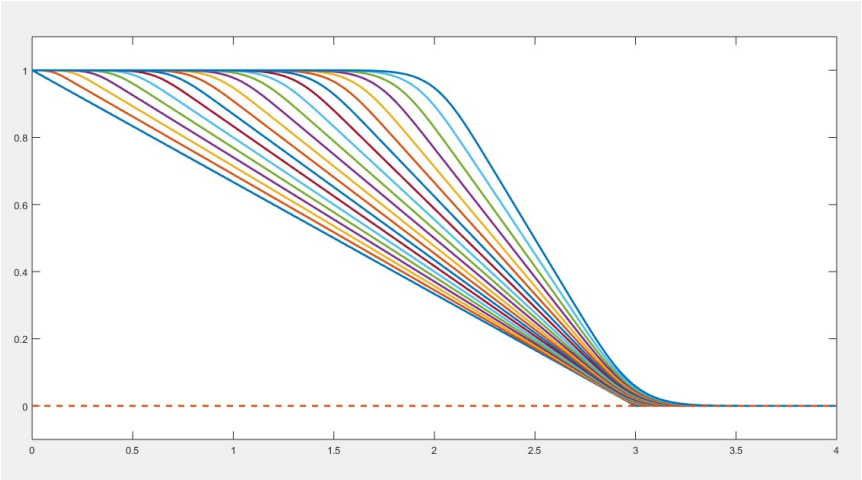

Figure-6: Response by Explicit Scheme

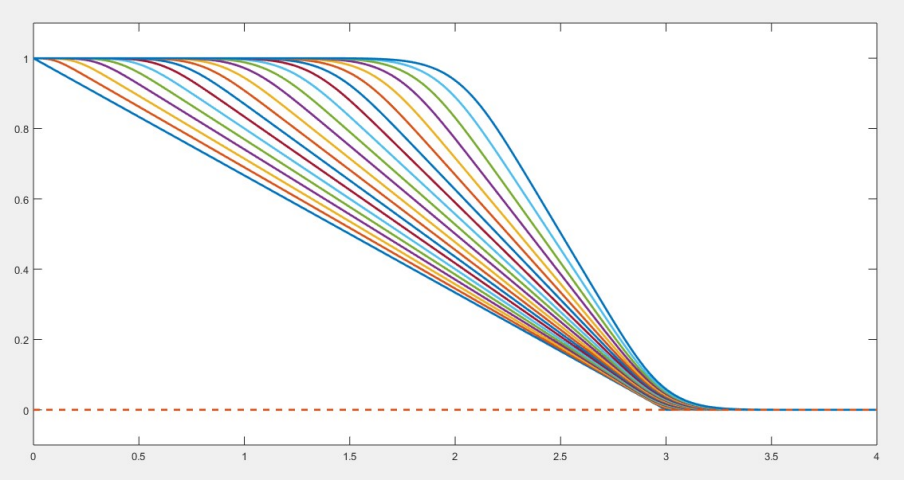

Figure-7: Response by Implicit Scheme (Picard)

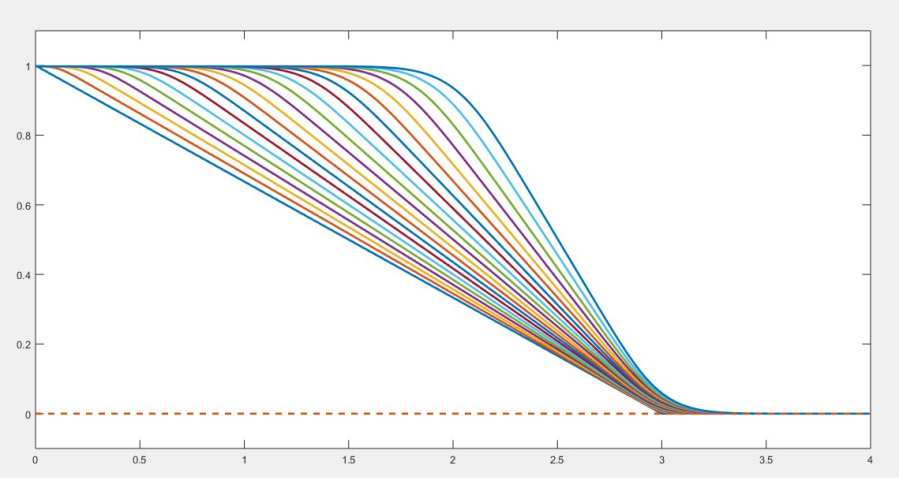

Figure-8: Response by Implicit Scheme (Newton Raphson)

b) Total Time t = 4

Let's consider the time step of *dt = 0.005* and $\epsilon = 0$. And compare the results for Explicit, Implicit Picard and Implicit Newton Raphson schemes.

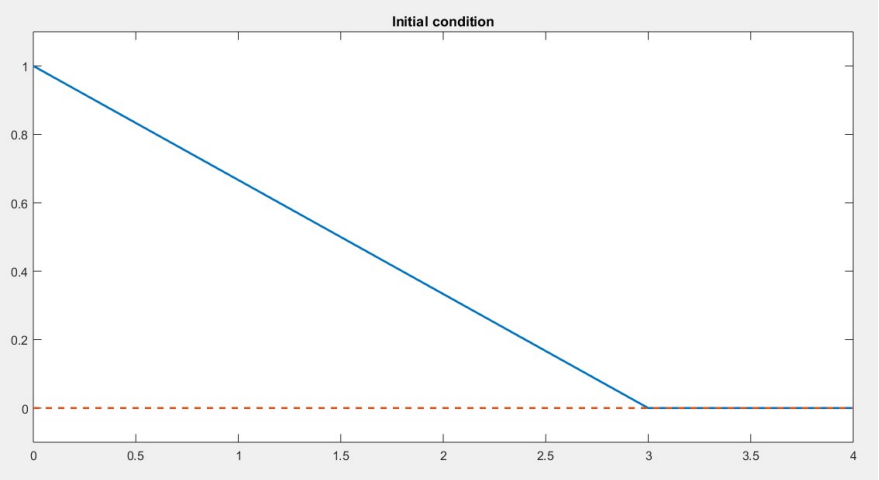

Figure-9: Initial conditions

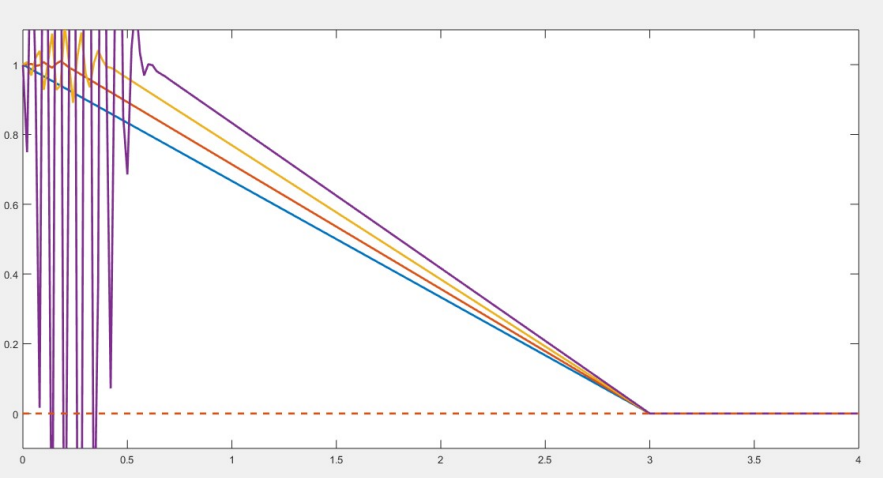

Figure-10: Response by Explicit Scheme

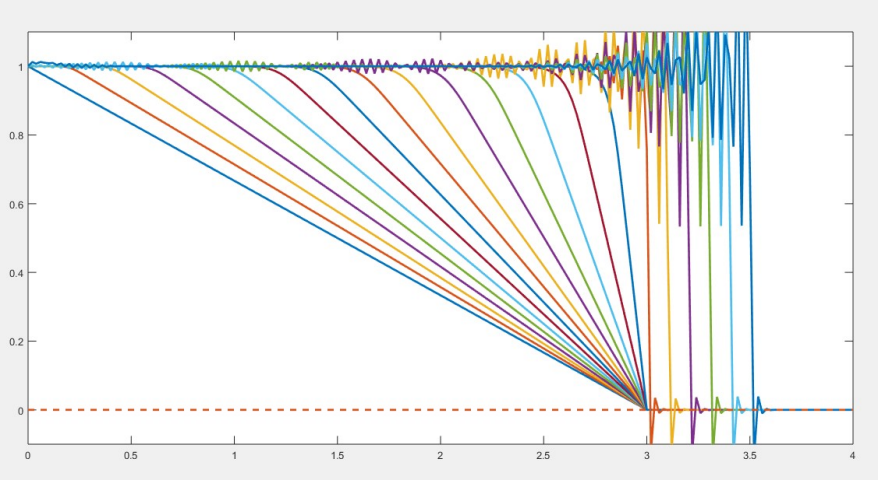

Figure-11: Response by Implicit Scheme (Picard)

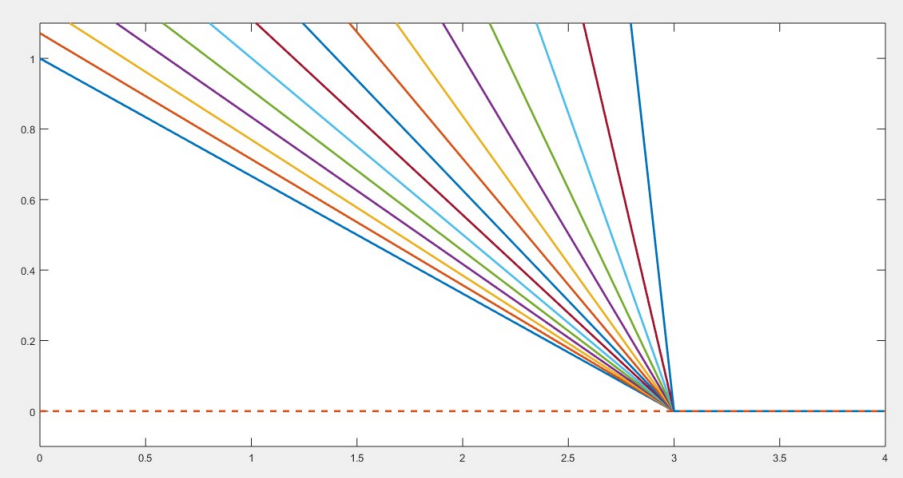

Figure-12: Response by Implicit Scheme (Newton Raphson)

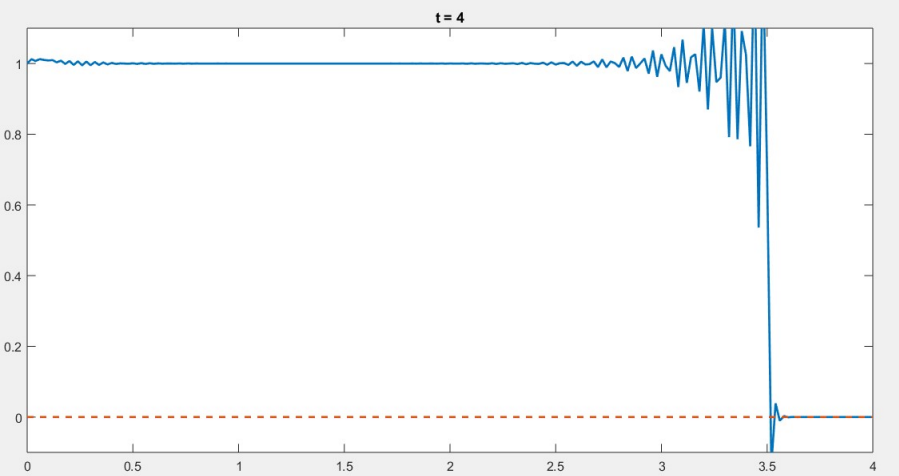

Figure-13: Total Response time

Let's consider the time step of *dt = 0.005* and *ϵ = 1e-2.* And compare the results for Explicit, Implicit Picard and Implicit Newton Raphson schemes.

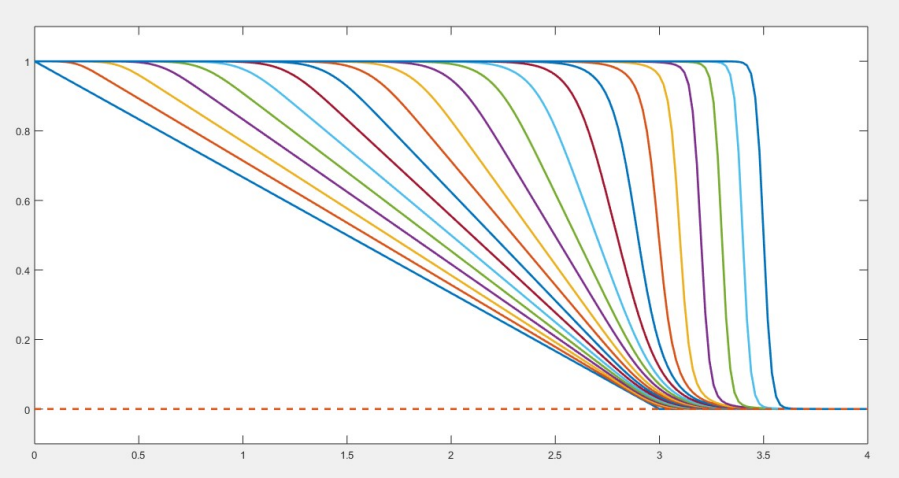

Figure-14: Response by Explicit Scheme

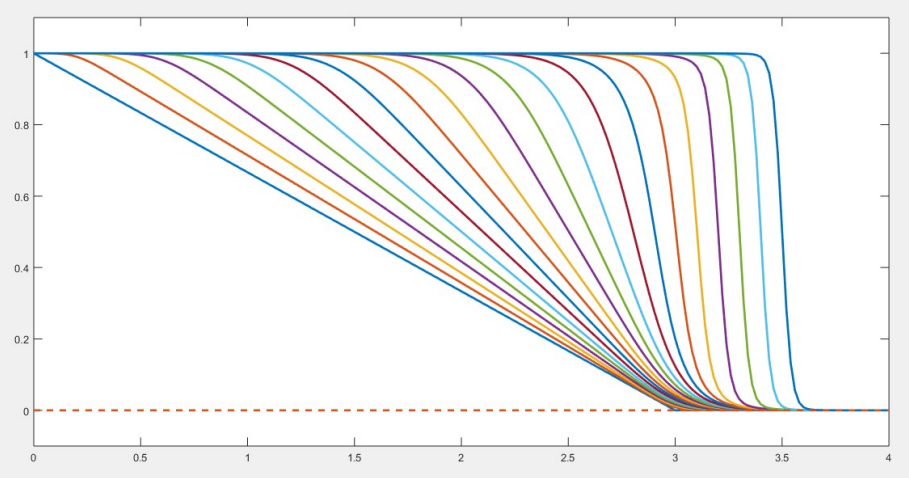

Figure-15: Response by Implicit Scheme (Picard)

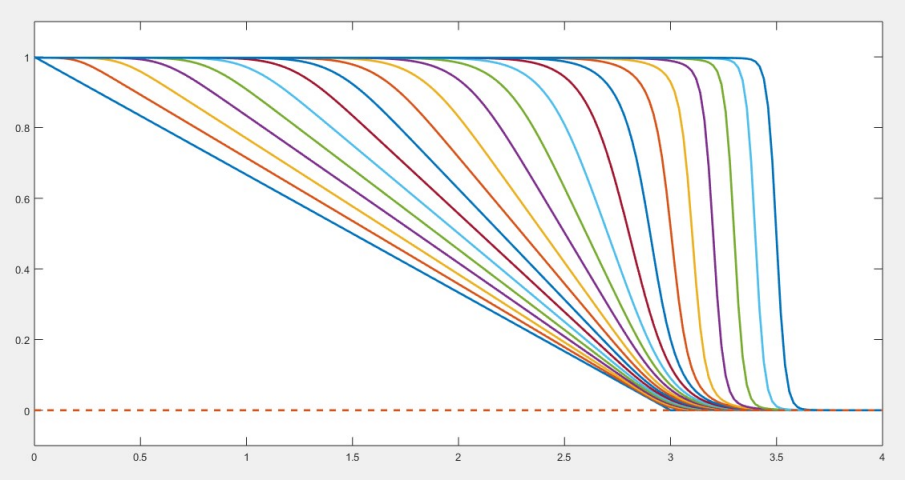

Figure-16: Response by Implicit Scheme (Newton Raphson)

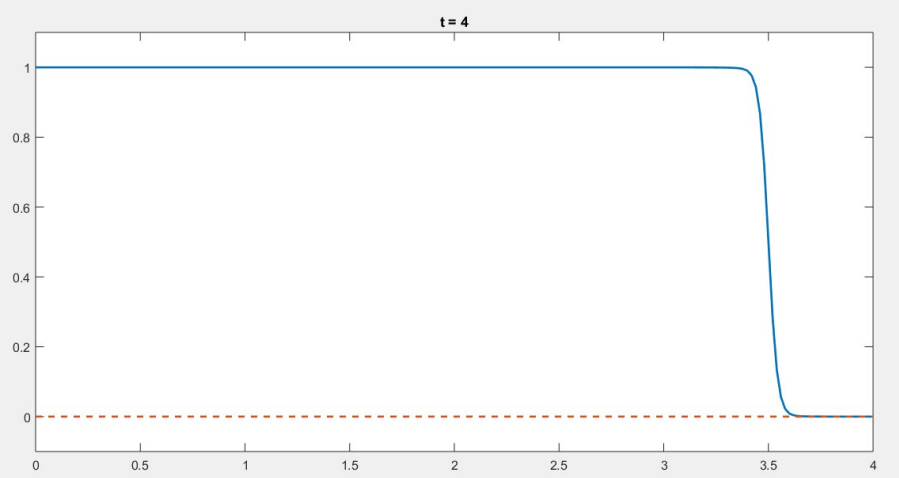

Figure-17: Total Response Time

Comment:

This can be observed through simulation's results that at **$\epsilon = 0$**, solution by explicit scheme exploded for both total time of t=2 & t=4 while both implicit scheme of Picard & Newton Raphson behaved well and gave smooth response at total time t=2. When initial data is increased (t=4), solution became discontinuous and uniqueness is not assured. In order to have smooth response, some artificial diffusion of **$\epsilon = 1e\text{-}2$** is added.