UPC - BARCELONA TECH
MSc Computational Mechanics
Spring 2018

Finite Elements in Fluids

Lab 5: Unsteady Convection and Convection-Diffusion Problem

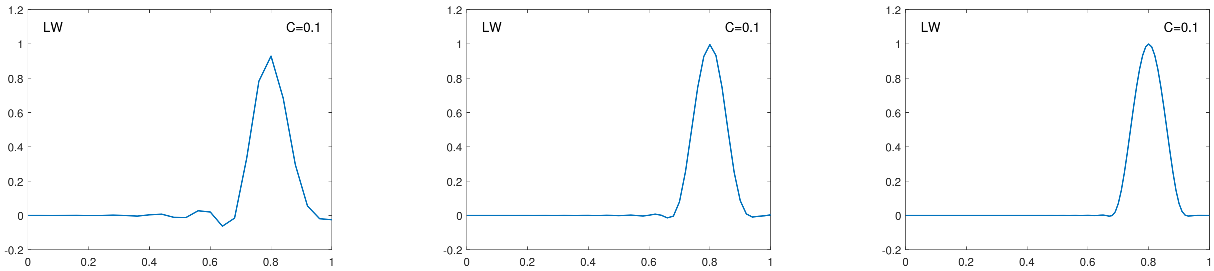
Due 02/04/2018

Prasad ADHAV

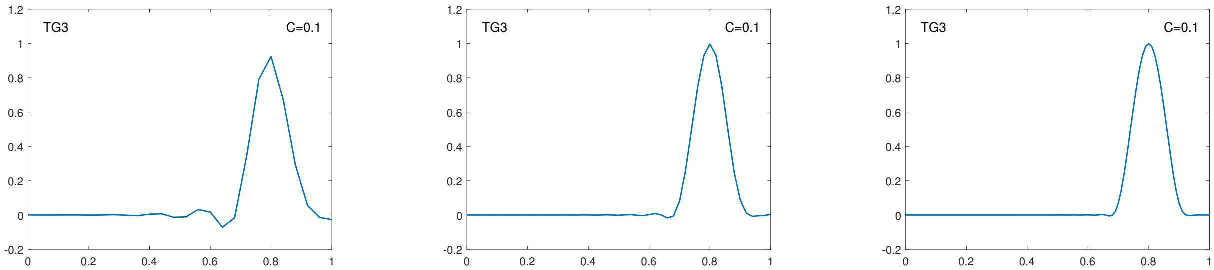
1 Part 1: Pure convection problem.

In the figures below first we observe the behaviour of the methods when the number of time steps is varied. The Courant number is kept low so as to ascertain that by default the solution is close. We just want to observe the effects of time steps in this part. All other inputs are kept to their default value.

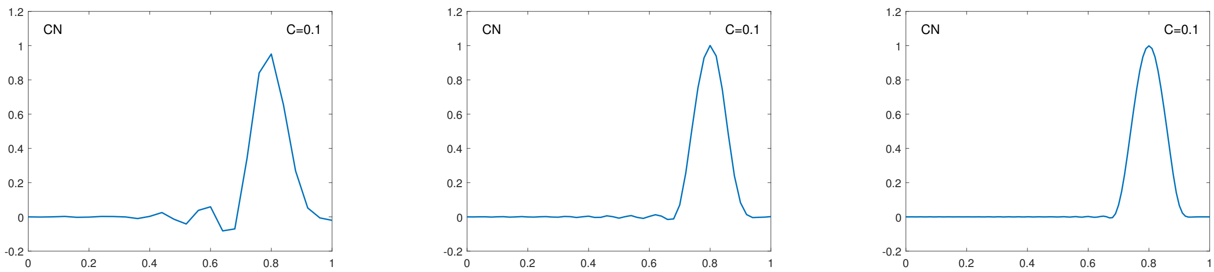
t = 25, c = 0.1   t = 50, c = 0.1   t = 100, c = 0.1



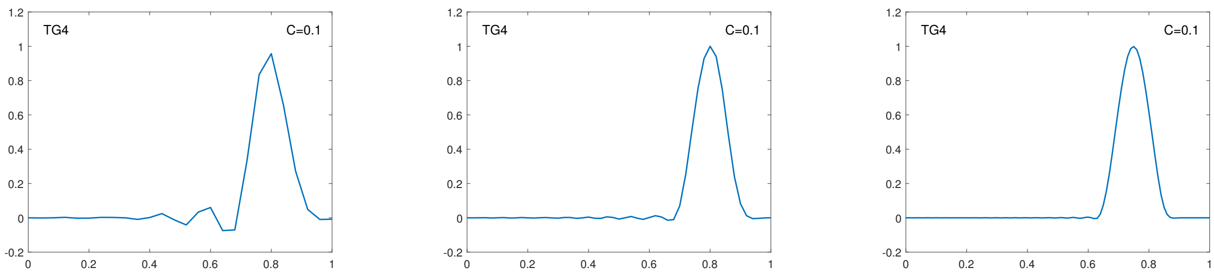
Lax Wandroff



Taylor Galerkin Third order explicit



Crank Nicoleson - Finite Element

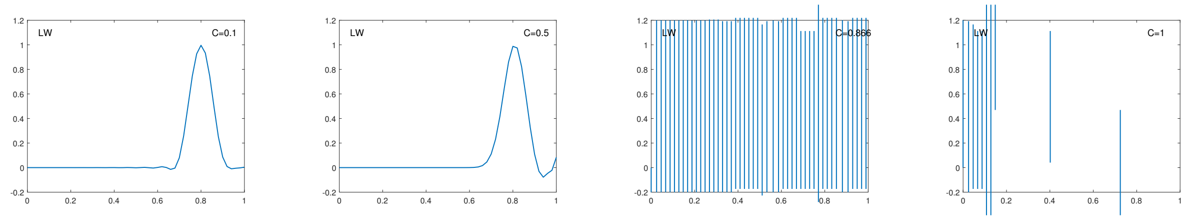


Taylor Galerkin Fourth Order

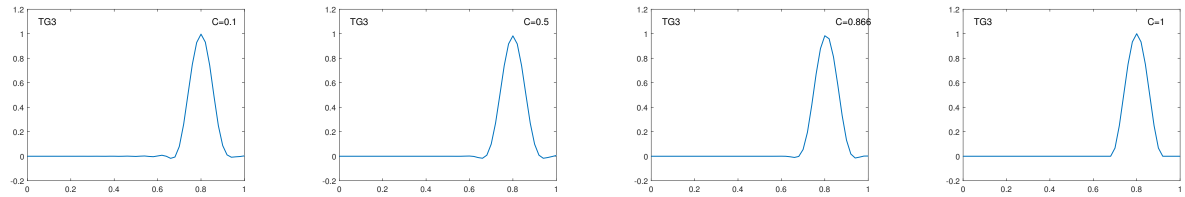
It is clear from the figures that as we increase the number of time steps we get a better numerical solution. Note that the exact solution is a cosine hill in this part. For a high time step all the methods perform well and give almost the similar solution which is very close to the exact solution. As for the lower time step values, we can clearly see that the numerical solution obtained ragged. Also, the Galerkin part of the methods induces oscillations, as there are less time steps for it to achieve the sudden change in slope. Finally, it can be seen that the methods behave as expected from the theory.

In the following figures we will vary the Courant number as observe its effects on the performance of the various methods. The other inputs are kept to there default value. It is known that different methods have different stability condition, hence we choose the highest value for courant number under which majority of methods are stable.

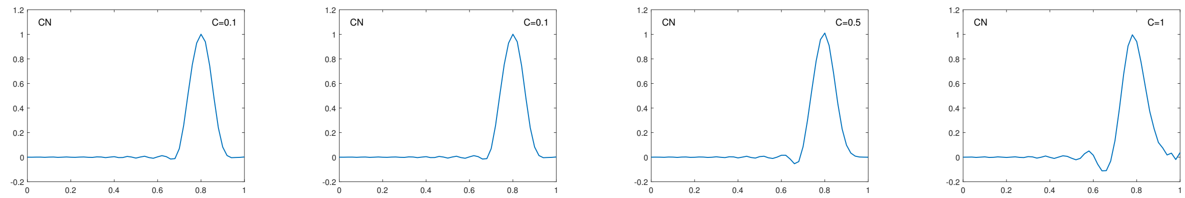
c = 0.1, t = 50   c = 0.5, t = 50   c = 0.8660, t = 50   c = 1, t = 50



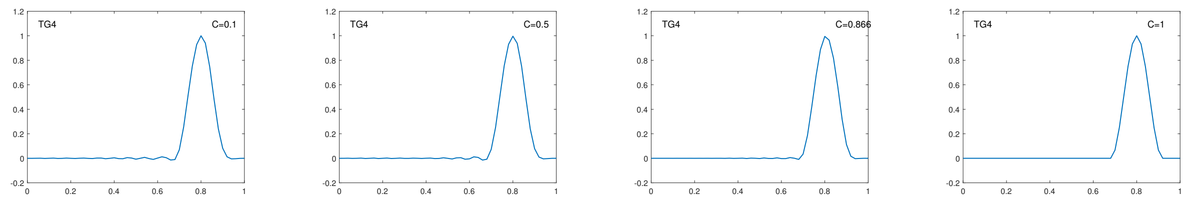
Lax - Wandroff »



Taylor Galerkin Third order explicit »



Crank Nicoleson - Finite Element »



Taylor Galerkin Fourth Order »

It can be clearly observed from the above plots that as we approach the critical Value of courant number, the accuracy for the solution decreases. This is with the time steps which are more that enough to get an almost exact solution. Crank Nicoleson method is simplest to observe this phenomenon. Conversely we can see that the Talyor Galerkin series (TG3 and TG4) give a better numerical solution as the Courant number approaches the critical value. As we can see Lax Wandrauf is more accurate throughout its stability range, but the problem is that its stability range is small compared to others. Overall we can conclude that the methods behave consistently with the expected results we predict from the theory.

2 Part 2: Propagation of steep front

The problem statement is as follows:

$$
\begin{aligned}
u_t + a u_x &= 0, && x \in (0,1), \quad t \in (0, 0.6] \\
u(x,0) &= u_0(x), && x \in (0,1) \\
u(0,t) &= 1, && t \in (0, 0.6]
\end{aligned}
$$

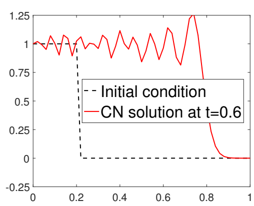
$$u_0 = \begin{cases} 1 & if x \leq 0.2 \\ 0 & otherwise \end{cases}$$

$a = 1, \quad \Delta x = 0.02, \quad \Delta = 0.015$
According to this problem statement the change in the code were done for the nElem = 50, nStep = 40, tEnd =0.6, dom = $[0, 1]$.

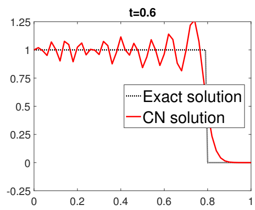
2.1 Compute Courant number

We know that Courant Number $= \dfrac{a \times dt}{h} = \dfrac{1 \times 0.015}{0.02} = 0.75.$

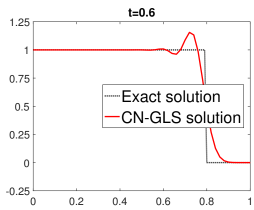
Solve using CN in time and Galerkin in space.



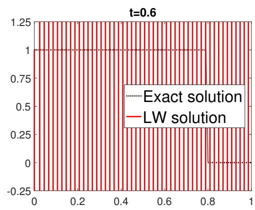
CN - time, G - in space
Fig 1



CN - time, G - in space
Fig 2



CN - time, GLS- in space
Fig 3

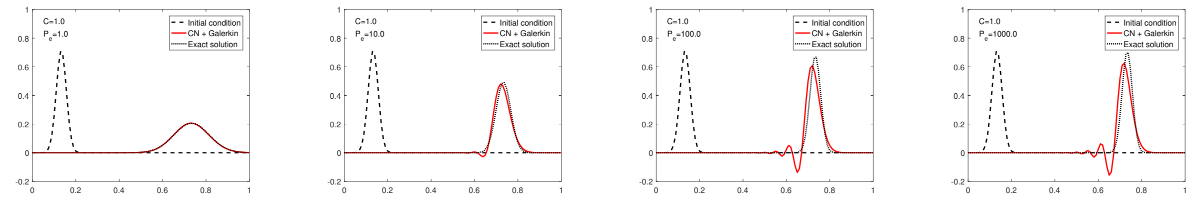


LW Method
Fig 4

As we can see from the plots above, it is clear that the Lax Wendroff method is unstable and it is inadequate in producing a solution. This is because the Courant number is higher than the stability condition value.
As for the other implementations presented, namely Crank Nicoleson in time, Galerkin in space and Crank Nicloeson in time, Galerkin Least Square in space. We can clearly identify the oscillatory behaviour induced due to Galerkin formulation (fig 2), because of which the numerical solution obtain is not good. We can the compare it with the solution obtained from CN-GLS (fig 3) where the stabilization technique has obviously worked and we get a stable upper part. Although the numerical solution obtained here is not as close to the exact solution, it is better than one obtained by CN-G.

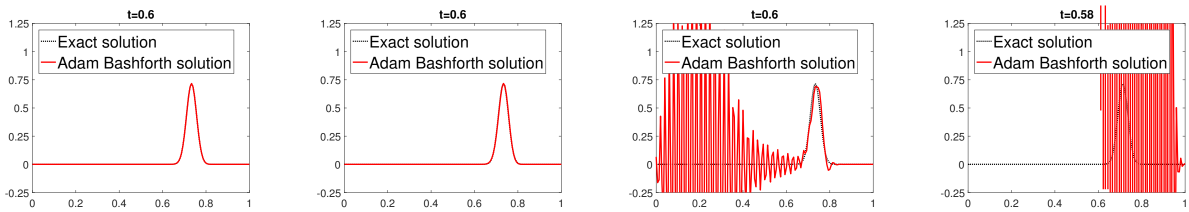
3 Part 3

Test the code for different viscosity proposed and comment the numerical results.
It is known that diffusion coefficient is inversely proportional to viscosity. Hence, as viscosity increases diffusion coefficient decrease and as viscosity decreases diffusion coefficient increases. Also, as we know that the Peclet number is affected inversely diffusion coefficient, we can say that Peclet number is proportional to viscosity. To study the effects of variation of viscosity on the numerical solution, we solve the problem by using Crank Nicoleson descritization in time and Galerkin descritization in space. The Courant number is constant for all the cases.



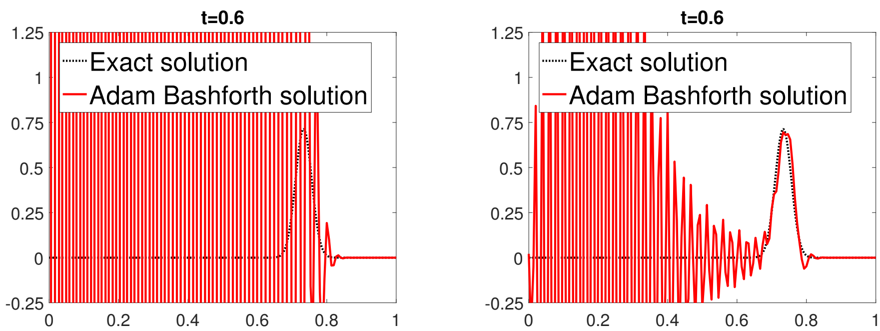
It is clear from the results that with lower viscosity values(thus resulting in lower Peclet numbers), the numerical solution achieved is better. As the value of viscoity is increased, we can clearly observe that the solution achieved is not stable.

The Adams-Bashforth scheme in time and Galerkin in space was implemented and the following results were obtained.



C = 0.08333 C = 0.15 C = 0.375 C = 0.75

It can be clearly observed that for lower Courant numbers the numerical solution Prodcued by Adam's Bashforth method in time and Galerkin in space, is better, and quiet accurate. As we increase the Courant number it can be seen that their is instability induced. And for C = 0.75 which above the stability limit of Adam's Bashforth, the numerical solution achieved is unstable and inaccurate.
Now we will see the effects of change in Peclet number on the above mentined methods implemented. The COurant number is same for the cases studied here.



nu $= 6.7e-04$ nu $= 3.3e-06$

The method is totally unstable for the first value of ν provided, and no solution is obtained on the plots. But for that later values of ν we can clearly observe that as the values reduces we get a better solution. But still this solution is not good compared to other methods.We can get a better solution by decreasing the Courant number further.