Finite Elements in Fluids

Homework 1

Task: Transform the Cauchy momentum equation from conservative into non-conservative form.

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \cdot \nabla) \vec{v} - \nabla \cdot \sigma = \rho \vec{b} \qquad \text{(non-conservative)}$$

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot (\rho \vec{v} \otimes \vec{v} - \sigma) = \rho \vec{b} \qquad \text{(conservative)}$$

- $$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot (\rho \vec{v} \otimes \vec{v}) - \nabla \vec{\sigma} = \rho \vec{b}$$

  Differentiation product rule

- $$\rho \frac{\partial \vec{v}}{\partial t} + \vec{v} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v} \otimes \vec{v}) - \nabla \vec{\sigma} = \rho \vec{b}$$

  using the following rule: $\nabla \cdot (\vec{u} \otimes \vec{v}) = (\nabla \cdot \vec{u}) \vec{v} + (\vec{u} \cdot \nabla) \vec{v}$, we obtain:

- $$\rho \frac{\partial \vec{v}}{\partial t} + \vec{v} \frac{\partial \rho}{\partial t} + (\nabla \cdot \rho \vec{v}) \vec{v} + (\rho \vec{v} \cdot \nabla) \vec{v} - \nabla \vec{\sigma} = \rho \vec{b}$$

  rewrite:

- $$\rho \frac{\partial \vec{v}}{\partial t} + \vec{v} \underbrace{\left(\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} \right)}_{=0} + \rho (\vec{v} \cdot \nabla) \vec{v} - \nabla \vec{\sigma} = \rho \vec{b}$$

  insert mass conservation equation: $\frac{\partial \rho}{\partial t} + \nabla (\rho \vec{v}) = 0$:

- $$\rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \cdot \nabla) \vec{v} - \nabla \vec{\sigma} = \rho \vec{b}$$