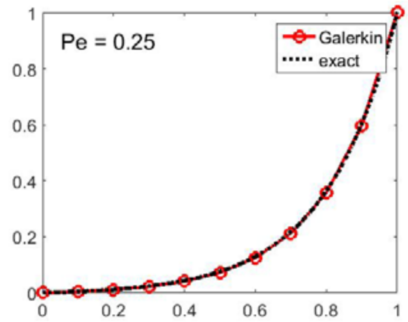
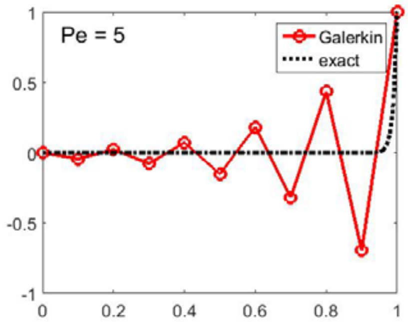
Steady convection-diffusion (1D)

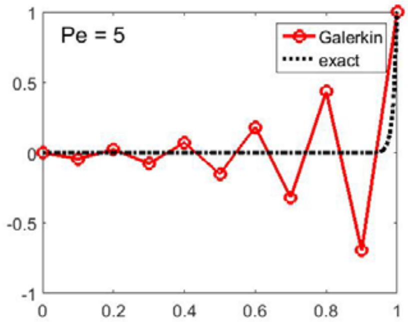
Equation $au_x - \nu u_{xx} = f$ where f=0 with BC u(0) = 0, u(1) = 1



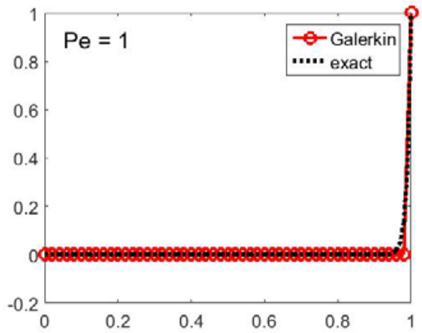
a = 1, ν = 0.2, 10 linear elements



a = 20, ν = 0.2, 10 linear elements

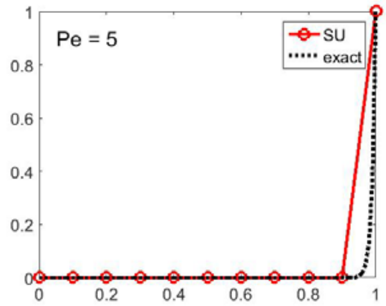


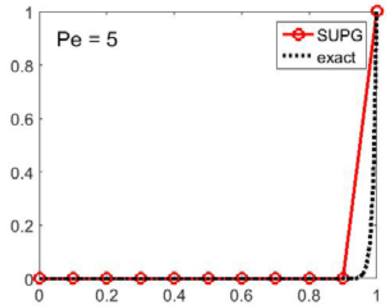
a = 1, ν = 0.01, 10 linear elements

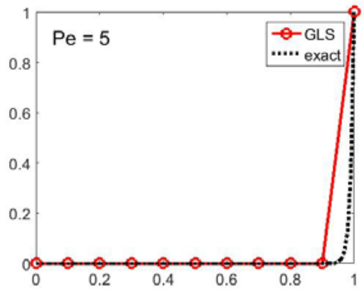


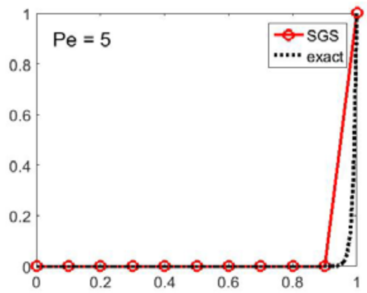
a = 1, ν = 0.01, 50 linear elements

Solving the equation with a = 1, ν = 0.01, 10 linear elements and optimal τ parameter

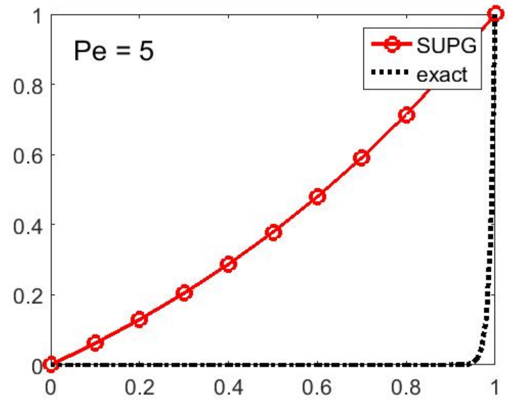




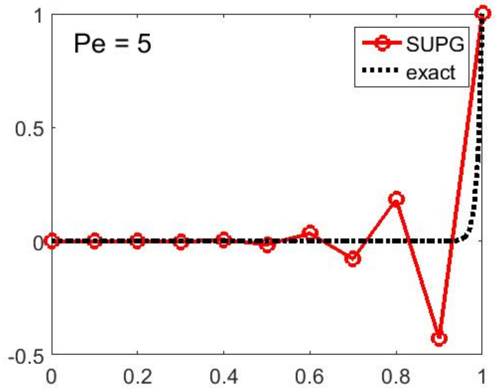




Solving the equation by SUPG method with a = 1, ν = 0.01, 10 linear elements and different τ parameters

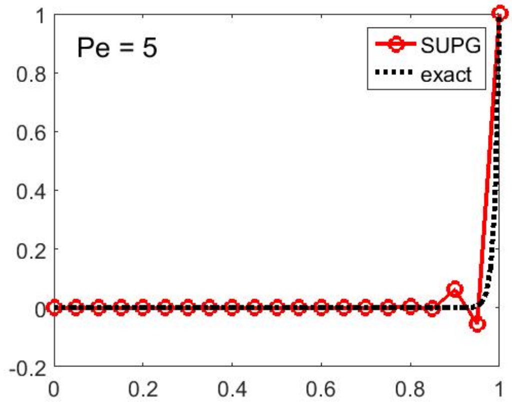


τ =1



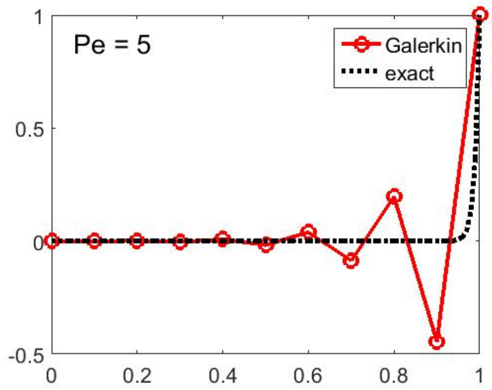
τ =0.01

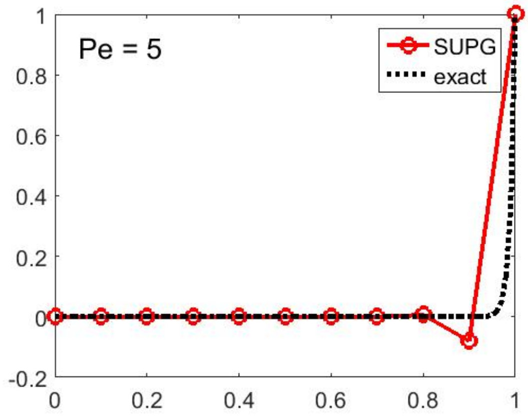
Solving the equation by SUPG method with a = 1, ν = 0.01, 10 linear elements, τ=0.01 and using quadratic elements



Equation $au_x - \nu u_{xx} + \sigma u = f$ where f=0 with BC u(0) = 0, u(1) = 1. Solving the equation with a = 1, ν = 0.01, $\sigma = 20$, 10 linear elements and optimal τ parameter solving as

$$\tau = \left(\left(\frac{2a}{h}\right)^2 + 9\left(\frac{4\nu}{h^2}\right)^2 + \sigma^2\right)^{-1/2} = \frac{h}{2a}\left(1 + \frac{9}{Pe^2} + \left(\frac{h}{2a}\sigma\right)^2\right)^{-1/2}$$





Unsteady transient convective problem (1D)

Solving the transient convection equation

$$u_t + a u_x = 0$$

With BC:

$$u(x,0) = \frac{5}{7}\left(- \left(\frac{x - x_0}{L}\right)^2 \right)$$

Analytical solution:

$$u(x,t) = \frac{5}{7\sigma}\left(- \left(\frac{x - x_0 - at}{\sigma L}\right)^2 \right)$$

Where

$$\sigma = \sqrt{1 + \frac{4\nu t}{L^2}}, \qquad x_0 = \frac{2}{15}, \quad L = \frac{7\sqrt{2}}{300}$$

Using Crank-Nicolson method for time and a Galerkin for space, receive: