Task 1

Solve the first example using Galerkin's method with

(a) a = 1, $\nu = 0.2$, 10 linear elements

(b) a = 20, $\nu = 0.2$, 10 linear elements

(c) a = 1, $\nu = 0.01$, 10 linear elements

(d) a = 1, $\nu = 0.01$, 50 linear elements

The following figures were obtained by using the Galerkin's method with varying input parameters according to the given task.

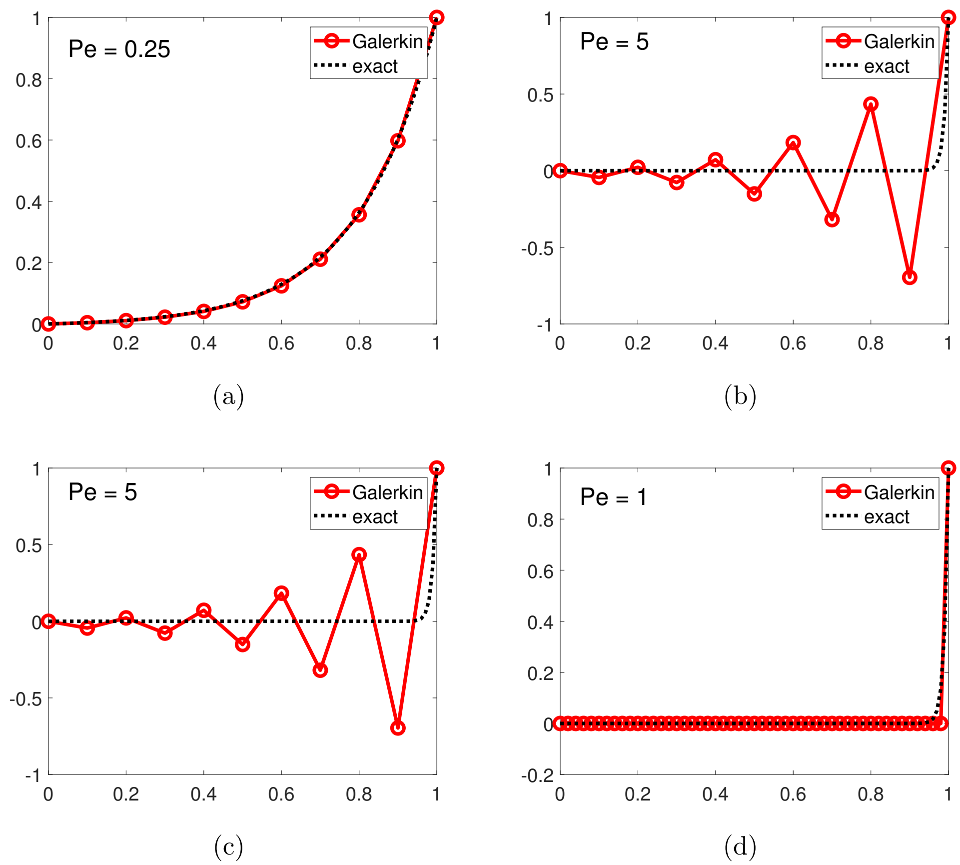


Figure 1: Task 1

Fig. 1 (b) & (c) show the expected instabilities of the Galerkin method when the Péclet number is above 1. These solutions underline the necessity of an introduction of stabilization techniques like the Streamline Upwind (SU), the Streamline Upwind Petrov-Galerkin (SUPG) or the Galerkin Least-Squares (GLS) method.

Task 2

Solve the third case above using

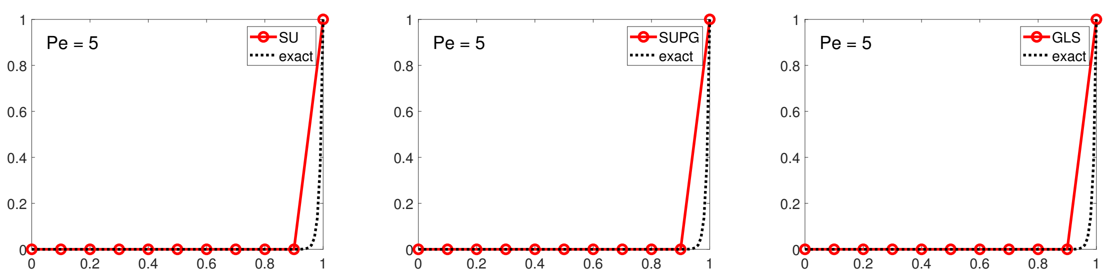
(a) Streamline upwind

(b) SUPG

(c) GLS

with the optimal stabilization parameter. Do you obtain the exact solution?

In order to obtain solutions for SUPG and GLS, both methods had first to be implemented in the provided MATLAB code. In comparison to the Garlekin method a good improvement according the stability can be observed. However, the solution is still not exact due to the linear interpolation between the nodes. Especially at the last element this inaccuracy becomes clearly visible. In the case of a 1D convection-diffusion problem $\sigma = 0$ holds. Additionally, the usage of linear shape functions lead to the fact, that second order derivatives in the equation of the SUPG as well as the GLS are vanishing. As as result of these simplifications, both methods display the same solution as the SU method.



(a) (b) (c)

Figure 2: Task 2

Task 3

Repeat the exercise with a source term of the form: $f = 10e^{-5x} - 4e^{-x}$

The implementation of the non-linear source term leads to similar results with the respect to the Galerkin method. For Péclet numbers $Pe > 1$ growing oscillations can be seen in Fig 3 (b) & (c).

Looking at the solutions of the other methods in Fig. 4 the stabilizing effect of the stabilization term is still present. Though, the SU solution is quite off compared to the exact solution. In this case SUPG and GLS are much closer to the exact solution only limited by the linearity of the shape functions.

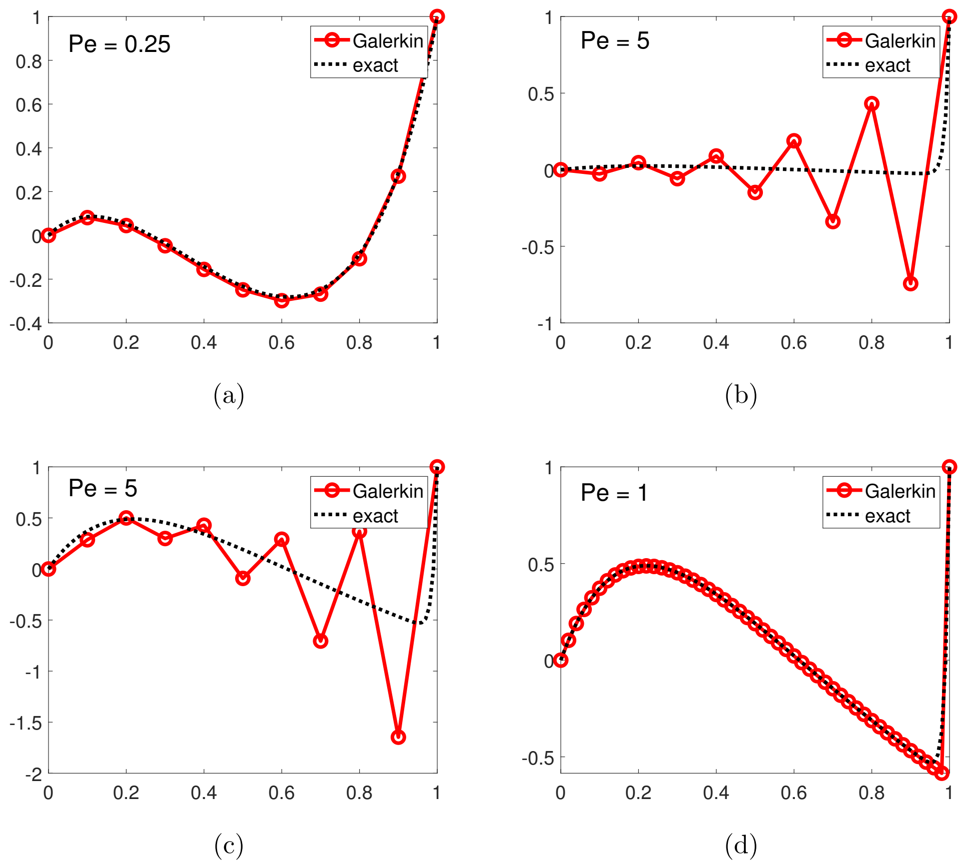


Figure 3: Task 3.1

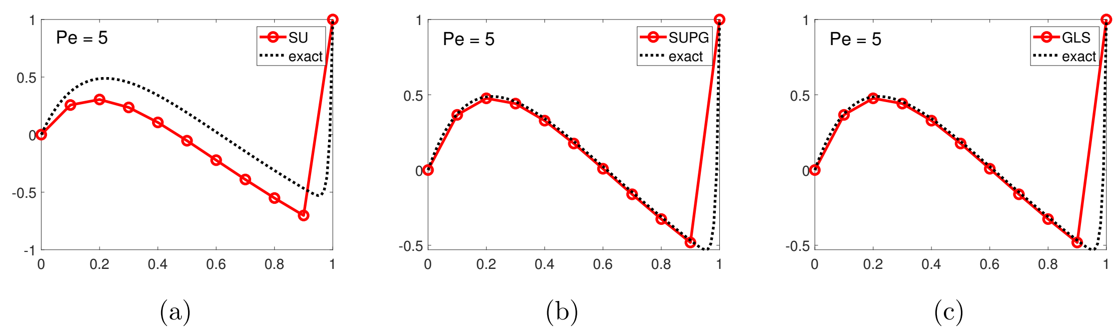


Figure 4: Task 3.2

Task 4

Include quadratic elements and discuss the changes in the code and in the solution of the problem.

In order to implement the quadratic shape functions some changes in the MATLAB code were necessary. In this case a matrix had to be assigned to the stabilization parameter of the SU, the SUPG and the GLS mehtod. The matrix assures the right execution of the different approaches on the middle and end nodes of the elements.

Figure 5 shows all four solutions for the Galerkin method. Even though the results for a Péclet number $Pe > 1$ still show oscillations, they seem to be of smaller magnitude. This is due to the higher number of elements introduced by the parabolic shape function. The number of elements has actually doubled and thereby the Péclet number has been decreased by a factor of 2.

The comparison of the results of the other methods in Figure 6 shows once again that the SU method is lacking accuracy. SUPG and GLS are getting even closer to the exact solution, but the sudden change close to the Dirichlet boundary condition could not be modelled correctly. A solution for this problem could be the usage of higher order shape functions or a higher element discretization.

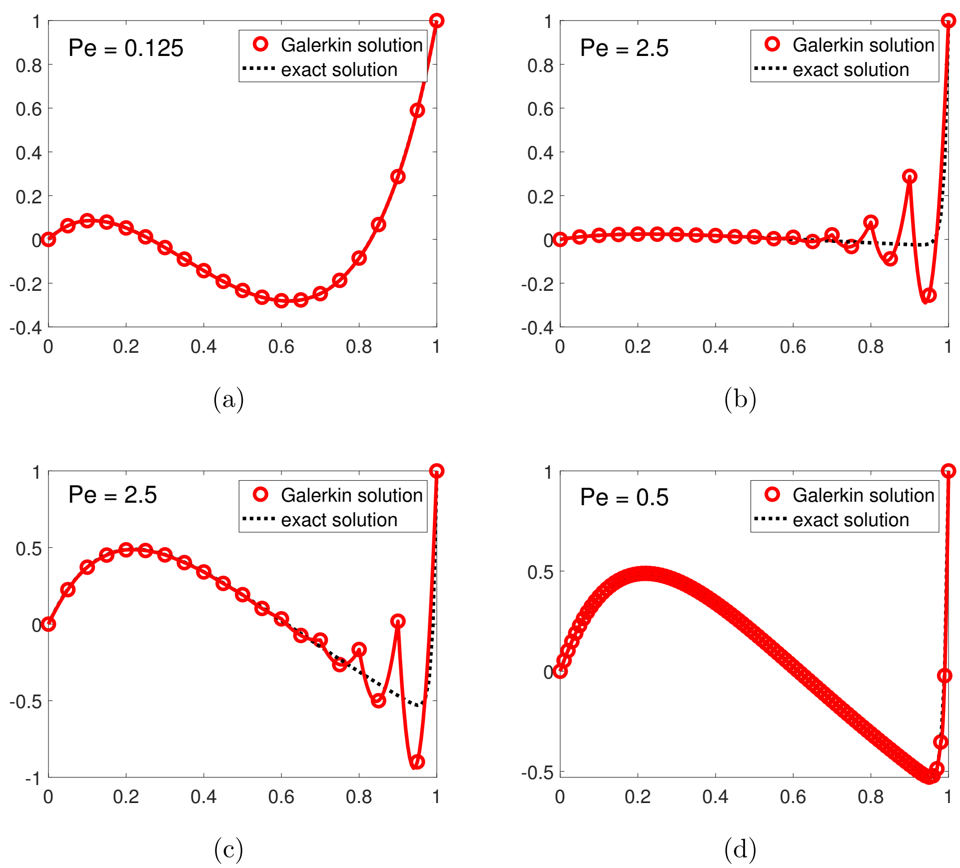
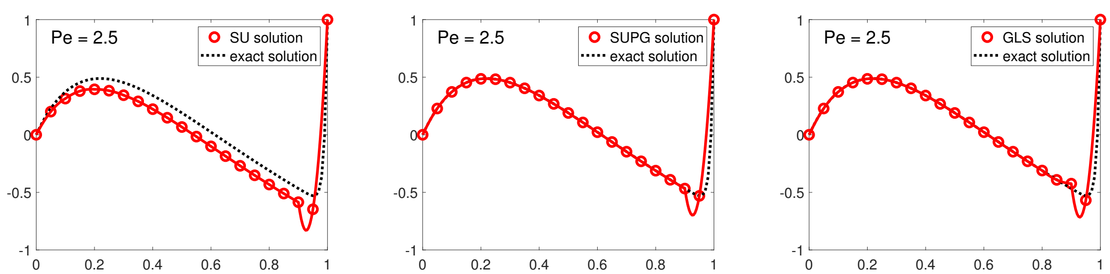


Figure 5: Task 4.1



(a) (b) (c)

Figure 6: Task 4.2