The Gaussian Hill

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1. The influence of the value of viscosity

In this part, I have chosen three methods and they are CN-Galerkin, CN-LS and CN-SUPG.

(1) CN-Galerkin method

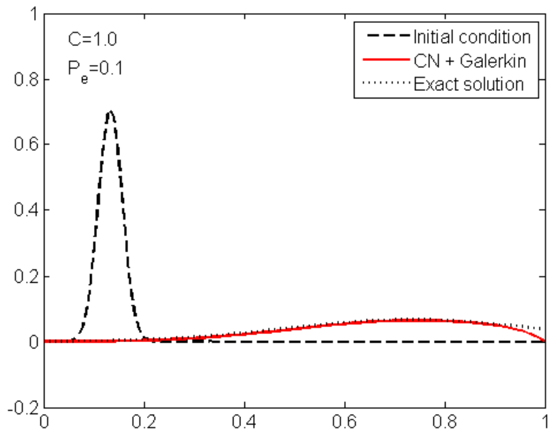


Figure 1. The result of CN-Galerkin with C=1 and Pe=0.1

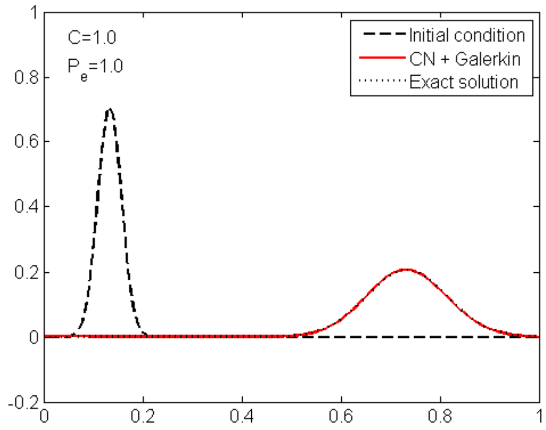


Figure 2. The result of CN-Galerkin with C=1 and Pe=1.0

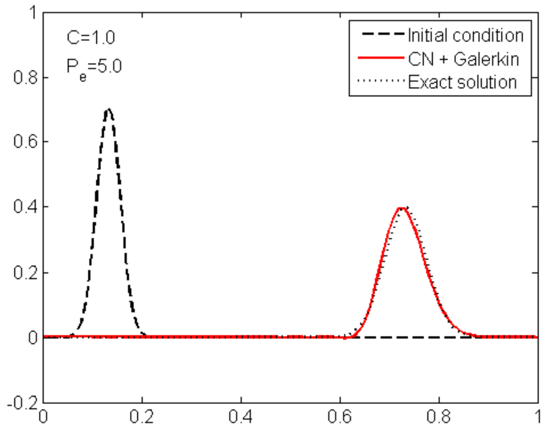


Figure 3. The result of CN-Galerkin with C=1 and Pe=5.0

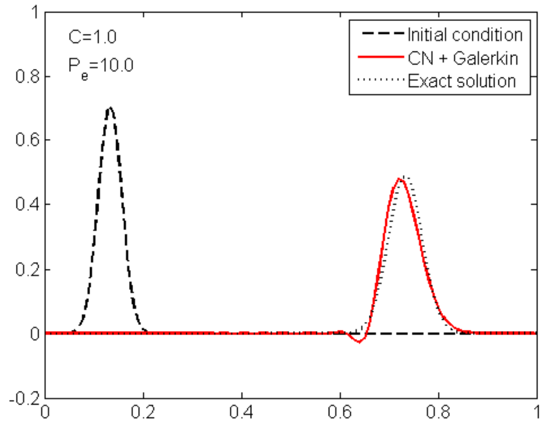


Figure 4. The result of CN-Galerkin with C=1 and Pe=10

As we can see, when the value of Pe increases which means that the viscosity is small, the curve will be more and more steep. Because the boundary conditions will lead to internal or boundary layers in advective problems and the time integration need to be able to present the role of characteristics, so we need appropriate time schemes and stabilization. In above figures, the result will be inaccurate when Pe increases. When Pe is large, the advective influence will be remarkable. Consequently, above problems have caused the inaccuracies.

(2) CN-SUPG method

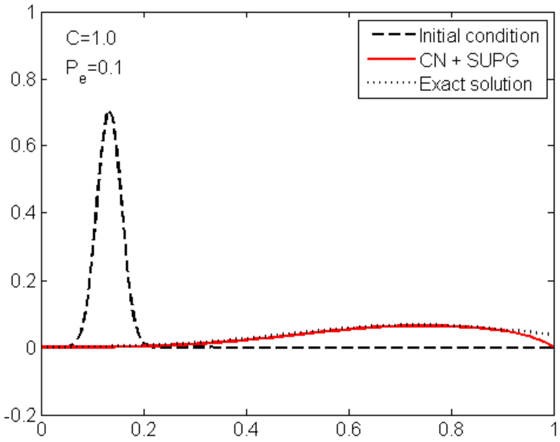


Figure 5. The result of CN-SUPG with C=1 and Pe=0.1

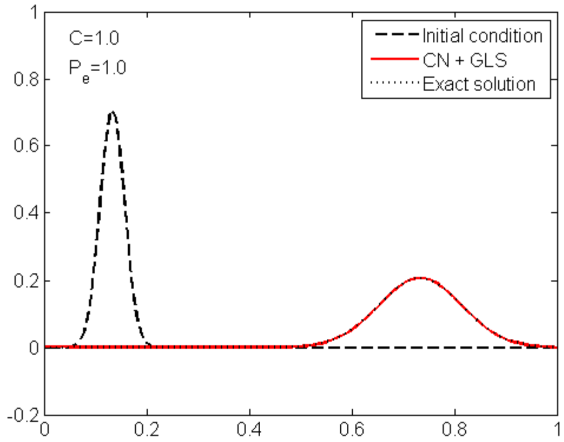


Figure 6. The result of CN-SUPG with C=1 and Pe=1.0

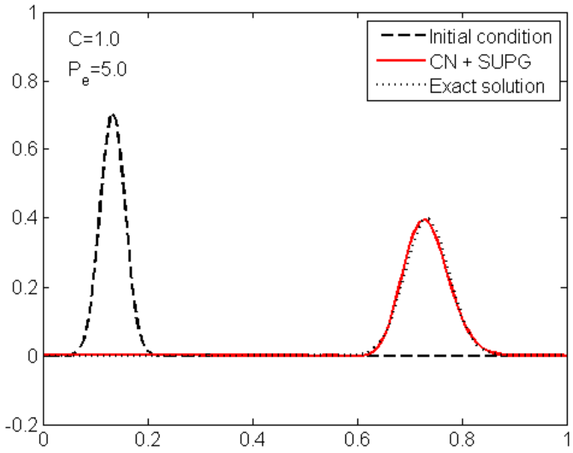


Figure 7. The result of CN-SUPG with C=1 and Pe=5.0

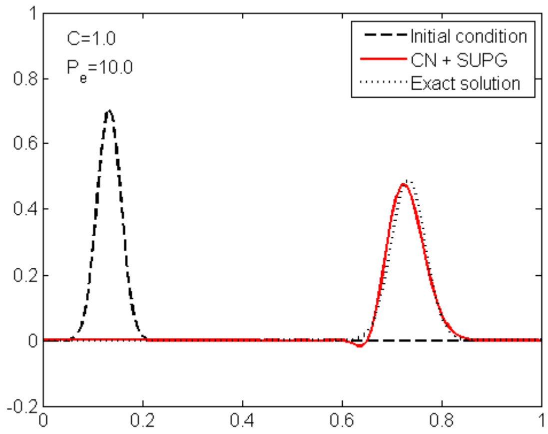
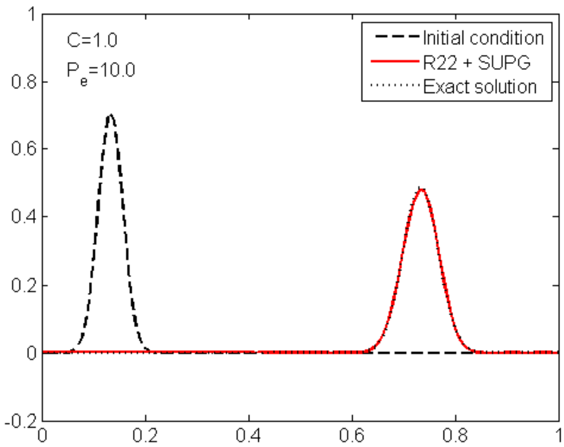


Figure 8. The result of CN-SUPG with C=1 and Pe=10

By using the stabilization, the results do not show much improvements.
We need to change the time schemes to make the results better.

(3) R22-SUPG and R33-SUPG



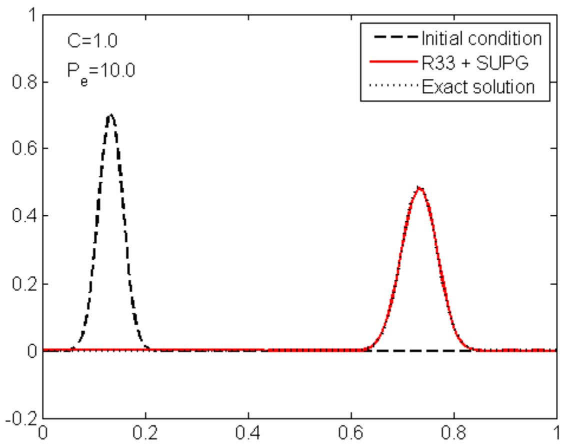
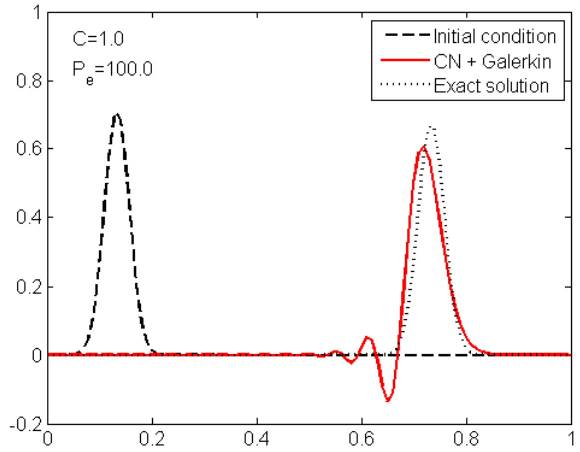


Figure 9. The result of R22-SUPG with C=1 and Pe=10

Figure 10. The result of R33-SUPG with C=1 and Pe=10

By using R22 time schemes and SUPG stabilization, we have remarkably improved the accuracies and the spurious oscillations also disappear.



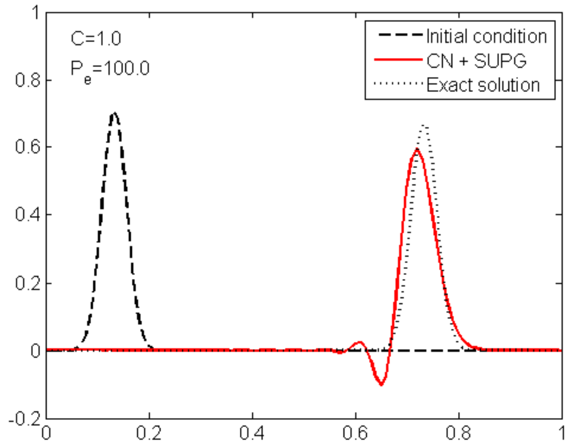
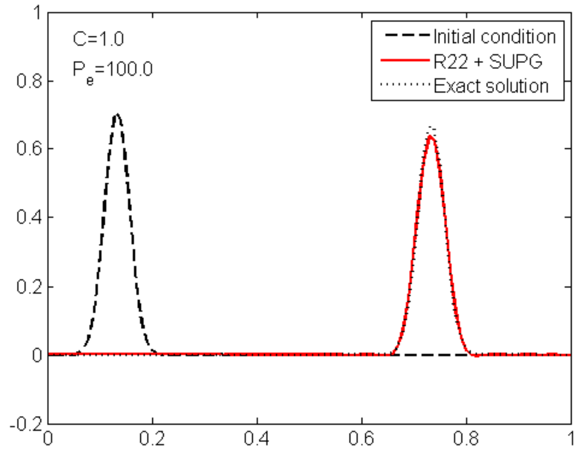


Figure 11. The result of CN-Galerkin with C=1 and Pe=100

Figure 12. The result of CN-Galerkin with C=1 and Pe=100

When we increase the Pe to 100 which is very high, the CN-Galerkin becomes more unstable and the effect of CN-SUPG is limited.



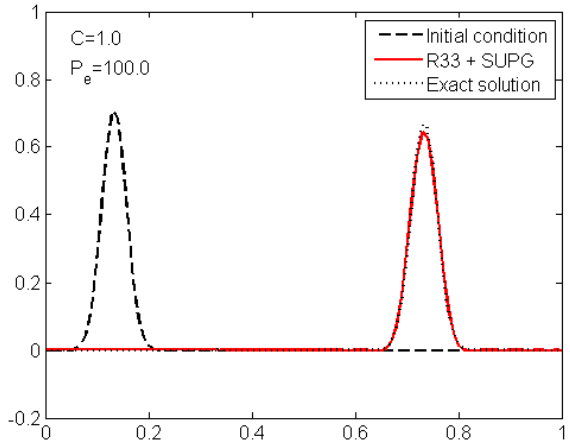


Figure 13. The result of R22-SUPG with C=1 and Pe=100

Figure 14. The result of R33-SUPG with C=1 and Pe=100

But R22-SUPG and R33-SUPG still work very well even though Pe is very high.
As a conclusion, in unsteady convection diffusion problems, we need to consider both time schemes and stabilization.

2. R22-Galerkin method

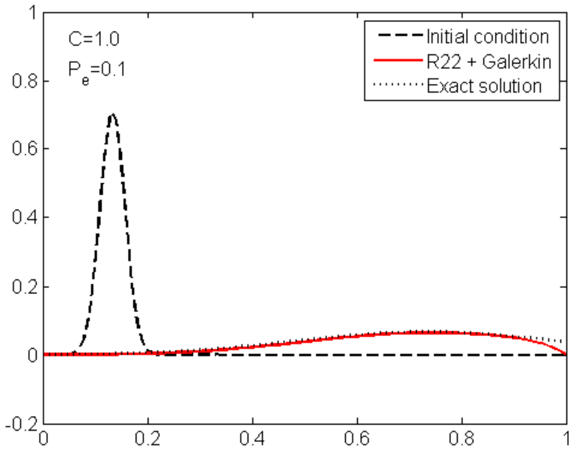


Figure 9. The result of R22-Galerkin with C=1 and Pe=0.1

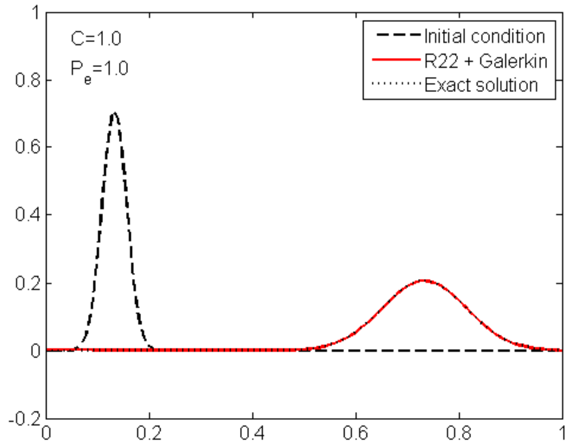


Figure 10. The result of R22-Galerkin with C=1 and Pe=1.0

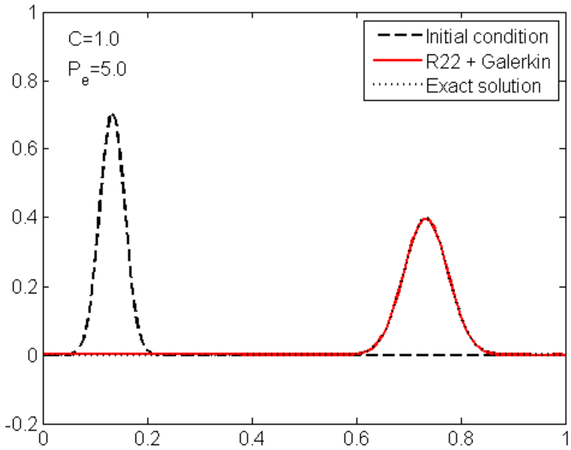


Figure 9. The result of R22-Galerkin with C=1 and Pe=5.0

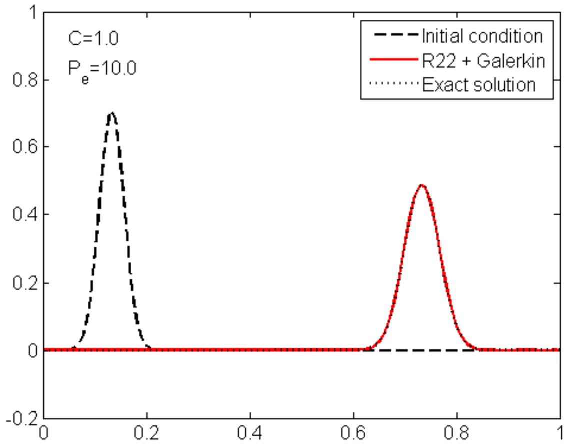


Figure 10. The result of R22-Galerkin with C=1 and Pe=10.0

The R22-Galerkin method only has implemented appropriate time schemes and it works very well. Its effect on improving behavior of the result is better than only using stabilization.

3. The time-discontinuous Galerkin formulation of the convection-diffusion equations

$$\left(\boldsymbol{M}+\frac{2}{3}\Delta t\boldsymbol{C}+\frac{2}{3}\nu\Delta t\boldsymbol{K}\right)\boldsymbol{u}^{n+1}-\left(\boldsymbol{M}-\frac{1}{3}\Delta t\boldsymbol{C}-\frac{1}{3}\nu\Delta t\boldsymbol{K}\right)\boldsymbol{u}^{n+}=0$$
$$\left(\boldsymbol{M}+\frac{1}{3}\Delta t\boldsymbol{C}+\frac{1}{3}\nu\Delta t\boldsymbol{K}\right)\boldsymbol{u}^{n+1}+\left(\boldsymbol{M}+\frac{1}{3}\Delta t\boldsymbol{C}+\frac{1}{3}\nu\Delta t\boldsymbol{K}\right)\boldsymbol{u}^{n+}=2\boldsymbol{M}\boldsymbol{u}^{n-}$$