Finite Elements in FLuids

Assignment 2

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Introduction

This assignment solves the cavity flow problem and analyzes the results obtained by adopting both the Stokes and the Navier Stokes equation in the domain given in the Figure. 1

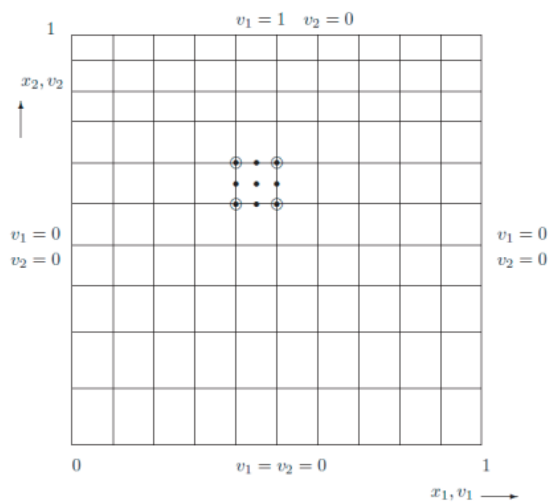


Figure 1: Domain and Boundary conditions

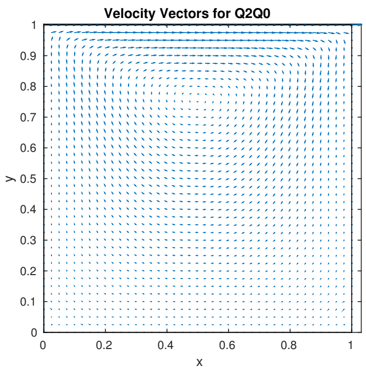
As all the boundary conditions are dirichlet, the Pressure is known upto a constant. Therefore the pressure is prescribed at an arbitrary point to be equal to 0.

a) Steady Stokes problem using uniform mesh

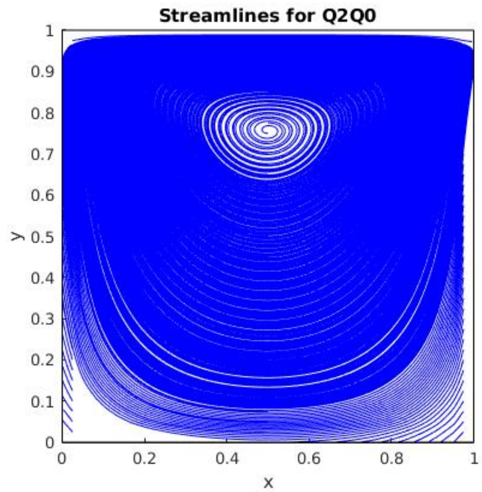
20 Elements are used in each side of the domain for the calculation.

Q_2Q_0 Element

Figure. 2 shows the velocity vectors and the streamlines.



(a) Velocity Vectors



(b) Streamlines

Figure 2: Velocity vectors and Streamlines for Q2Q0 element

Figure. 3 shows the Pressure.

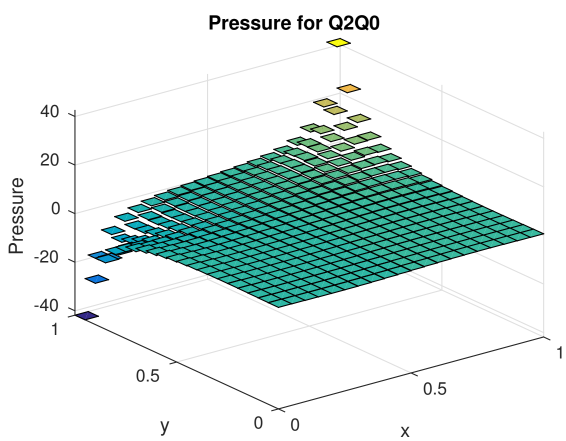
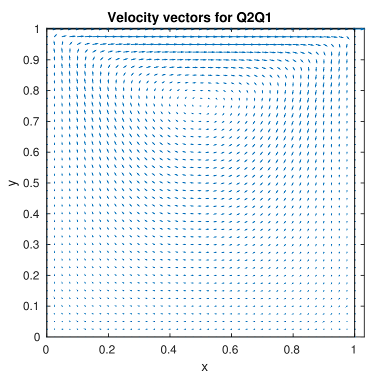


Figure 3: Pressure for Q2Q0 element

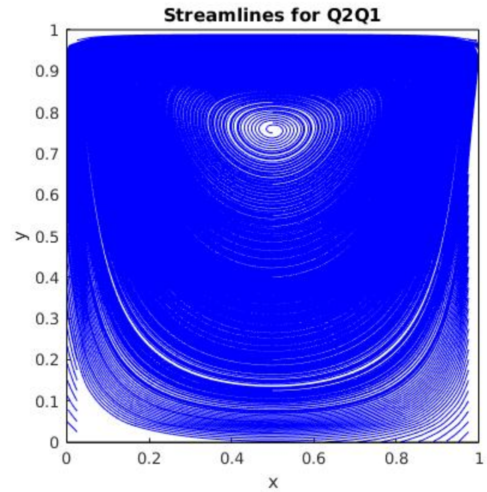
The pressure is not oscillating except at the top 2 corners where there is a variation. This variation is not clearly depicted by a constant pressure in each element.

Q_2Q_1 Element

Figure. 4 shows the velocity vectors and the streamlines.



(a) Velocity Vectors



(b) Streamlines

Figure 4: Velocity vectors and Streamlines for Q2Q1 element

Figure. 5 shows the Pressure.

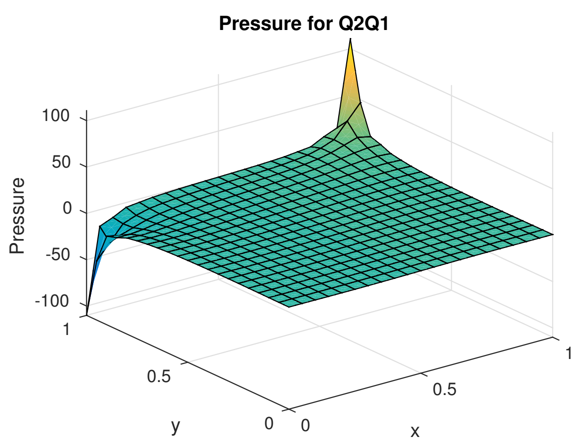
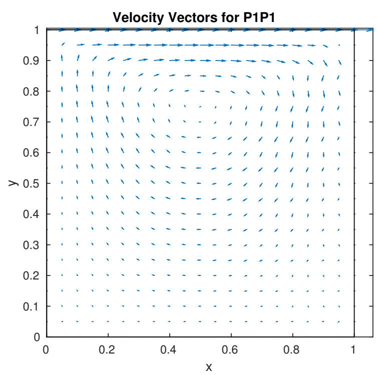


Figure 5: Pressure for Q2Q1 element

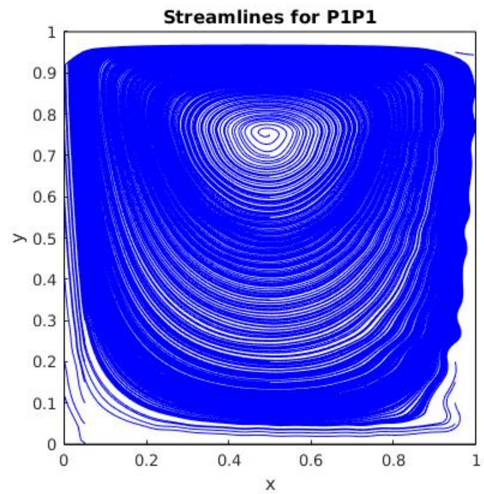
The pressure is not oscillating except at the top 2 corners where there is a variation, however the pressure variation in the corners is better represented by the Q2Q1 compared to the Q2Q0.

P_1P_1 Element

Figure. 6 shows the velocity vectors and the streamlines.



(a) Velocity Vectors



(b) Streamlines

Figure 6: Velocity vectors and Streamlines for P1P1 element

Figure. 7 shows the Pressure.

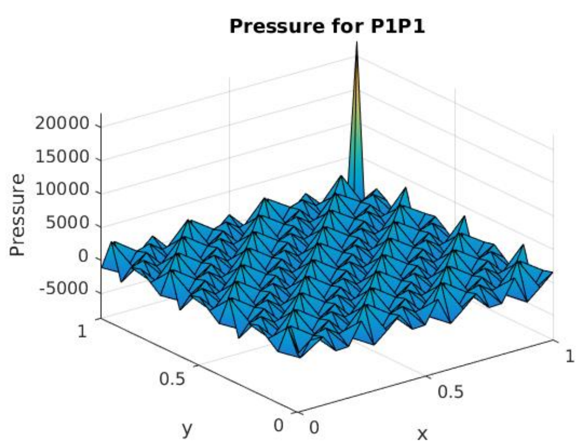
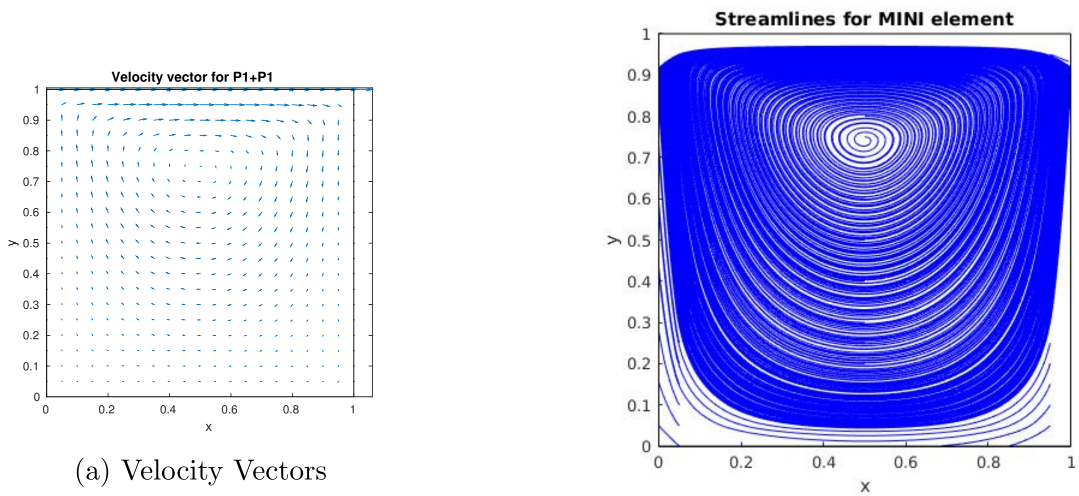


Figure 7: Pressure for P1P1 element

The pressure is oscillating from element to element with a very high value on the top right corner. However, from the velocity plots it is seen that the velocity is accurately calculated and streamlines are similar to the Q2Q1 and Q2Q0.

MINI ($P_1^+P_1$) Element

Figure. 8 shows the velocity vectors and the streamlines.



(a) Velocity Vectors

(b) Streamlines

Figure 8: Velocity vectors and Streamlines for MINI element

Figure. 9 shows the Pressure.

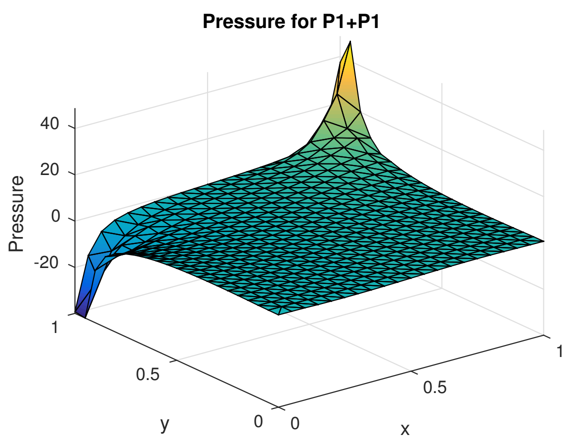


Figure 9: Pressure for MINI element

The addition of the bubble node to a P1P1 element gives a stable solution for Pressure.

The streamlines are symmetric to the vertical mid domain axis similar to the earlier cases.

From the 4 cases it is seen that the streamlines are symmetric with respect to the mid domain vertical axis. The pressures at the top two corners are not found accurately, this is because of the discontinuity in the boundary condition at these 2 corners. The LBB stable elements Q2Q1 and MINI give good result for the pressure. The P1P1 element is not LBB stable as seen by its pressure plot which shows oscillations.

b) Steady Stokes problem using non-uniform mesh

A non-uniform mesh with 20 elements on each side which is refined near the walls is used to analyze the cavity problem. The mesh is shown in figure. 10.

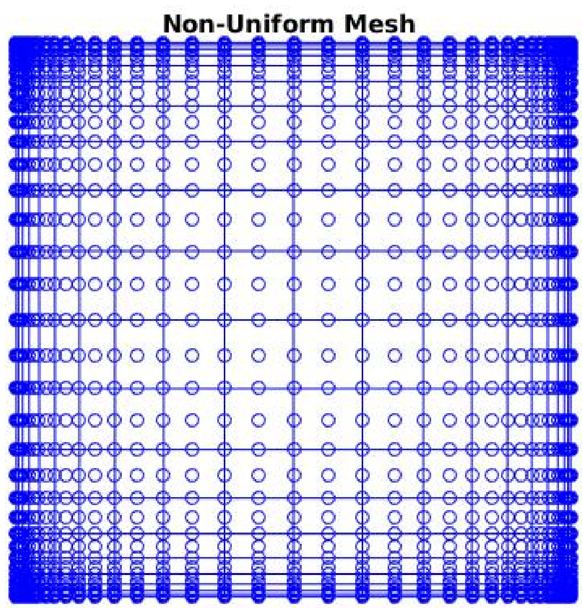
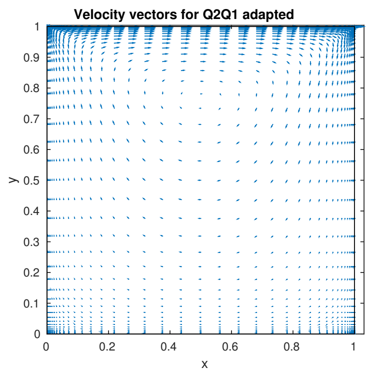
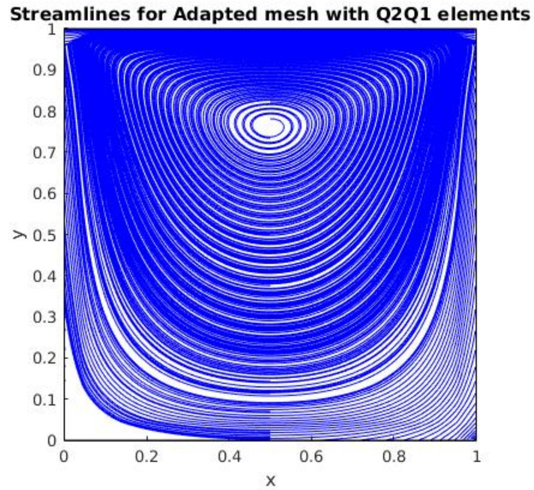


Figure 10: Non-Uniform mesh with velocity nodes

Figure. 11 shows the velocity vectors and the streamlines.



(a) Velocity Vectors



(b) Streamlines

Figure 11: Velocity vectors and Streamlines for adapted mesh with Q2Q1 element

Figure. 12 shows the Pressure.

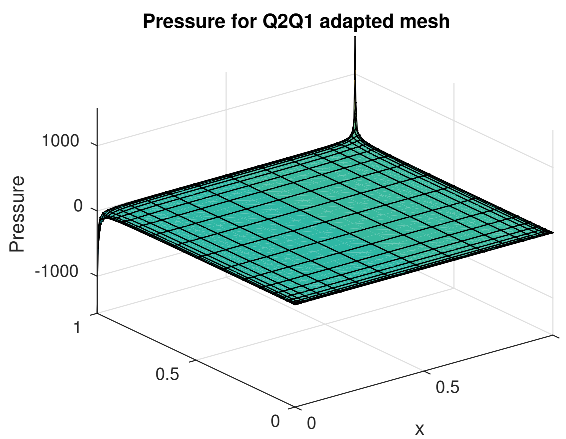


Figure 12: Pressure for adapted mesh with Q2Q1 element

The results are improved after using an adapted mesh compared to the uniform mesh in the section a for Q2Q1 elements. The pressure is undefined in a smaller region near the top two corners. This is because for the adapted mesh, there are more elements near the boundary, where there is maximum variation near the top two corners. Hence with more nodes there, the variation can be captured accurately. The velocity vectors are also much clearer with a higher velocity near the top side.

c) Steady Stokes problem using GLS Stabilization

GLS stabilization is used to stabilize the solution for the P1P1 element which is not compliant with the LBB condition. The weak form of the momentum equation is not affected because the terms involving the second derivative of the weighting function vanish. The incompressibility equation is modified by adding the stabilization term to give the following equation:

$$b(v^h, q^h) - \sum_{e=1}^{n_{el}} \tau_e(\nabla q^h, \nabla p^h)_{\Omega_e} = -\sum_{e=1}^{n_{el}} \tau_e(\nabla q^h, b^h)_{\Omega_e}$$

where the notations used are from the book 'Finite element methods for flow problems, J.Donea, A.Huerta, 2003'.

In this problem, the body force(b) is taken as 0,hence the matrix system which governs the discrete stokes problem is given by:

$$\begin{bmatrix} \boldsymbol{K} & \boldsymbol{G^T} \\ \boldsymbol{G} & \boldsymbol{S} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{p} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}$$

where $\boldsymbol{S}$ is the stabilization matrix. Hence the diagonal terms of the matrix in the above equation are no longer 0 and the solution is stabilized. The stabilization parameter τ_e is taken as $\dfrac{\alpha h_e^2}{4\nu}$ where α is taken as $\dfrac{1}{3}$ for linear elements, h_e is the measure of element size and ν is the kinematic viscosity. The GLS stabilized P1P1 element gives a unique solution for the pressure and there are no spurious pressure modes. Figure. 13 shows the pressure for the P1P1 element after GLS stabilization.

As seen in the figure, the pressure is stable and similar to the MINI element. Hence, the stabilization allows to circumvent the LBB condition and use more convenient elements from a computational point of view.

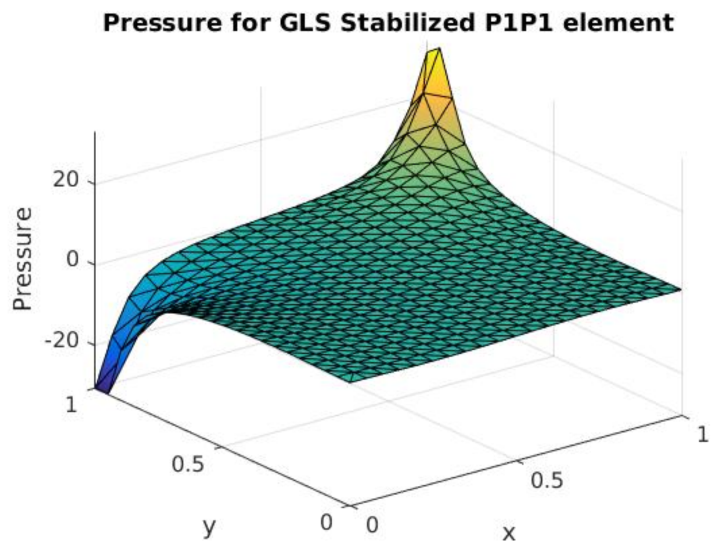


Figure 13: Pressure for P1P1 element after GLS stabilization

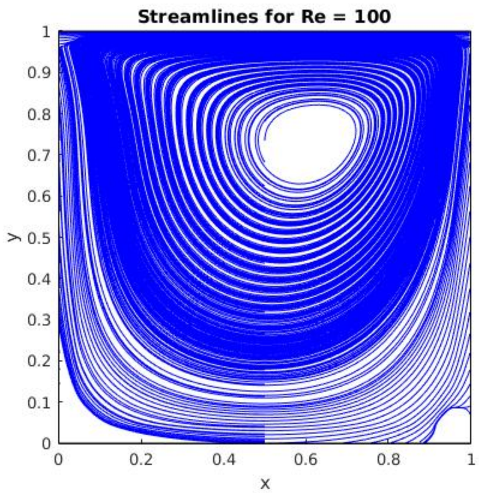
d) Steady Navier Stokes problem using Picard Method

The Navier-Stokes equations using a structured mesh of Q2Q1 elements with 20 elements per side is solved, for 4 cases of Reynolds number = 100, 500, 1000 and 2000.

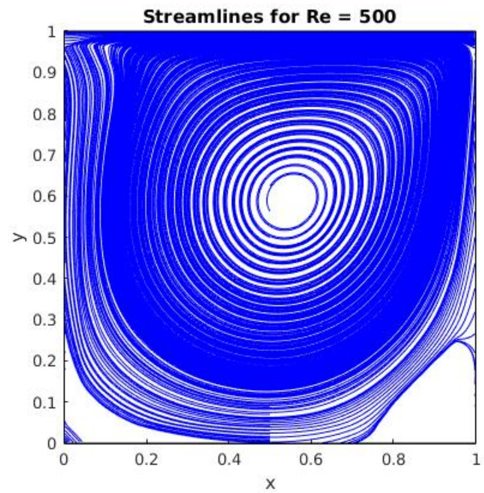
Figure. 14 shows the Streamlines for the 4 cases. It is seen that as the Reynolds number increases, the main vortex of velocity moves towards the center, from its initial position, which was to the right and above the center of the domain for Re = 100. Also the strength of the main vortex increases because the streamlines are much closer to each other with increase in the Reynolds Number.

Figure. 15 shows the Pressure for the 4 cases. There is not much change in the pressure in the domain, except in the top right corner, where the pressure appears to increase around the corner point as the Reynolds number is increased.

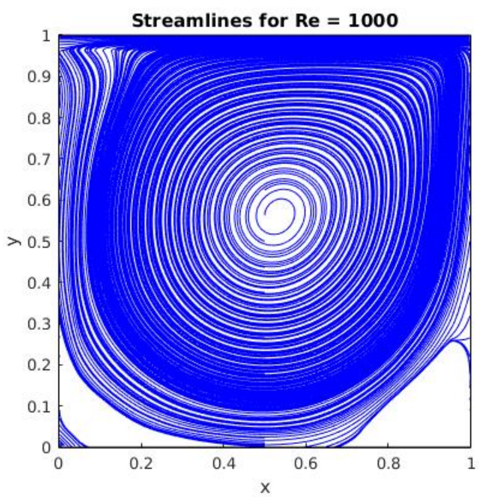
It is seen from Table. 1 that as the Reynolds number is increased, there is an increase in the Number of Iterations required. This is because, a galerkin formulation without stabilization is used for the analysis. So, as the Reynolds number is increased, the flow becomes more convection dominated and hence takes longer to converge. The results agree with the plots in the book 'Finite element methods for flow problems, J.Donea, A.Huerta, 2003'.



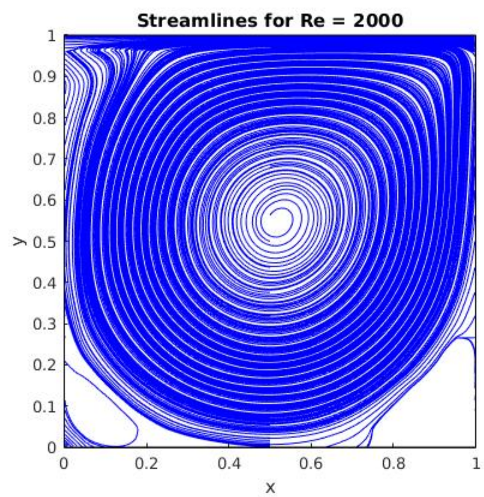
(a) Re = 100



(b) Re = 500

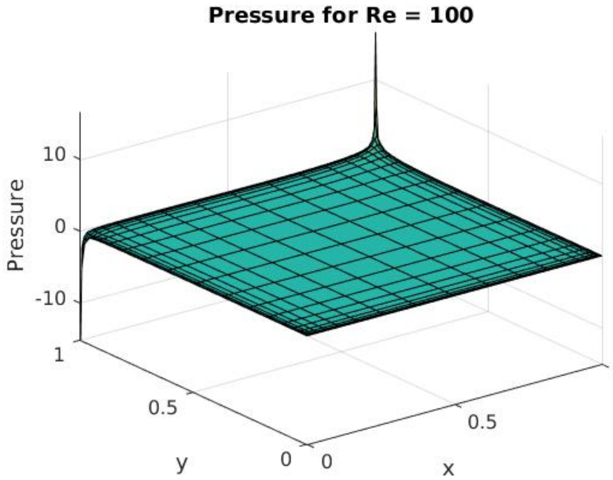


(c) Re = 1000

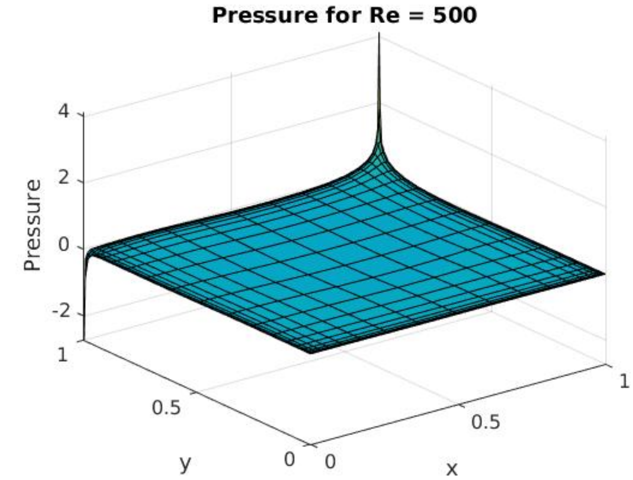


(d) Re = 2000

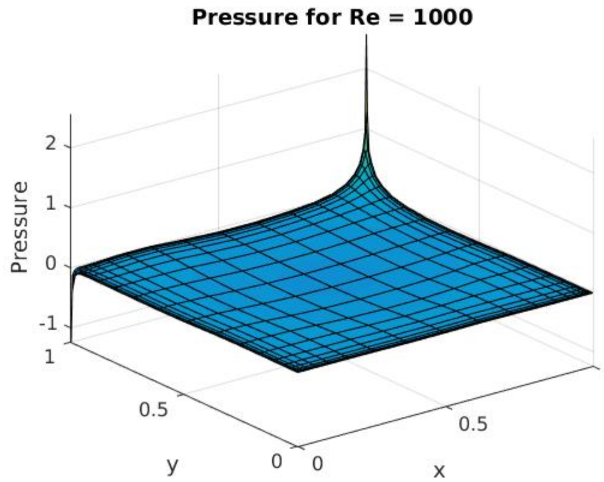
Figure 14: Streamlines for the 4 cases of Reynolds number



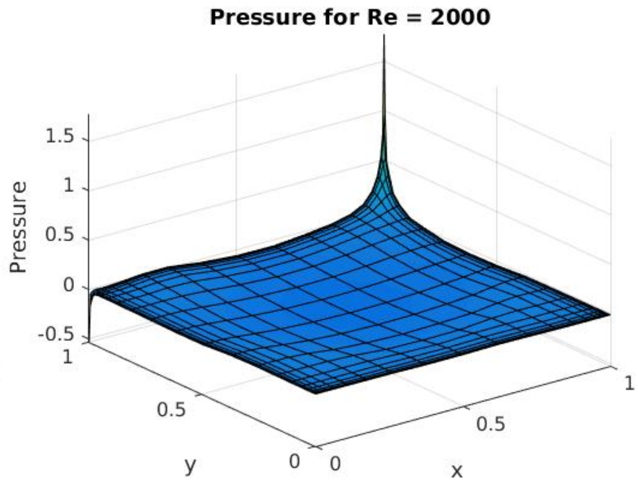
(a) Re = 100



(b) Re = 500



(c) Re = 1000



(d) Re = 2000

Figure 15: Pressure for the 4 cases of Reynolds number

Table 1: Number of Iterations required

Reynolds Number	**Number of Iterations**
100	13
500	29
1000	35
2000	69