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HOMEWORK # 1 FET

Prove that
$$\begin{cases} \dfrac{\partial(\rho\bar{v})}{\partial t} + \nabla\cdot(\rho\bar{v}\otimes\bar{v} - \sigma) = \rho b \\ \nabla\cdot\bar{v} = 0 \end{cases} \quad \text{— (A)}$$

is inconvertable to the form
$$\begin{cases} \dfrac{\partial v}{\partial t} + (\bar{v}\cdot\nabla)\bar{v} - \nu\nabla\cdot(\nabla\bar{v}) + \nabla p = b \\ \nabla\cdot\bar{v} = 0 \end{cases} \quad \text{— (B)}$$

Take Eq (A) and divide by ρ, incompressibility condition.

$$\frac{\partial\bar{v}}{\partial t} + \nabla\cdot(\bar{v}\otimes\bar{v}) - \frac{1}{\rho}\nabla\cdot\bar{\sigma} = \bar{b}$$

For Newtonian fluid we know that $\sigma = -PI + \mu(\nabla\bar{v} + (\nabla\bar{v})^T)$

Constitutive Equation $\sigma = -PI + 2\mu\nabla^s\bar{v}$

So,
$$\frac{\partial\bar{v}}{\partial t} + \nabla\cdot(\bar{v}\otimes\bar{v}) - \frac{1}{\rho}\nabla\cdot[-PI + 2\mu\nabla^s\bar{v}] = \bar{b}$$

As $[\bar{v}\otimes\bar{v}]_{ij} = v_i v_j \Rightarrow \nabla\cdot(\bar{v}\otimes\bar{v}) = (\bar{v}\cdot\nabla)\bar{v}$

and $2\mu\nabla^s\bar{v} = \mu(\nabla\bar{v} + (\nabla\bar{v})^T)$ where $\nabla\cdot(\nabla\bar{v})^T = 0$

$$\text{So,}\quad \frac{\partial\bar{v}}{\partial t} + (\bar{v}\cdot\nabla)\bar{v} + \frac{1}{\rho}\nabla p - \nu\nabla^2\bar{v} - \nabla\cdot(\nabla\bar{v})^T = \bar{b}$$

Hence
$$\frac{\partial\bar{v}}{\partial t} + (\bar{v}\cdot\nabla)\bar{v} + \nabla p^* - \nu\nabla\cdot(\nabla\bar{v}) = \bar{b}$$

So it's same as Eq (B)
$$\begin{cases} \dfrac{\partial v}{\partial t} + (\bar{v}\cdot\nabla)\bar{v} - \nu\nabla\cdot(\nabla\bar{v}) + \nabla p^* = \bar{b} \\ \nabla\cdot\bar{v} = 0 \end{cases}$$