Finite Elements in Fluids

Assignment-2

Assignment on ‘‘Viscous Incompressible Flows’’

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Cavity Flow Problem

The cavity flow problem is a standard benchmark test for incompressible flows. The goal of this exercise is to analyse the results obtained when adopting either the Stokes or the Navier-Stokes equations. Using the code in (*HW2Files-Cavity*) to compute the Finite elements approximation of these problems, the questions of this assignment are answered.

a) Using the script *mainStokes.m* the solution of the Stokes problem is computed using a uniform, structured mesh of Q_2Q_0, Q_2Q_1, P_1P_1 and MINI ($P_1^+P_1$)) elements, with 20 elements per side. The following figures (Fig. 1 to Fig. 16) illustrates the plots of meshes and the results. Mesh for velocity is shown in figure 1 to 4. And figure for pressure is shown in Figure 5 to 8.

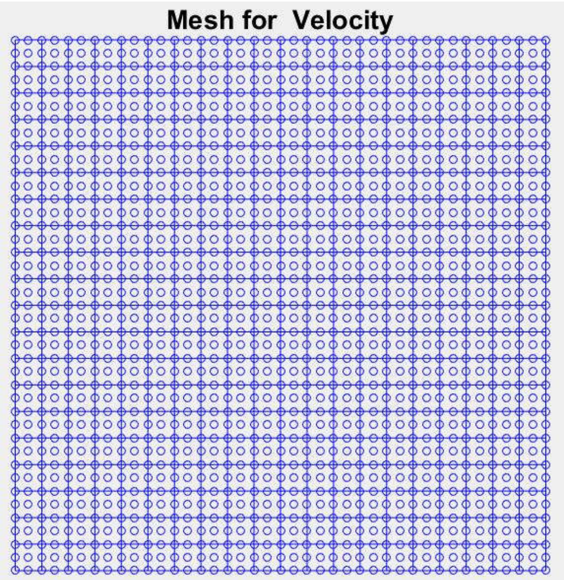

Figure 1. Mesh For Velocity of Q_2Q_0

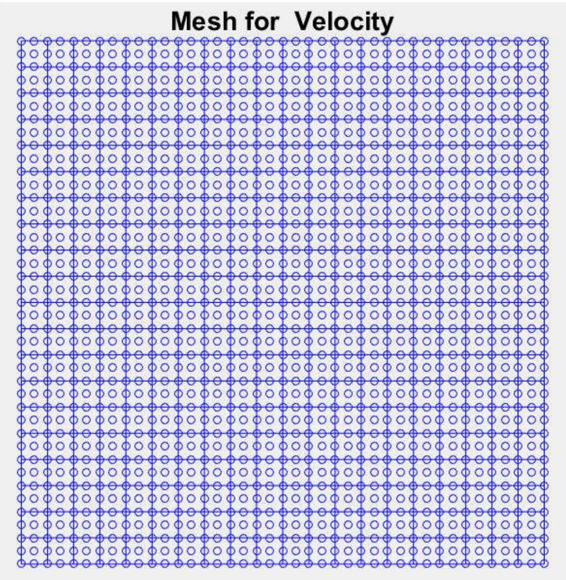

Figure 2. Mesh For Velocity of Q_2Q_1

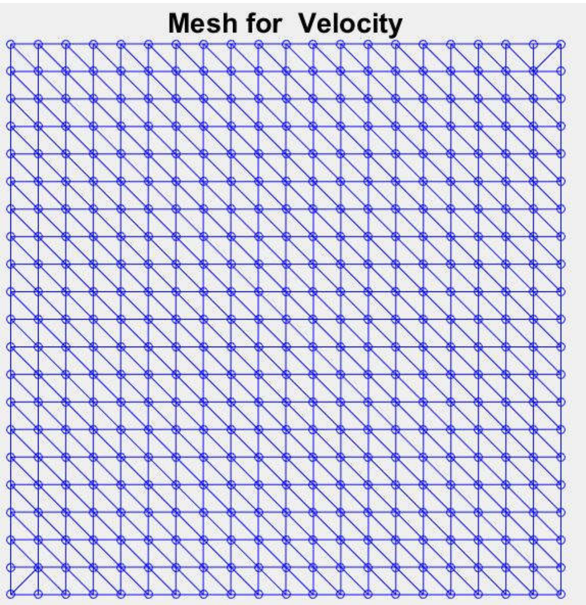

Figure 3. Mesh For Velocity of P_1P_1

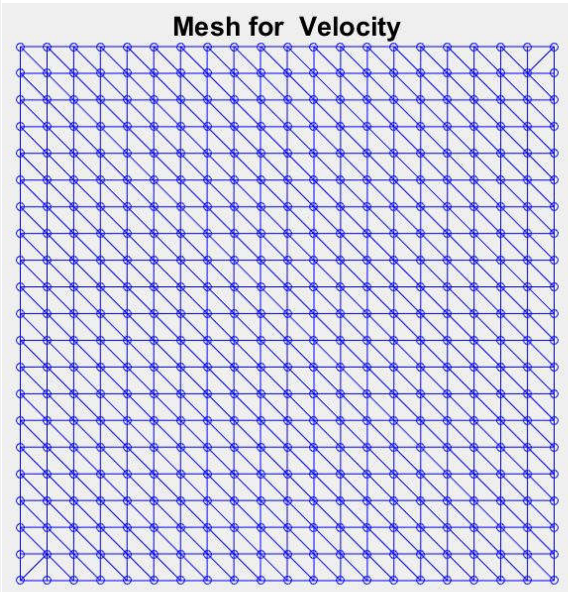

Figure 4. Mesh For Velocity for MINI($P_1^+P_1$)

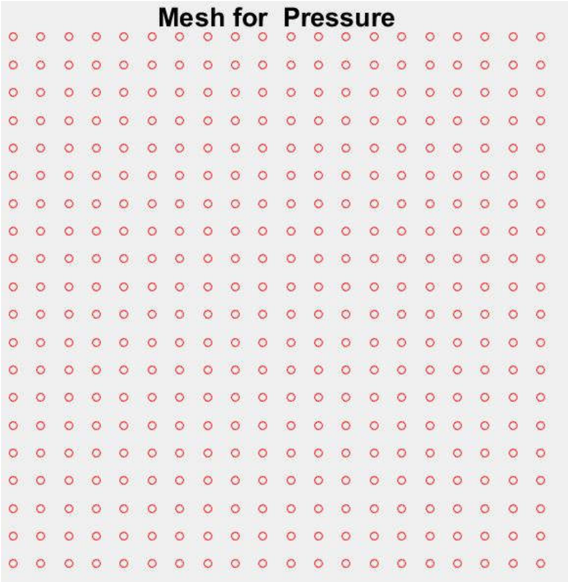

Figure 5. Mesh For Pressure of Q_2Q_0

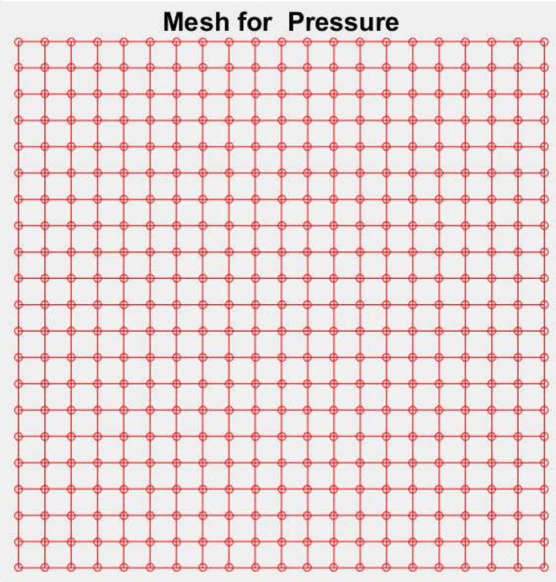

Figure 5. Mesh For Pressure of Q_2Q_1

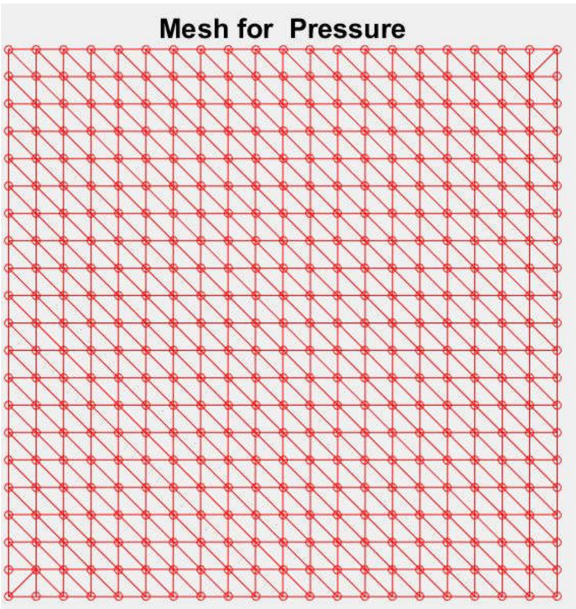

Figure 7. Mesh For Pressure of P_1P_1

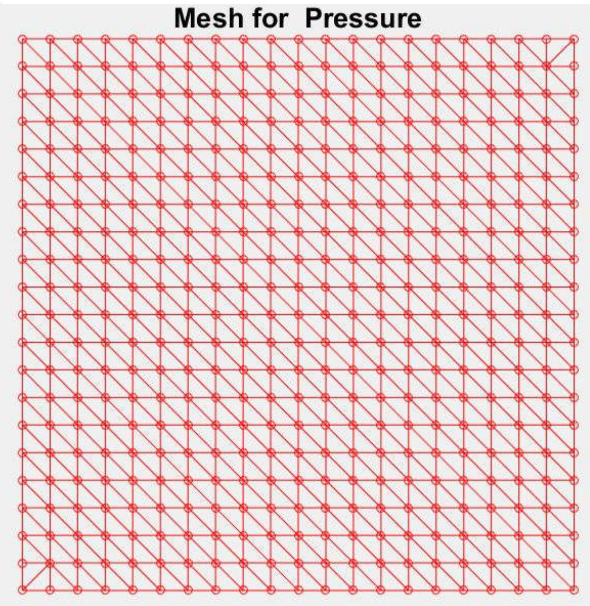

Figure 8. Mesh For Pressure of MINI ($P_1^+P_1$)

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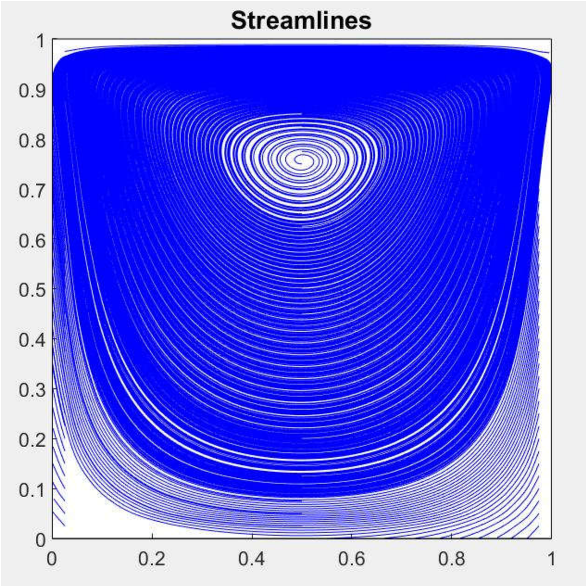


Figure 9. Streamlines of Q_2Q_0

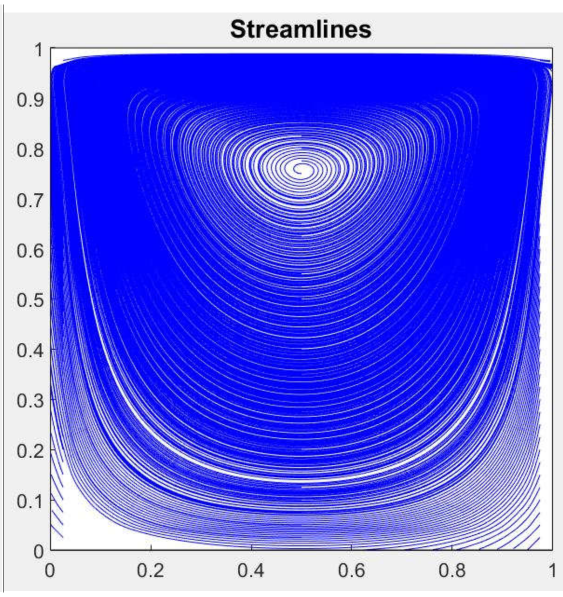


Figure 10. Streamlines of Q_2Q_1

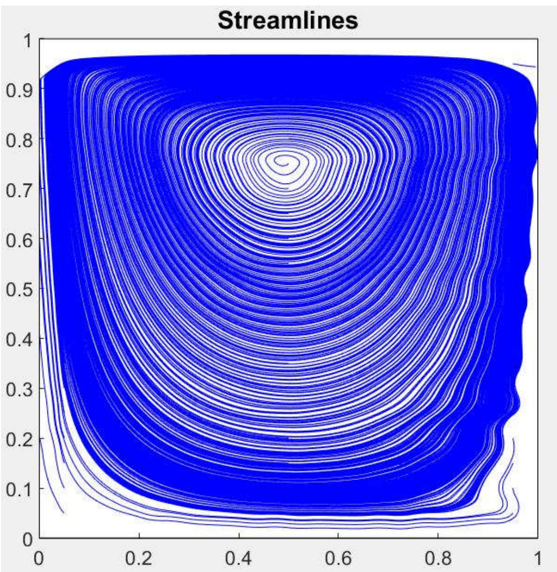


Figure 11. Streamlines of P_1P_1

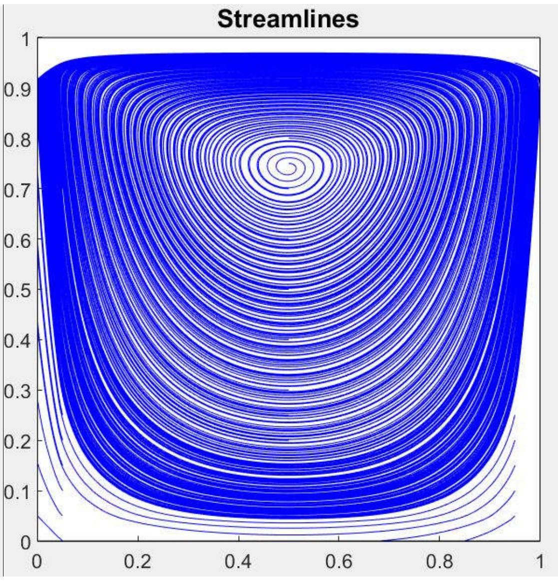
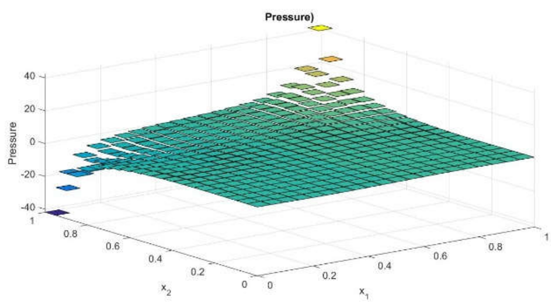


Figure 12. Streamlines of MINI ($P_1^+P_1$)

Figures 9 to 12 shows the streamlines for all four elements. For Q_2Q_0 and Q_2Q_1 the streamlines are smoother and better, as the velocity is approximated using biquadratic interpolation polynomials. For solution using P_1P_1 element (Figure 10) there is slight oscillations near the boundaries, owing to linear approximation of velocity. This problem is overcome in $P_1^+P_1$ due to the cubic bubble function



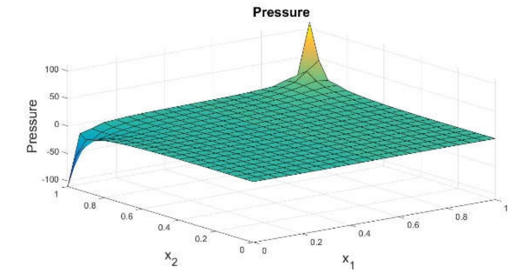
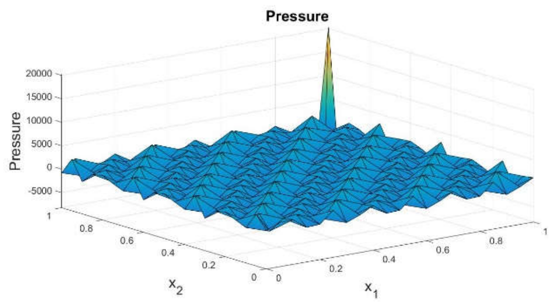


Figure 13. Pressure Field for Q_2Q_0

Figure 14. Pressure Field for Q_2Q_1



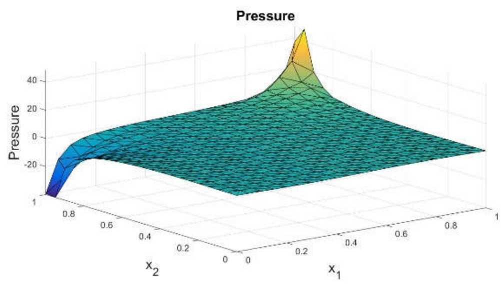


Figure 15. Pressure Field for P_1P_1

Figure 16. Pressure Field for MINI ($P_1^+P_1$)

The above four graphs illustrates the solution for pressure. In Q_2Q_0 (Figure 13) pressure is discontinuous between elements, this is due to constant pressure approximation. There are spurious oscillations in solution with P_1P_1, owing to the linear approximation in triangular elements. In solutions from Q_2Q_1 (Figure 14) and MINI ($P_1^+P_1$) (Figure 16) , which LBB, pressure is continuous and there are no spurious oscillations.

b) The solution of the Stokes problem is computed considering, (i) a structured, uniform mesh of Q_2Q_1 elements with 20 elements per side, (ii) a structured mesh of 20 ×20 Q_2Q_1elements refined near the walls. The meshes and results are compared in following figures (17 to 24).

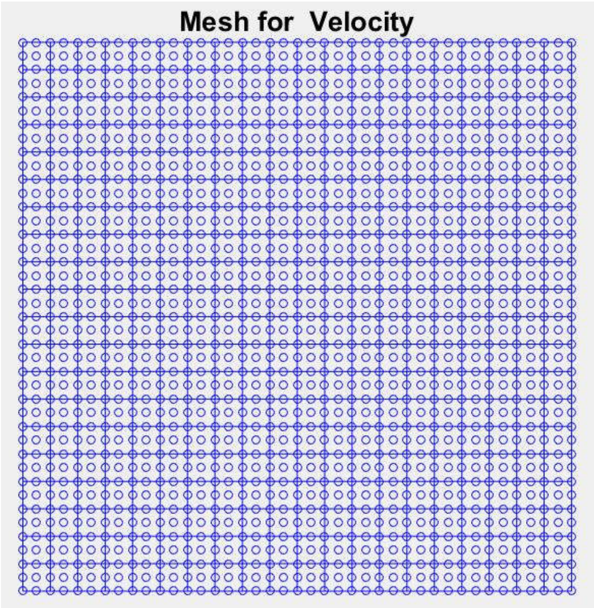


Figure 17. For of Q_2Q_1 (uniform mesh)

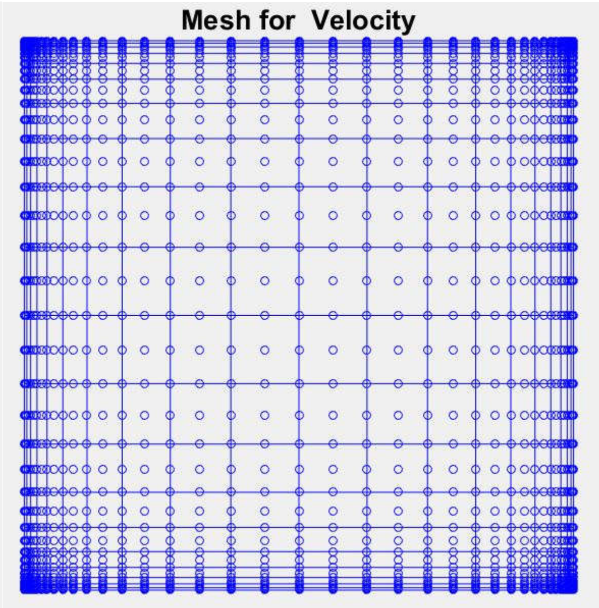


Figure 18. For of Q_2Q_1 (refined mesh)

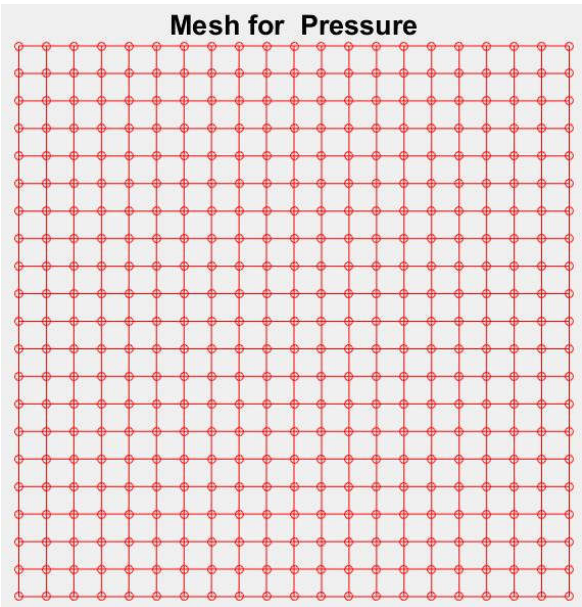


Figure 19. For of Q_2Q_1 (uniform mesh)

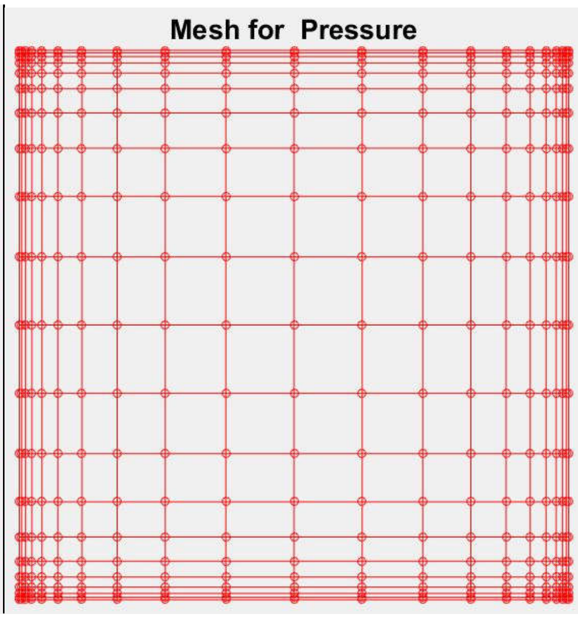


Figure 20. For of Q_2Q_1 (refined mesh)

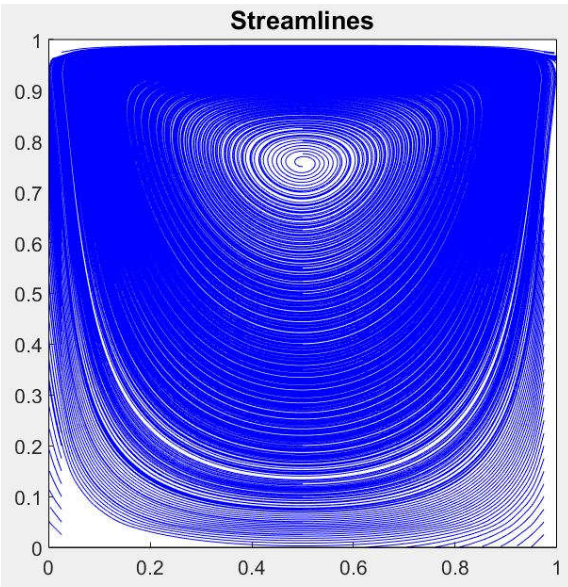


Figure 17. Streamlines of Q_2Q_1 (uniform mesh)

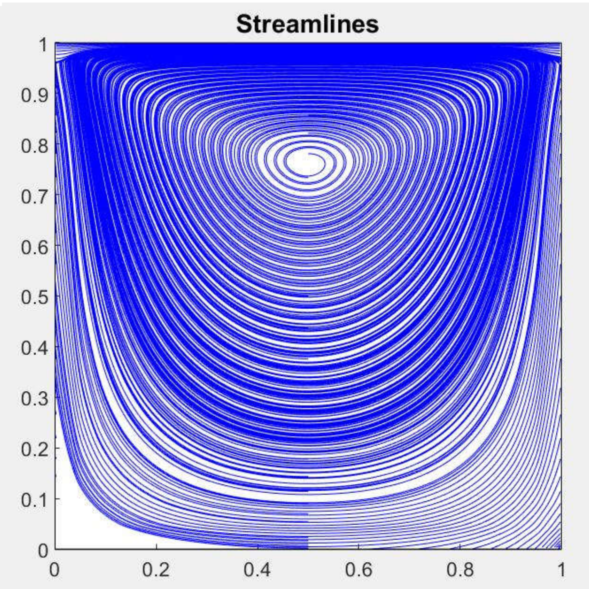


Figure 18. Streamlines of Q_2Q_1 (refined mesh)

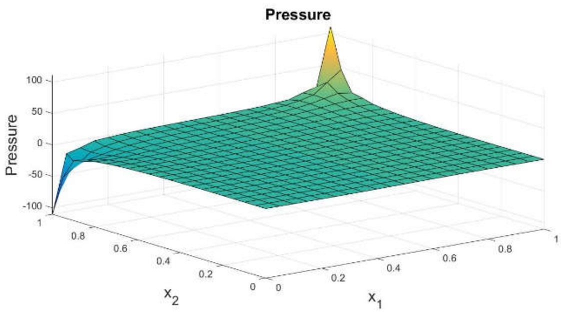


Figure 19. Pressure of Q_2Q_1 (uniform mesh)

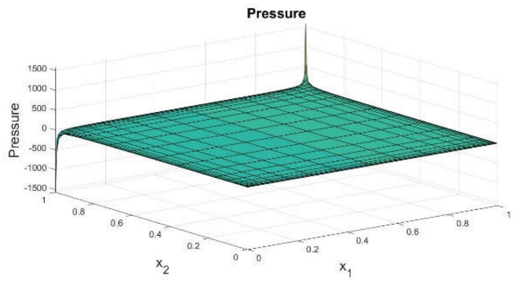


Figure 20. Pressure of Q_2Q_1 (refined mesh)

Consider the results. There is not much difference in velocity fields or streamlines. Whereas pressure is more refined at the boundaries. The results are improved in adaptive mesh (refined mesh). Especially, at the boundaries solution is better captured in adaptive mesh. The pressure at the boundaries is of the range of ±1500, while it is of the range of ±100 in the case of the uniform mesh. Since, the results are accurate with adaptive mesh and computational costs are same both the methods, adaptive meshing is best option. Considering practical aspects, in many problems, boundaries are the most critical regions and hence mesh is to be refined at boundaries. Therefore, adaptive mesh is better option for such applications.

c) The Stokes code is modified to solve the problem using a GLS stabilized formulation with P_1P_1 elements. The stabilization of the Stokes problem is obtained by adding stabilisation term to the Galerkin weak form of the Stokes equations. The following stabilisation terms are considered,

$$\begin{cases} (-\nu\nabla^2 \boldsymbol{w}, -\nu\nabla^2\boldsymbol{v} + \nabla p - \boldsymbol{b}) = 0 & \forall \boldsymbol{w} \in \boldsymbol{\mathcal{V}} \\ (\nabla q, -\nu\nabla^2\boldsymbol{v} + \nabla p - \boldsymbol{b}) = 0 & \forall q \in \mathcal{Q} \end{cases}$$

The reduced GLS form is given by: find $v^h \in \mathcal{S}^h$ and $p^h \in \mathcal{Q}^h$, such that, for all $(w^h, q^h) \in \mathcal{V}^h \times \mathcal{Q}^h$,

$$\begin{cases} \boldsymbol{a}(\boldsymbol{w}^h, \boldsymbol{v}^h) + \boldsymbol{b}(\boldsymbol{w}^h, p^h) = (\boldsymbol{w}^h, \boldsymbol{b}^h) + (\boldsymbol{w}^h, \boldsymbol{t}^h)_{\Gamma_N}, \\ \boldsymbol{b}(\boldsymbol{v}^h, q^h) - \sum_{e=1}^{n_{el}} \tau_e (\nabla q^h, \nabla p^h)_{\Omega^e} = -\sum_{e=1}^{n_{el}} \tau_e (\nabla q^h, \boldsymbol{b}^h)_{\Omega^e} \end{cases}$$

The stabilization parameter chosen is,

$$\tau_e = \alpha \frac{h_e^2}{4\nu}$$

Where,

$\alpha = 1/3$ is optimal for linear elements.

h_e is measure of element size.

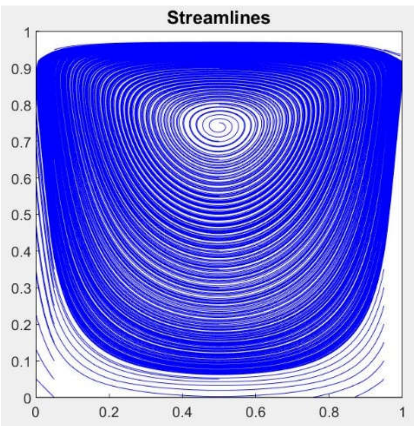


Fig 21. Streamlines for GLS (P_1P_1)

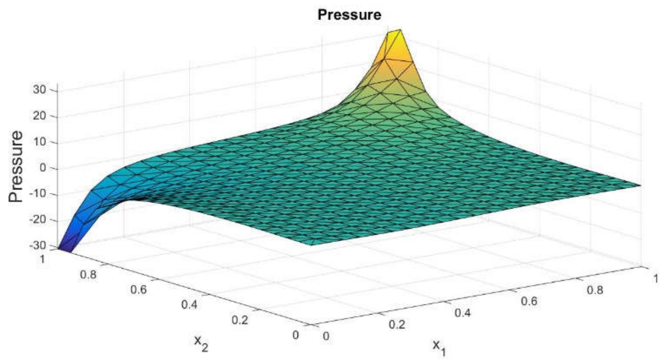


Figure 22. Pressure for GLS with P_1P_1 elements

Above two figures illustrates the results using GLS stabilised formulation for P_1P_1 element. Comparing with the galarking form (Fig.11 & Fig.15), the velocity and pressure solution are improved by GLS stabilisation for P_1P_1 elements. The oscillations near the boundary are reduced and the spurious oscillations of pressure are absent in the stabilised solutions. The optimal value $\alpha = 1/3$, is considered for plotting.

The following plots (Fig. 23 to Fig. 28) compare the results for higher values of 'α' or stabilisation parameter 'τ'. From the below graphs we can observe that for higher value stabilisation parameter the solution is deviated from the actual solution. Hence, smaller and optimal value of the stabilisation parameter is to be chosen for accurate results.

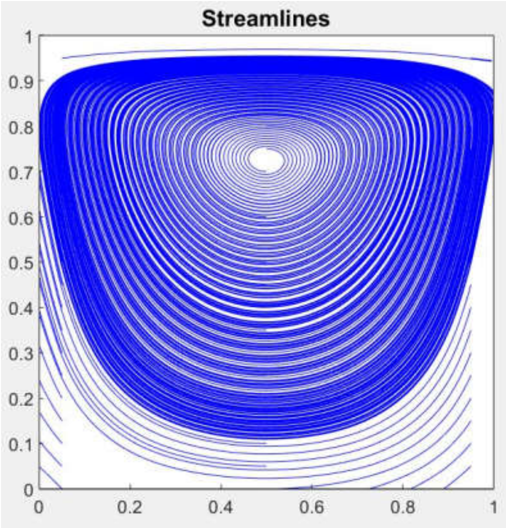


Fig 23. Streamlines for GLS (α=3)

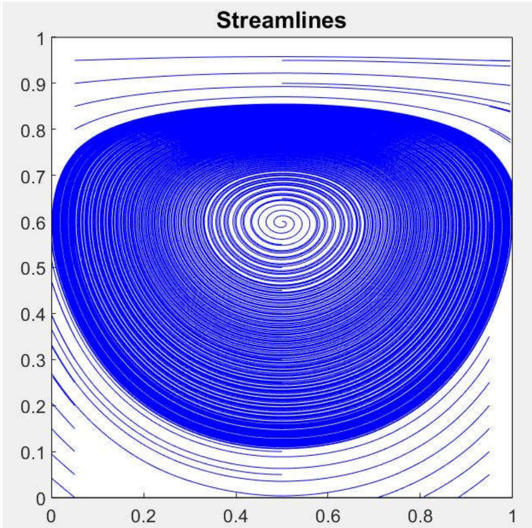


Fig 24. Streamlines for GLS (α=30)

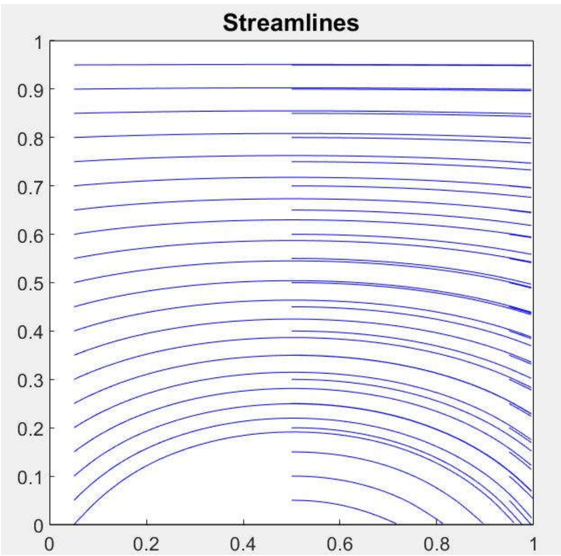


Fig 25. Streamlines for GLS (α=300)

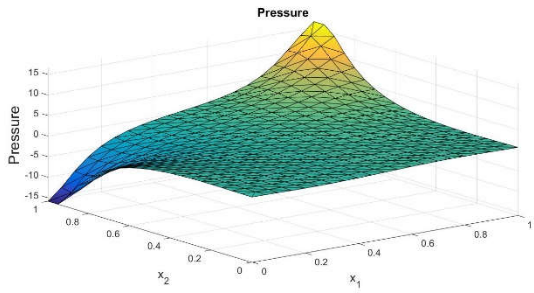


Fig 26. Pressure output for GLS (α=3)

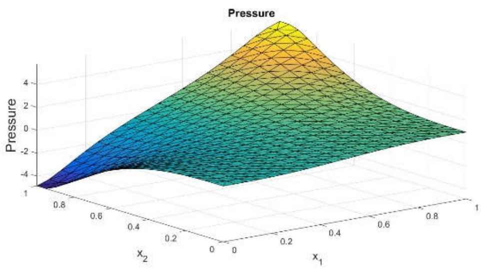


Fig 27. Pressure output for GLS (α=30)

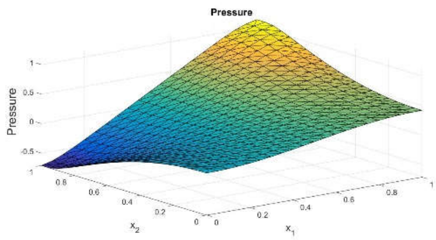


Fig 28. Pressure output for GLS (α=300)

d) The script *mainNavierStokes.m* is used to solve the Navier-Stokes equations with Picard method. Matlab function *ConvectionMatrix.m* is coded to evaluate the matrix arising from the discretization of the convective term. The Navier- Stokes equations is solved using a structured mesh of Q_2Q_1 elements with 20 elements per side, considering the Reynolds numbers Re = 100, 500, 1000 and 2000. The number of iterations required for convergence of Picard method are tabulated as,

Re	No. of Iterations
100	13
500	29
1000	35
2000	69

From the table we can conclude that the number of iterations required for convergence increases with increase in Re.

The flowing plots (Fig. 29 to Fig. 36) illustrates the solutions using naiver stokes for different Re. It can be observed the plots of streamlines that as Re is increased the position of the main vortex moves towards the centre of the cavity. Observing the streamlines it can be concluded that, for higher Re the stabilisation of the galarkin is required.

The range of pressure decreases with increase in Re. This is matching with the reference solution shown in Fig. 37. Reference solution is taken from the textbook. (Finite Element Methods for Fluid Flow Problems, Donea and Huerta, Wiley 2003)

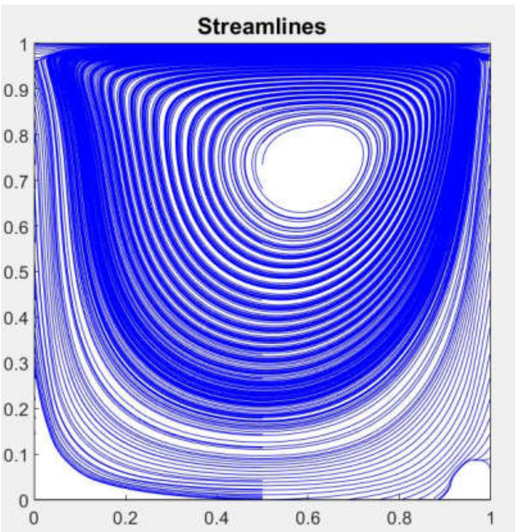


Fig.29 Streamlines for Re=100

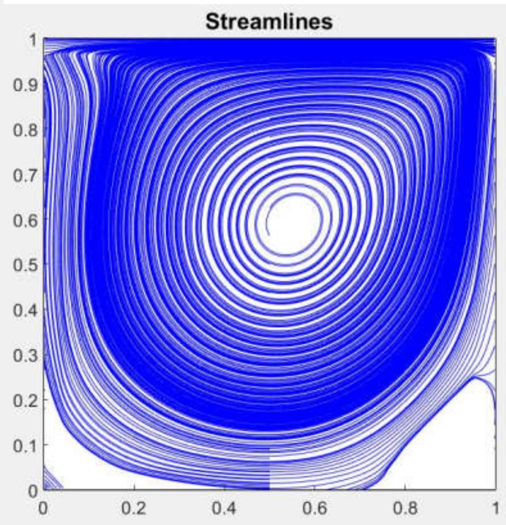
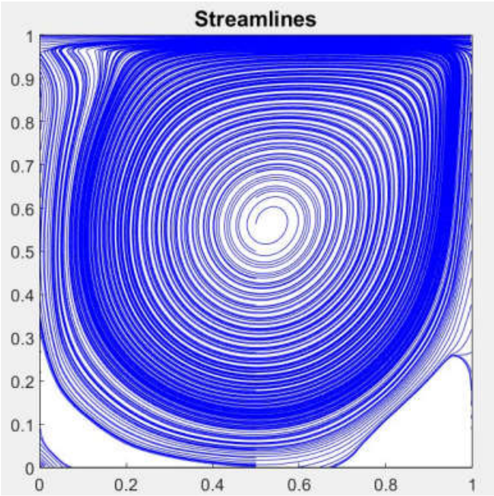


Fig. 30 Streamlines for Re=500



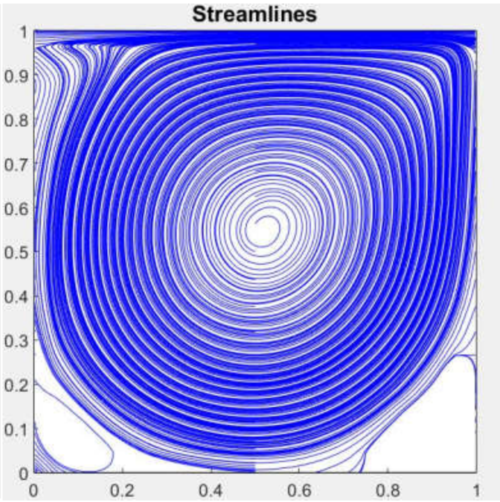
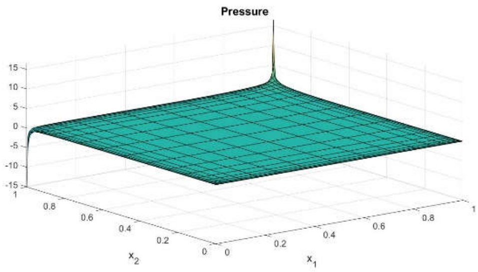


Fig.31 Streamlines for Re=1000

Fig. 32 Streamlines for Re=2000



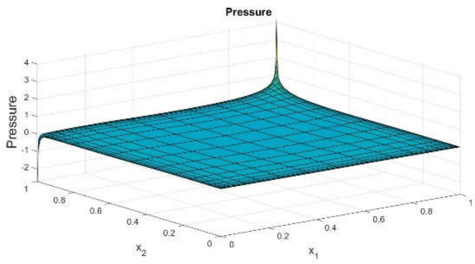
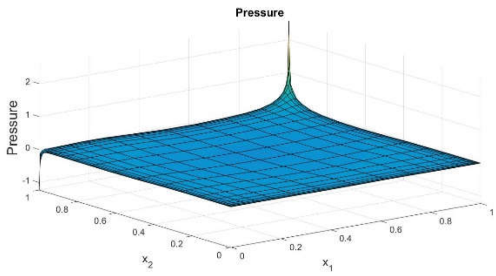


Fig.33 Pressure for Re=100

Fig. 34 Pressure for Re=500



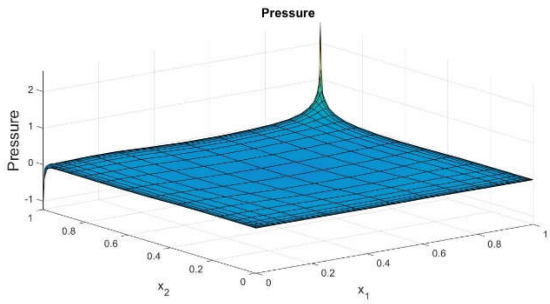


Fig.35 Pressure for Re=1000

Fig. 36 Pressure for Re=2000

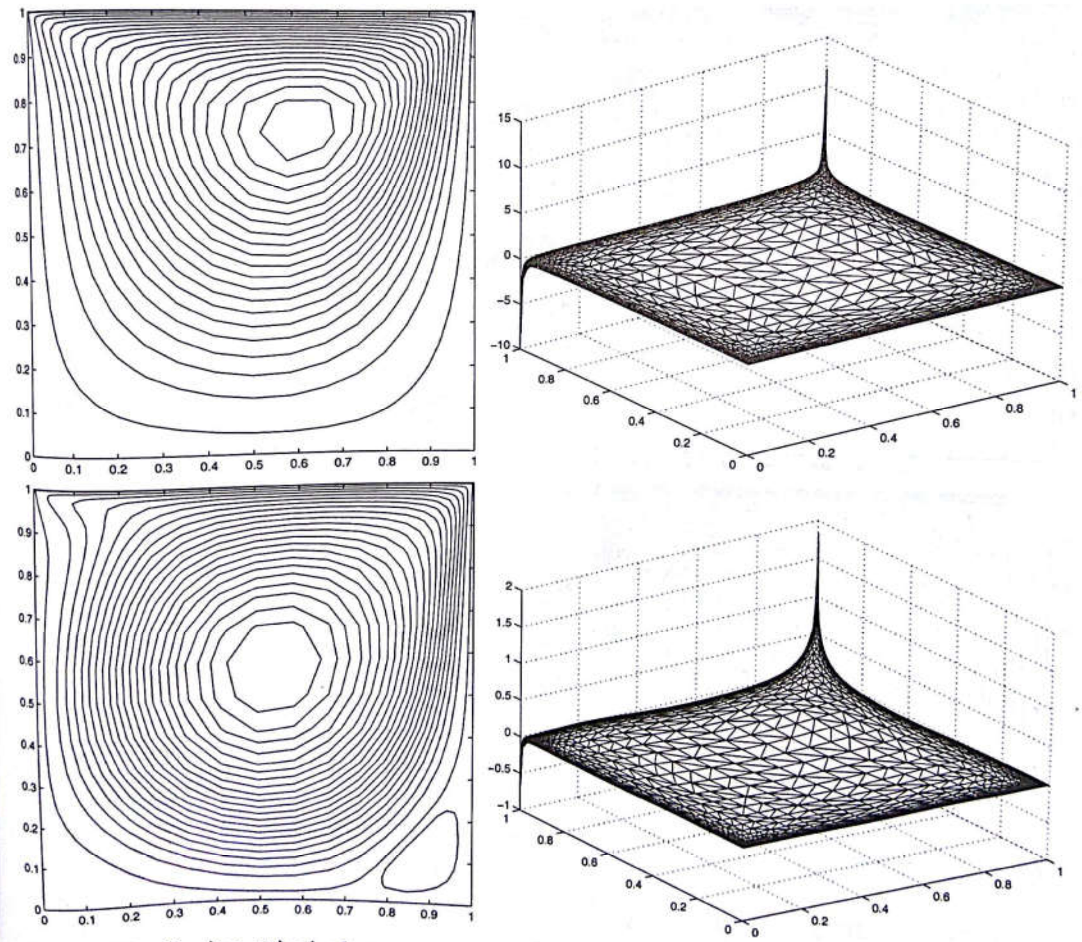


Fig. 6.12 Cavity: `Mini` element, streamlines and pressure for $Re = 100$ (top) and 1000 (bottom).

Fig. 37 Reference Solution from Text Book