2D Steady Transport Problem

Lei Pan

1. Problem 1

$$\mathbf{a} = (\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4})$$
$$v = 0.0001$$
$$\sigma = 1$$

The The Peclet number: **Pe** $= 500$

In problem 1, the velocity is much bigger than the value of diffusion and the Peclet number is very high, so we can regard this situation as pure convection problem.
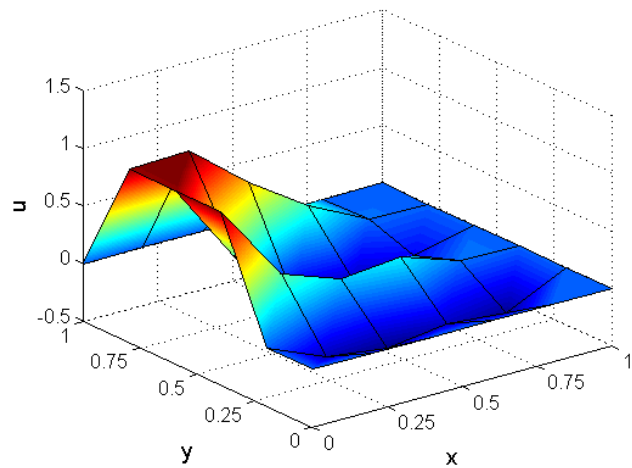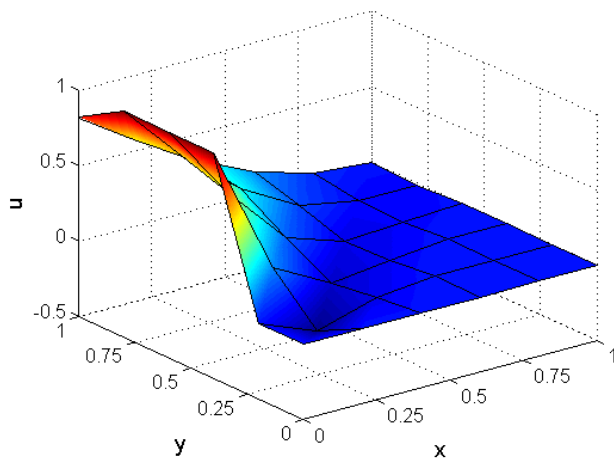
(1) Galerkin Method

Figure 1. u field of Galerkin Method with Neumann BCs   Figure 2. u field of Galerkin Method with zero Dirichlet BCs

When Neumann BCs are imposed on the outlet boundaries, the Galerkin method works well. But in the situation of applying zero Dirichlet BCs on the outlet boundaries, the Galerkin method shows spurious oscillations and inaccuracy.
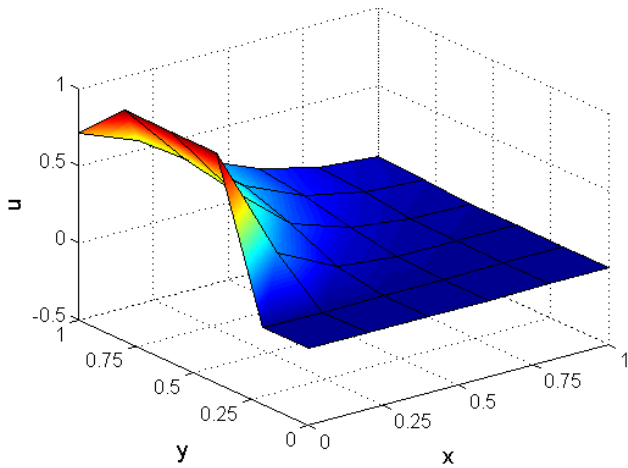
(2) Artificial Diffusion Method

Figure 3. u field of AD Method with Neumann BCs

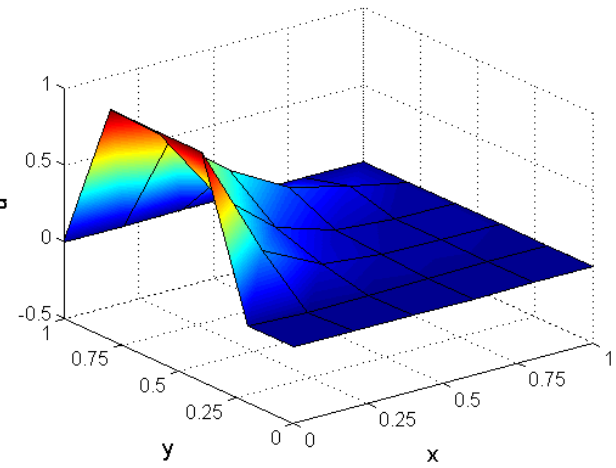

Figure 4. u field of AD Method with zero Dirichlet BCs

The Artificial Diffusion Method has added artificial diffusion which can make the Galerkin method more stable. And the oscillations happened in Galerkin method when the zero Dirichlet BCs are imposed on the outlet boundaries disappear in Artificial Diffusion Method. The Artificial Diffusion Method works well in both situations. Bu in 2D situation, the artificial diffusion is only added on the streamline direction which can not promise that the result is smooth on all directions. Consequently, the stabilization parameter methods will be better choices. And another problem of artificial diffusion method is that it has introduced excessive diffusion.
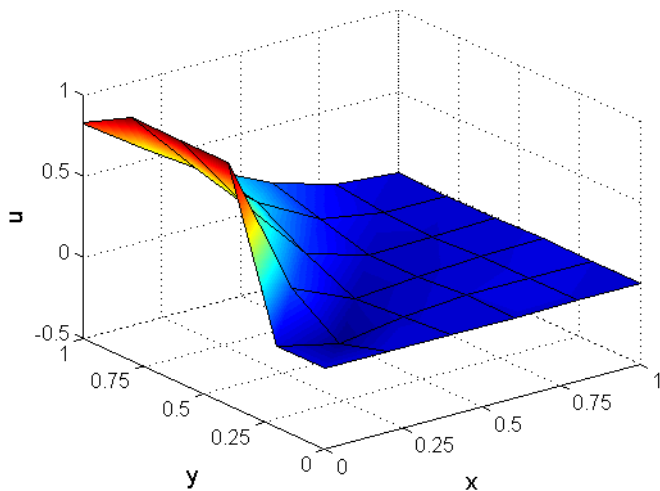
⑶ **SUPG**
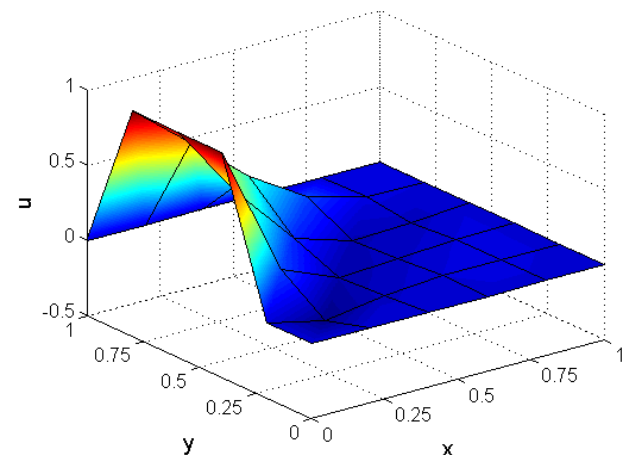

Figure 5. u field of SUPG Method with Neumann BCs

Figure 6. u field of SUPG Method with zero Dirichlet BCs

The SUPG method is different from Artificial Diffusion Method when it coms to making the result stable. The SUPG method has added the stabilization parameter to avoid the oscillations. And the result of SUPG is smooth in all directions.
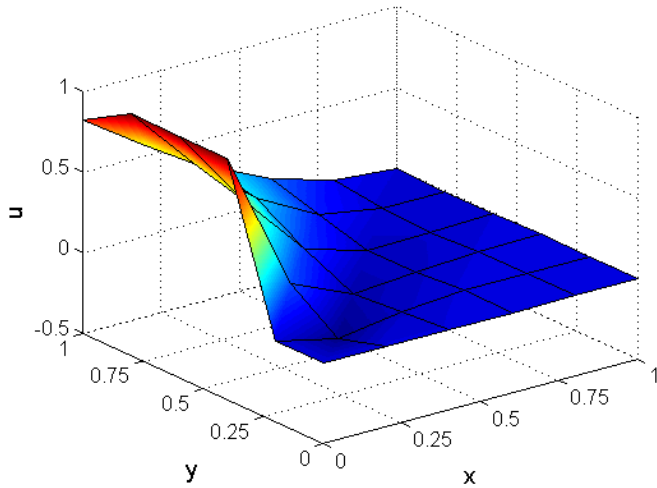
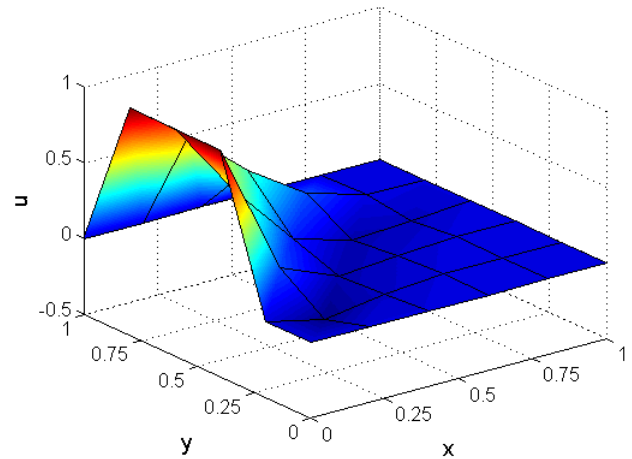## ⑷ **GLS**

Figure 7. u field of GLS Method with Neumann BCs　　Figure 8. u field of GLS Method with zero Dirichlet BCs

The drawback of SUPG is that the added stabilization term is not symmetric and this drawback will produce difficulties in establishing the stability. In order to solve this problem, the GLS method has added symmetric stabilization term. But when applying the linear element, SUPG and GLS are equal. From the results, we can see that GLS works as well as SUPG in both situations.

2. Problem 2

$$\mathbf{a} = (\sqrt{2}/2 \times 10^{-3}, \sqrt{2}/2 \times 10^{-3})$$

$$\nu = 0.0001$$
$$\sigma = 1$$

The Peclet number: $\mathbf{Pe} = 1$

In problem 2, the velocity value is close to diffusion value, so the influence of diffusion part will be important.
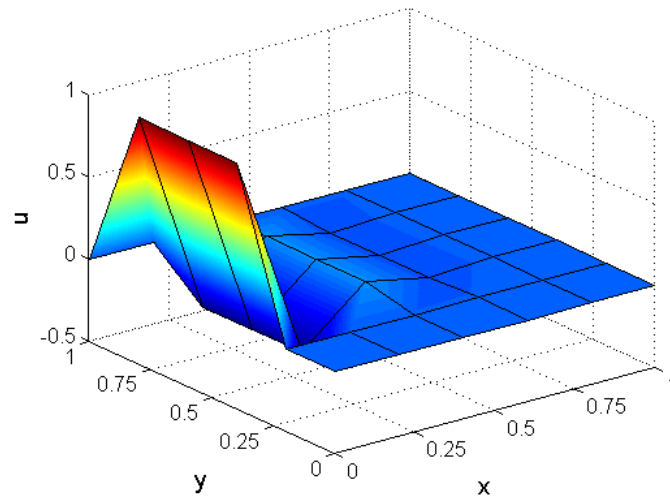
(1) Galerkin Method

Figure 9. u field of Galerkin Method with zero Dirichlet BCs

For problem 2, the Galerkin method doesn't work well. The result has spurious oscillations.
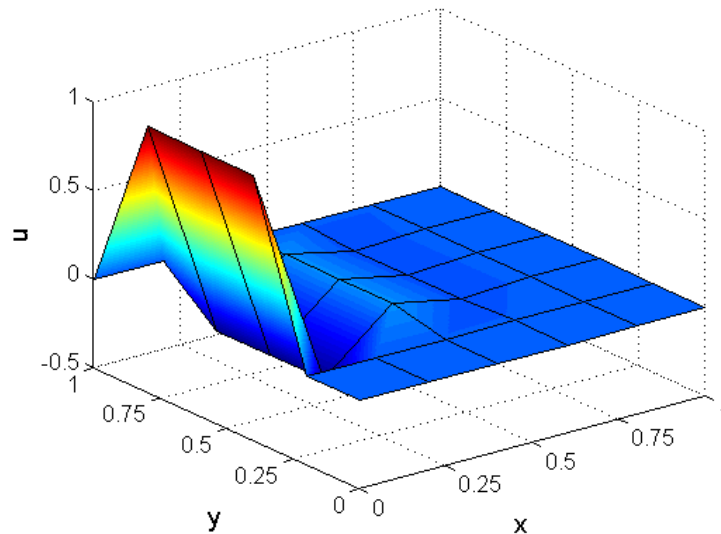
⑵ **Artificial Diffusion Method**

Figure 10. u field of AD Method with zero Dirichlet BCs ($\bar{v} = 3.2273e - 11$)

The Artificial Diffusion method doesn't solve the problem of spurious oscillations that has happened in Galerkin method. According to the recommended artificial diffusion value is 3.2273e-11, which can be neglected compared to the diffusion value. As a result, the effect of artificial diffusion is small.
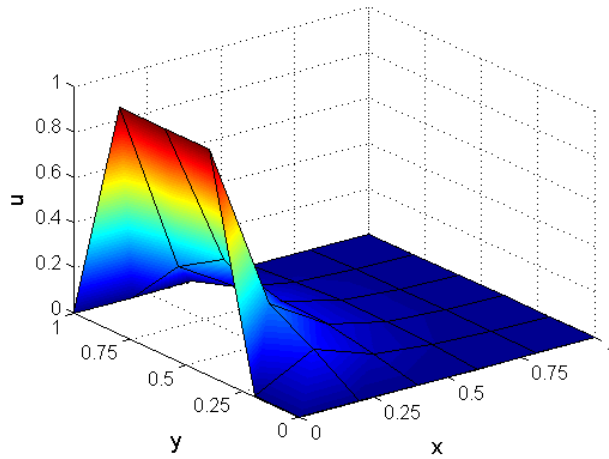
Figure 11. u field of AD Method with zero Dirichlet BCs ($\bar{v} = 0.05$)

When I change the artificial diffusion value to 0.05, the Artificial Diffusion method works well and there are no spurious oscillations.
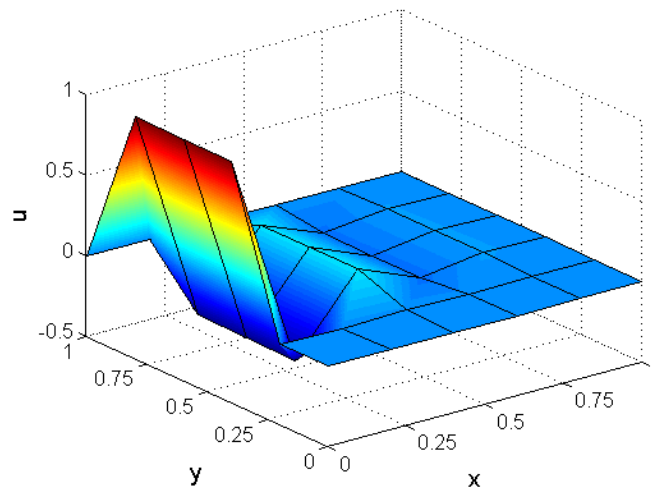
⑶ SUPG

Figure 12. u field of SUPG Method with zero Dirichlet BCs

The SUPG method also works badly when using the recommended stabilization parameter which is equal to 31.6228. The result also shows the spurious oscillations. When I change the stabilization parameter, the result still shows oscillation. So the best stabilization parameter is needed to put more researches.
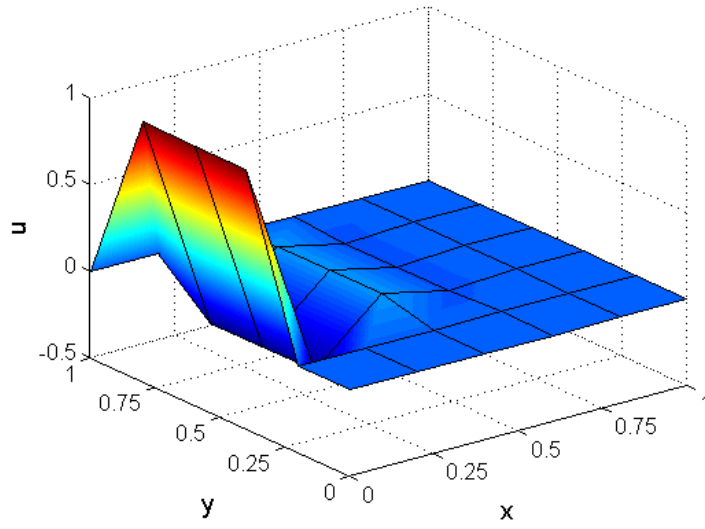
⑷ GLS

Figure 13. u field of GLS Method with zero Dirichlet BCs

The GLS is same to SUPG.

3. The modification of the code

```
if method == 0
    % Galerkin
    Ke = Ke + (nu*(Nx'*Nx+Ny'*Ny) + N_ig'*(ax*Nx+ay*Ny)+N_ig'*N_ig)*dvolu;
    aux = N_ig*Xe;
    f_ig = SourceTerm(aux);
    fe = fe + N_ig'*(f_ig*dvolu);
elseif method == 1
    % Artifficial diffusion
    Ke = Ke + (nu*(Nx'*Nx+Ny'*Ny) + N_ig'*(ax*Nx+ay*Ny)+N_ig'*N_ig)*dvolu;
    aux = N_ig*Xe;
    f_ig = SourceTerm(aux);
    fe = fe + N_ig'*(f_ig*dvolu);
elseif method==2
    % SUPG
    Ke = Ke + (nu*(Nx'*Nx+Ny'*Ny) + N_ig'*(ax*Nx+ay*Ny)+N_ig'*N_ig + ...
        tau*(ax*Nx+ay*Ny)'*((ax*Nx+ay*Ny)+N_ig))*dvolu;
    aux = N_ig*Xe;
    f_ig = SourceTerm(aux);
    fe = fe + (N_ig+tau*(ax*Nx+ay*Ny))'*(f_ig*dvolu);
else
    %GLS
    Ke = Ke + (nu*(Nx'*Nx+Ny'*Ny) + N_ig'*(ax*Nx+ay*Ny)+N_ig'*N_ig + ...
    tau*((ax*Nx+ay*Ny)+N_ig)'*((ax*Nx+ay*Ny)+N_ig))*dvolu;
    aux = N_ig*Xe;
    f_ig = SourceTerm(aux);
    fe = fe + (N_ig+tau*((ax*Nx+ay*Ny)+N_ig))'*(f_ig*dvolu);
end
```

Figure 14. The modification of the code

I have added the σ term and GLS method in the code.