UPC

Master Numerical Methods in Engineering

Industrial Trainig Report:

Analysis and Possible Improvements of the Material Point Method, MPM

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1 Introduction

A simulation of different test using the Material Point Method (MPM) are performed. The MPM is a numerical technique used to simulate the behavior of solids, liquids, gases, and any other continuum material. Especially, it is a robust spatial discretization method for simulating multi-phase (solid-fluid-gas) interactions. In the MPM, a continuum body is described by a number of small Lagrangian elements referred to as 'material points'. These material points are surrounded by a background mesh/grid that is used only to calculate gradient terms such as the deformation gradient. Unlike other mesh-based methods like the finite element method, finite volume method or finite difference method, the MPM is not a mesh based method and is instead categorized as a meshless/meshfree or continuum-based particle method.

During the Industrial training our objective was to see if the method works for different cases. We have analysed different results for Compression, Brasilian and Shear test.It has been analysed the method in 2D and 3D in order to compare the different results and prove that the method works and can be used for different applications. Also these different results will be compared with results of DEM and laboratory analysis. Note that all 2D cases have been calculated considering Plane Strain and concrete H-30. The use of plane strain is not really true, it is only true for the case of Brasilian Test, the other results are only an approximation. Although it can be seen that the results we get are quite good. This document is structured in the way that the analysis in 2D is shown first and then in 3D.

2 MPM Compression Test

A compression breaking test using the Material Point Method (MPM) was performed. In the simulation it is intended to perform a test that is similar to that of the laboratory, so the tests have been performed on a cylindrical specimen having the same dimensions as in the laboratory: 100 mm in diameter and 200 mm in length. The simulation has been carried out for different types of meshes and the results have been compared with the stress-strain laboratory and DEM curves.

2.1 Analysis of 2D compression test

Before analyzing the results obtained, note that the calculations with square meshes give better results than with triangular meshes, as can be seen below. In the beginning, several calculations have been made with triangular meshes, and the area of the specimen in which damage was expected was refined. However the results for this type of meshes have not been as expected and it was decided to use meshes with squares.

2.1.1 Mesh typology for the calculations

To perform an analysis of the method we have used different types of meshes. The use of meshes with different sizes and elements allows us to observe which meshes adapt better with the MPM formulation. To this end, several analyzes have been carried out with triangular and square meshes, each of them analyzed for different sizes of elements. At the end of the analysis you can see which mesh gives the best results. Then the different meshes used are shown:

Meshes used for the calculations:

Mesh Type		Unstructured	Structured
Triangular	T1	0.0075	0.0075
Triangular	T2	0.005	0.005
Triangular	T3	0.0025	0.0025
Quadrilateral	Q1	-	0.0075
Quadrilateral	Q2	-	0.0065
Quadrilateral	Q3	-	0.005

In spite of having used all these meshes it is observed that meshes with triangular elements do not give good results. The distributions of damages that are obtained are not those expected, that do not correspond with the lab tests. Therefore, it is decided to use the square elements that provide the expected results. Throughout the report the results with the elements are shown, while the triangular ones can be seen in the Annex.

The correct values of the different test performed with quadrilateral elements are:

Mesh Type	Mesh Size	Displacement	Steps	N Particles
Q1	0.0075	-1.00E-06	300	2
Q2	0.0065	-1.00E-07	2000	2
Q3	0.005	-2.00E-06	300	2

The displacement indicated in the table is the displacement made by the steel material on the specimen for each step, to simulate the compression test. While the number of steps indicated is the approximate number of the step in which the specimen breaks. As we know in laboratory tests, a force is applied to the test piece while in our simulation we apply a displacement. That displacement will exert a pressure and with it a force.

In order to minimize the extension of the report it is shown inside of this the results with the mesh that gives better results.

Results for case Q3

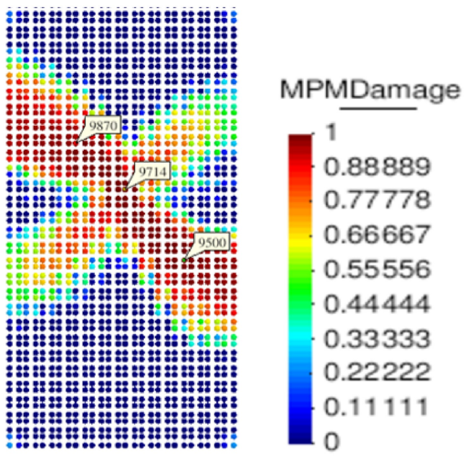

Figure 1: MPM final damage for a Compression test Q3

In this case a level of damage equal to 1 is reached. The rupture of the probe occurs because a level of damage equal to 1 in a node means that the resistance of the probe in that node is 0. Again a Cross-rupture surface, although the result for this case is more real since it can be seen that in one of the diagonals a greater level of damage is reached and the specimen breaks. Of all the cases analyzed this is the most real because it reaches a damage equal to 1 and the surface of rupture makes sense.

On the other hand in the results of GiD it is observed that the tensions vary in the different layers of elements. This is due to how our MPM model is defined because the voltages are not well discretized (calculated) and depend on the mesh and the element, so for each line of quadrilaterals the voltages vary. In order to obtain a better solution, in which these tensions were not dependent on the mesh or the particles, an MPM analysis could be performed, but with UP formulation, ie where we not only take into account the displacements (as in this case) but also the Pressures. We could also test with FEM or mesh-less.

Some examples that show that this type of formulation does not give good results regarding the distribution of tensions can be seen in the following graphics.As mentioned below, it can be seen that the stress distribution is not uniform and follows the direction of the elements (ie depends on the mesh type). It is recommended to improve the results with an UP formulation, using FEM or mesh-less.

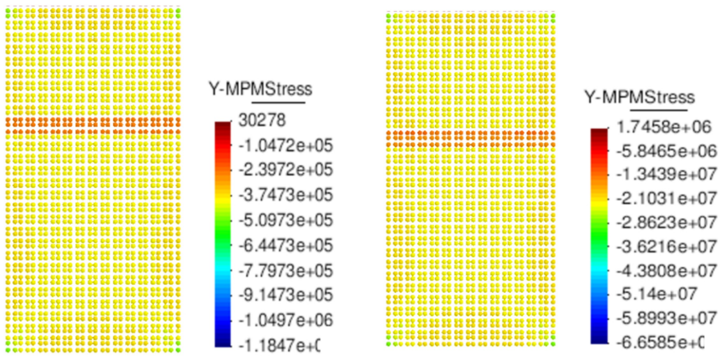

Figure 2: MPM vertical stress at the steps 1 and 56 respectively with Q3 mesh

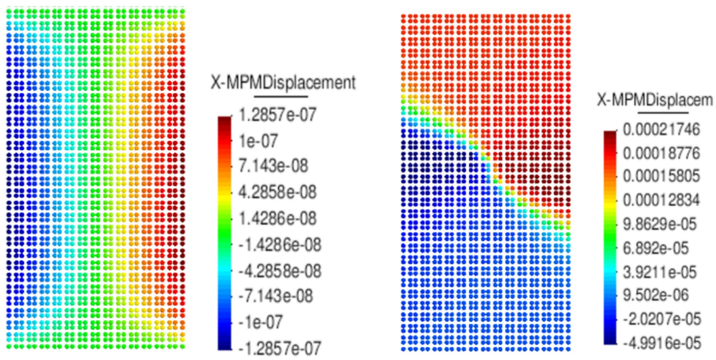

Figure 3: MPM Initial and final horizontal displacement for Q3

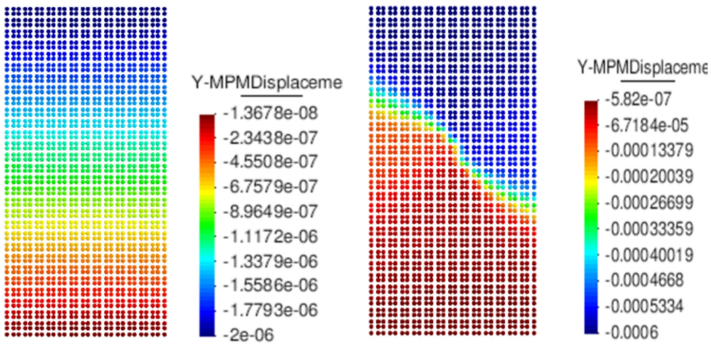

Figure 4: MPM Initial and final vertical displacement for Q3

With the results obtained for this method an analysis of the stress strain curve has been carried out, taking different nodes of the test tube that have a damage equal to 1. The results have been compared with the laboratory curves.

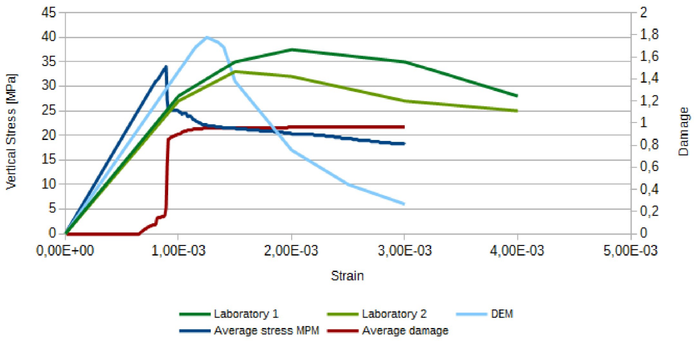

Figure 5: MPM Initial and final vertical displacement for Q3

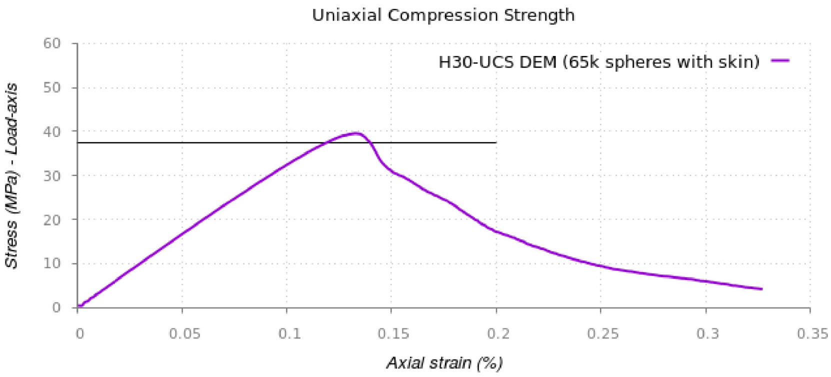

Figure 6: DEM Stress-Strain curve in a compression test

With the MPM calculation higher values of elastic limit are obtained than in the laboratory, but lower than those of the DEM analysis. The graph shows a sudden drop in compressive strength when the damage goes from 0.1 to 0.9 in a very short time. After that the test piece still has some resistance that is losing until breaking (damage = 1).

In conclusion, it can be observed that the results obtained with MPM make a lot of sense. The strain-strain curve has an elastic zone, reaching the elastic limit of 35MPa approximately and then begins the plastic zone in which a softening curve is observed until the specimen ruptures. This mode of rupture can be compared with the results obtained with the DEM formulation. Below are the results provided in 3D using DEM. In the following graphs it could be seen some results of DEM analysis.

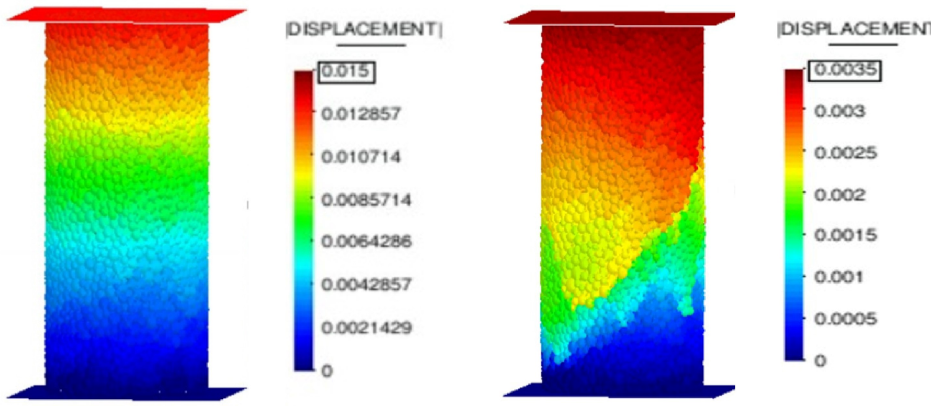

Figure 7: 3D DEM total displacements for a Compression test

2.2 Analysis of 3D compression test

After having made the analysis in 2D and check that it works correctly, we wanted to perform the same test in 3D to verify that the code also works in three dimensions.

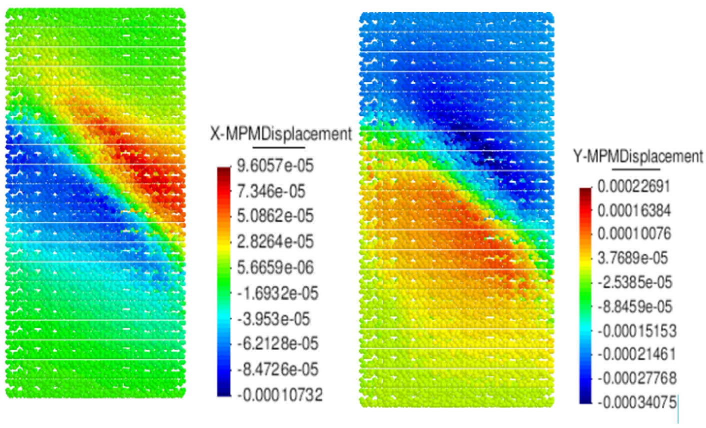

Figure 8: 3D MPM displacements for a Compression test

and the total displacement is shown in the following graph

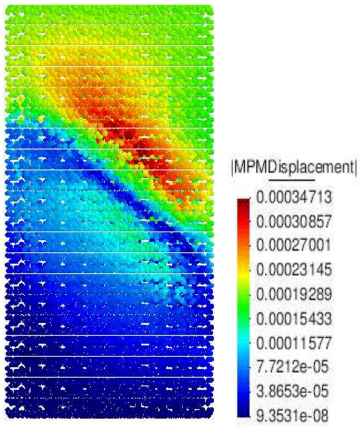

Figure 9: 3D MPM total displacement for a Compression test

On the other hand the horizontal and vertical stresses distributions are as shown:

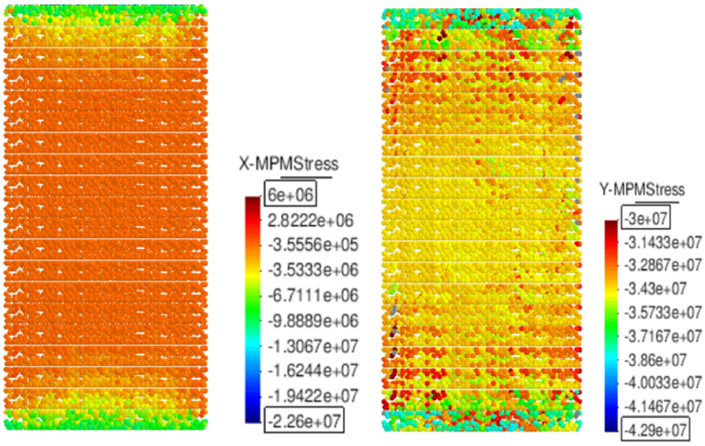

Figure 10: 3D MPM stresses for a Compression test

One of the ways to verify if our code works correctly is to observe if it is a break mode that corresponds to the one we obtain in laboratory tests. The following figure shows the damage of the compression test specimen.

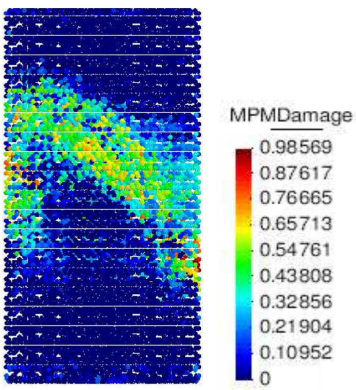

Figure 11: 3D MPM Damage for a Compression test

Below are the stress-strain curves that are obtained. It is shown a comparison between the stress-strain curves using DEM and MPM

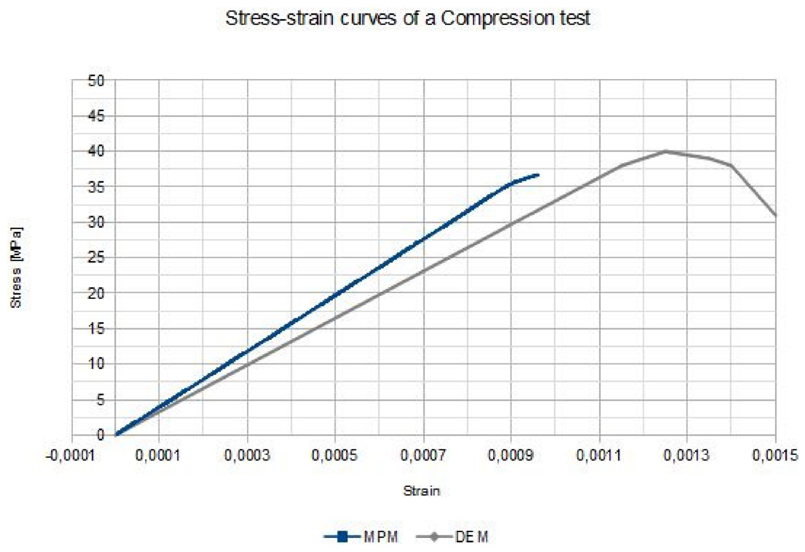

Figure 12: DEM and MPM stress-strain curves for a Compression test

On the other hand it has been verified if the MPM method analysis varied greatly if the study was carried out in 2D or 3D. In the following graph you can see the stress-strain curves for both cases of study.

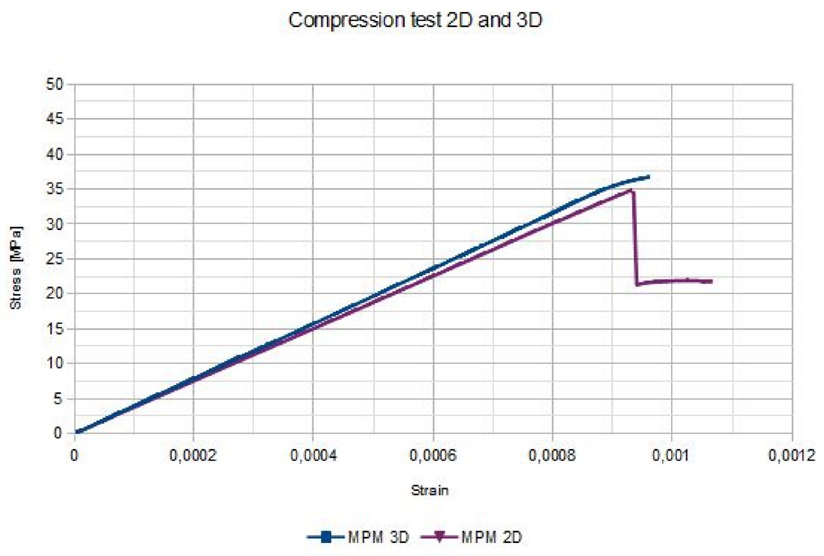

Figure 13: 2D and 3D stress-strain curves for a Compression test

2.3 Laboratory results for a compression test

An other form to see if the way of fracture is the expected, is the comparison with lab results. In the pictures below could be seen how breaks a concrete samples under compression.

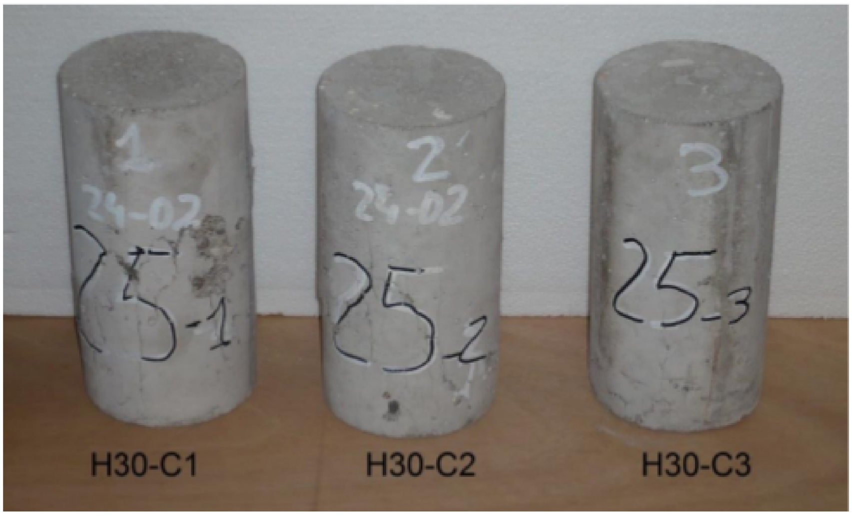

Figure 14: Fracture of the samples of concrete 30 in a Compression test

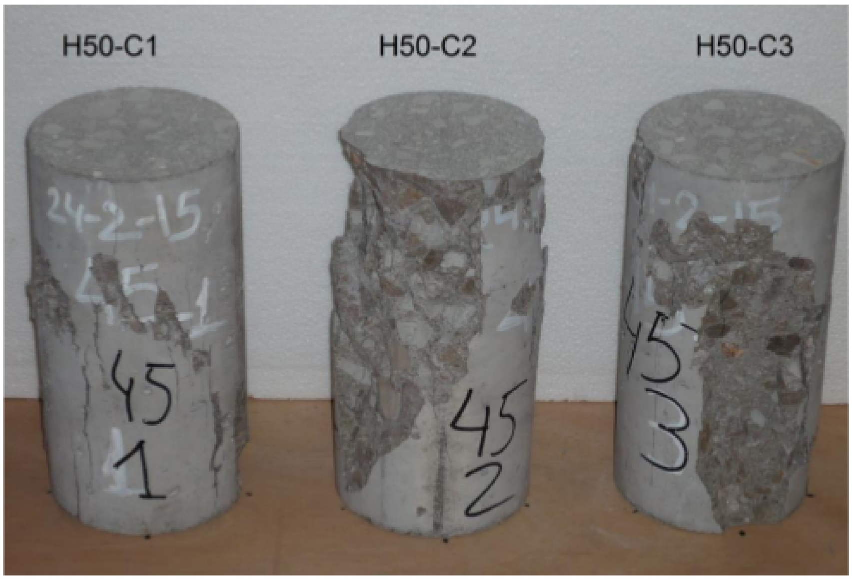

Figure 15: Fracture of the samples of concrete 50 in a Compression test

3 MPM Brazilian Test

A simulation of a Brazilian test (or indirect traction) has been performed using the Material Point Method (MPM). As in the compression test, this test has been performed on a cylindrical sample of a a diameter of 100 mm and a lenght of 200 mm.

3.1 Analysis of 2D brazilian test

For this, different types of meshes have been used and the results have been compared with the stress-time and damage-time curves of DEM. For all the cases a rectangular mesh of background and a triangular one for the test tube has been used. In total, 4 cases have been compared.
DEM results for a Brazilian test

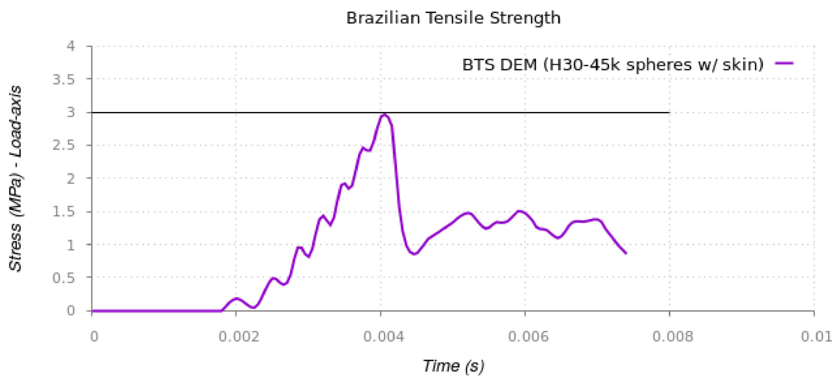

Figure 16: DEM Stress-strain curves for a Brazilian test

In the analysis made with DEM a loading speed of 0.1 m/s in compression was used. While in indirect traction (Brazilian test) the velocity was 0.05 m/s but applied on both sides, in this way the imposed displacement was the same in both tests.

3.2 MPM results for number of particles=3 in a brazilian test

Meshes used for the calculations:

Mesh type	Mesh size	Displacement	Steps	N Particles
Case 1	0.005	-1.00E-6	1000	3
Case 2	0.005	-2.00E-6	1000	3
Case 3	0.005	-5.00E-7	1200	3
Case 4	0.005	-1.00E-6	1000	6

Case 1: Using an element size of 0.005. Number of particles = 3 and a displacement imposed by each step equal to -1E-6, performing 1000 steps. The distribution of the damage obtained in the sample is as follows

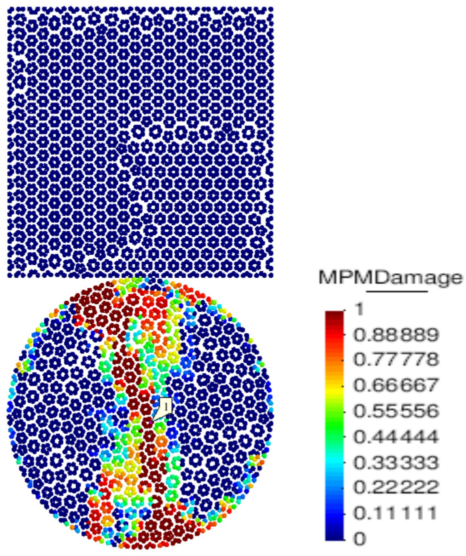

Figure 17: MPM damage distribution for Brazilian test Case 1

On the other hand the results get with this mesh and using FEM are

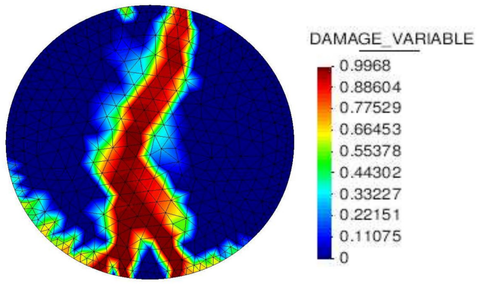

Figure 18: FEM damage distribution for Brazilian test

We have analyzed the results at one of the points where the most rapid damage is reached (damage = 1). The vertical stress-time (on loading axis) and damage-time curves are compared with the ones obtained in the laboratory.
Results with MPM analysis

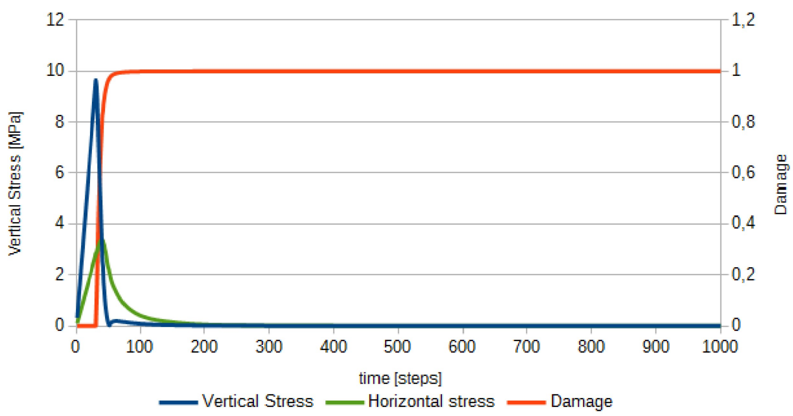

Figure 19: Stresses curves for Brazilian test Case 1

It can be observed that when the elastic limit is exceeded, the damage begins, until it reach a damage equal to 1, which means that the resistance is zero and the sample breaks. In this test the sample breaks by traction, because the tensile strength of the concrete is smaller.
Variation of vertical and horizontal stresses over time

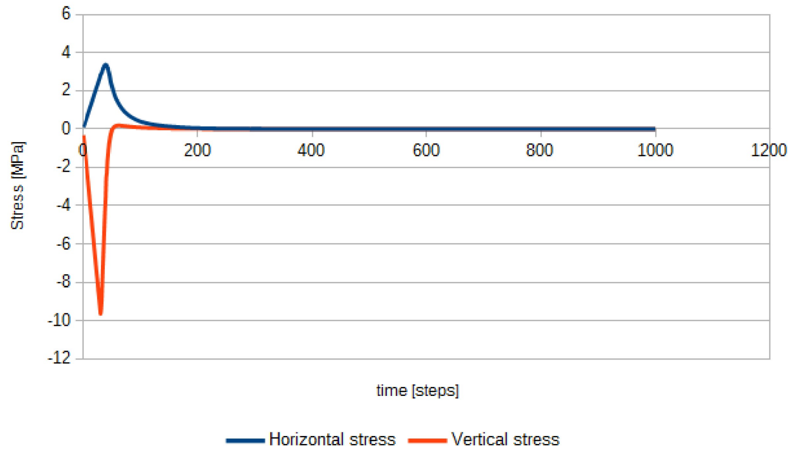

Figure 20: Stresses curves for Brazilian test Case 1

This variation makes sense because as the force on the sample is increased (compression on the Y axis generates greater vertical tension),so the horizontal tension increases, generating an indirect tension. It is also possible to observe how a linear tension distribution is first obtained because it is in the elastic zone and then the plastic zone (curve distribution in the graph) begins. Until the tension reaches the limit (either compression or tension) and the material damages until it breaks. It can be seen how a softening curve is generated. Remember that the tensile strength of the concrete is much lower than the compression, so in the simulations the sample breaks earlier with this test (Brazilian test), because breaks in tension.

As it was said in the graph it can be seen that the elastic limit of the tensile strength (corresponding to the horizontal stress, indirect tension) is of the order of 3 MPa while the elastic limit to compression (vertical stress) is around 9.5 Mpa.
Rupture simulation of the sample (Damage and vertical displacement respectively

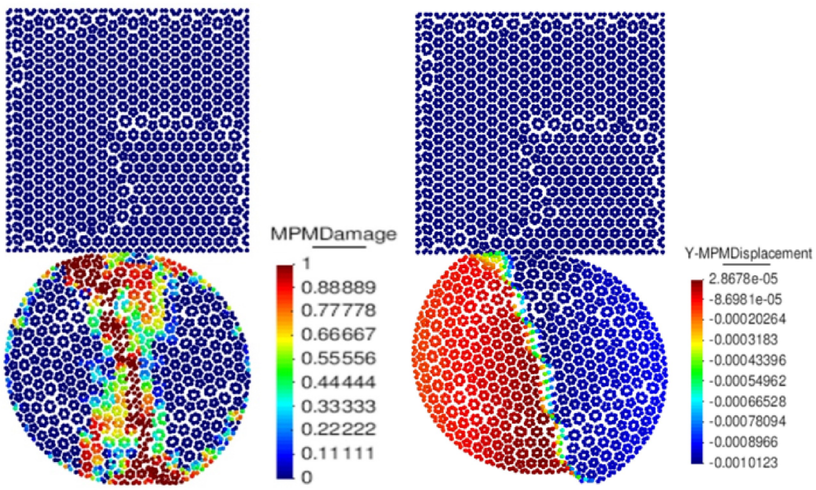

Figure 21: MPM Damage and vertical displacement for a Brazilian Test Case 1

Now it has been shown the results we get if we use more number of particles, in this case 6.
Case 4: Using an element size of 0.005. Number of particles = 6 and a displacement imposed by each step equal to -1E-6, performing 1000 steps.
In this case the number of particles has been increased in order to see if it is get the same

solution or what happen.

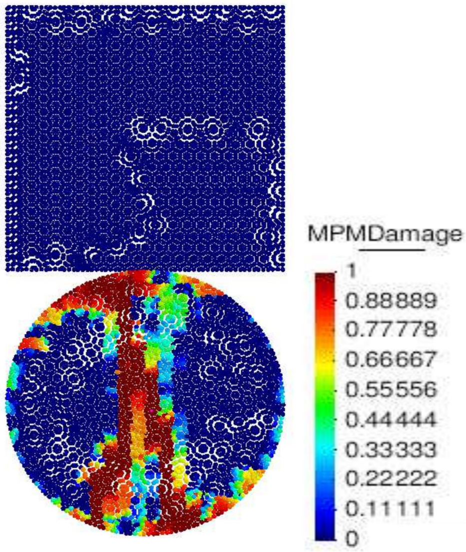

Figure 22: MPM Damage distribution for a Brazilian Test Case 4

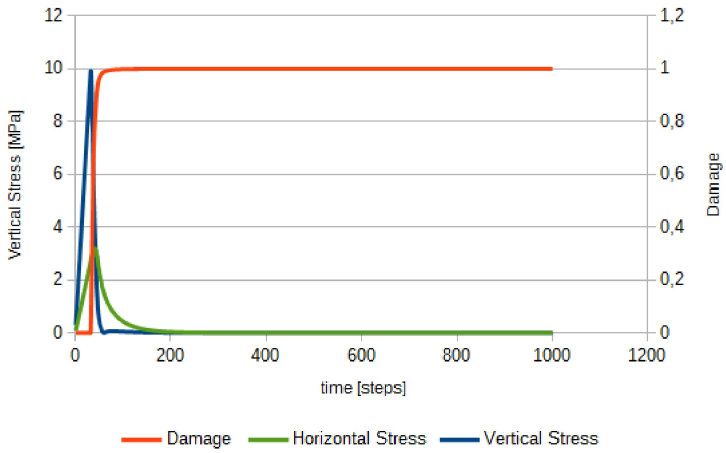

Figure 23: MPM stresses evolution along time for a Brazilian Test Case 4

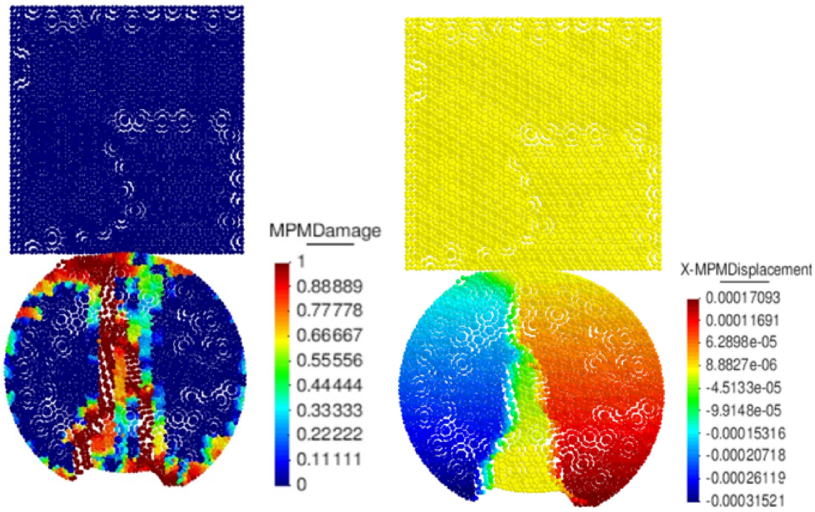

Figure 24: MPM damage and horizontal displacement for Case 4

Note: the deformation of the structure is increased.

It could be observed that the way of rupture is different depending on the number of particles. In the graphs above it can be seen that the sample breaks at the middle but there is also a piece on the bottom of the sample, so the sample breaks in 3 pieces. But this way of fracture is also a solution that can be seen in lab (see figures below).

Note: the deformation of the structure is increased.

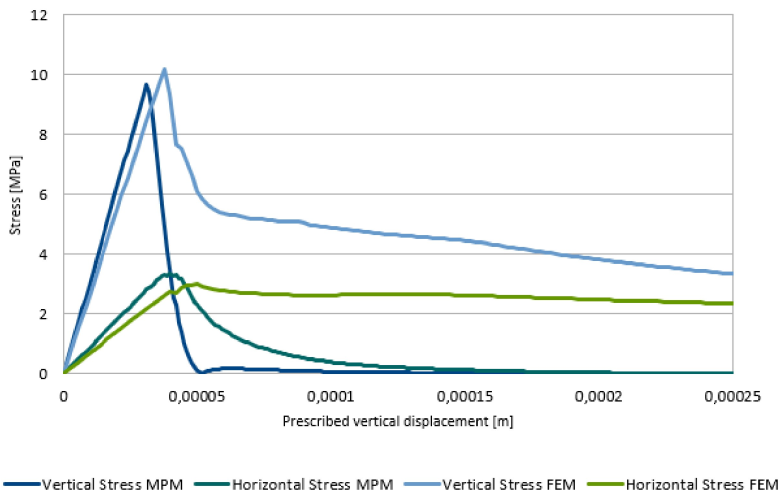

Figure 25: Comparison between the stress evolution with FEM and MPM analysis

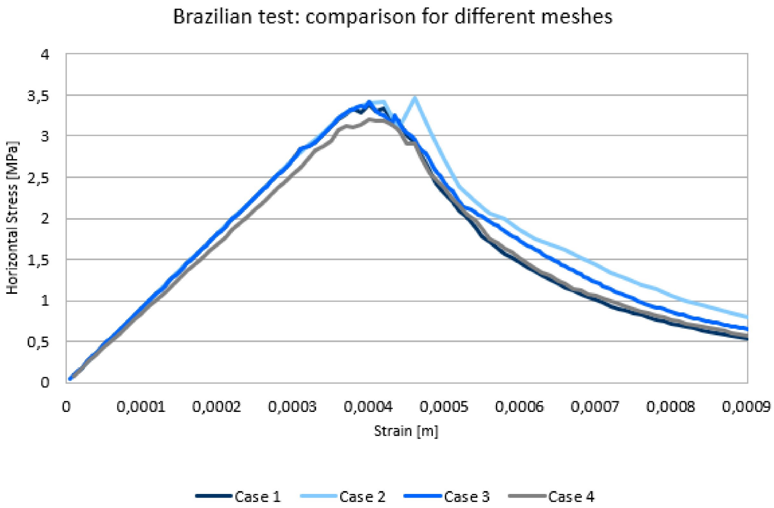

Figure 26: Comparison horizontal stress evolution for different meshes analysis

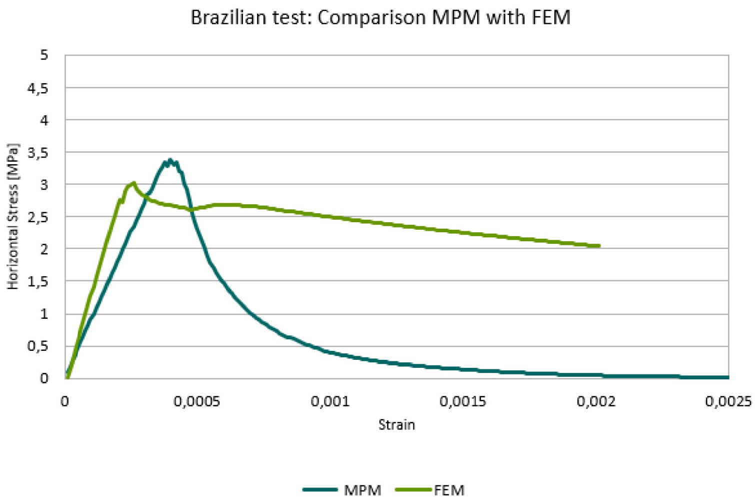

Figure 27: Comparison horizontal stress evolution for FEM and MPM analysis

The way of fracture is what we expected, this results could be compared with the ones get with a DEM formulation and also with the real fracture in lab.

3.3 Analysis of 3D brazilian test

After proving that we get good results with this test in a 2D analysis, we have simulated a 3D test in order to verify that the code works also well. In the following lines it could be seen some graphs with the results obtained.

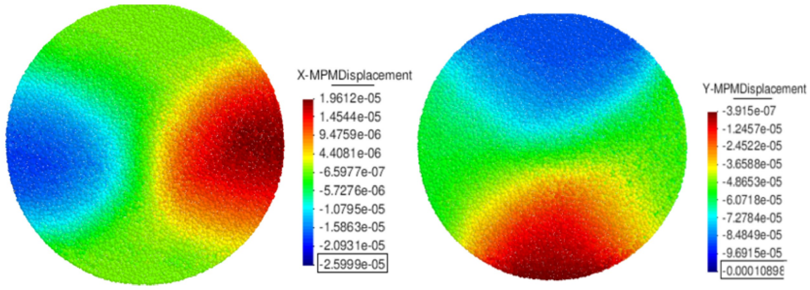

Figure 28: MPM horizontal and vertical displacement for 3D analysis

As can be seen the displacements are as we expected.Now we are going to check if the stresses distributions correspod to the real ones, below can be seen the results we get.

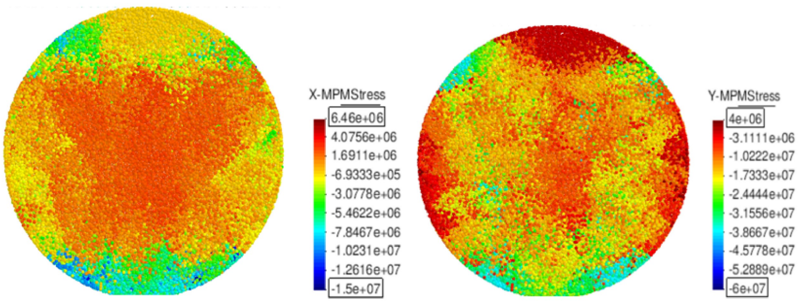

Figure 29: MPM horizontal and vertical stresses for 3D analysis

The most interesting part of the analysis is to see if the way of fracture is as the cross section that we get in 2D and in lab analysis. Here could be seen that a kind of cross is actually generated.

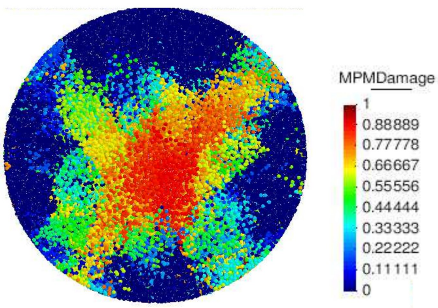

Figure 30: MPM damage for a 3D Brazilian Analysis

is the same that the one produced with DEM. Here is shown a simulation with the displacements get in DEM.

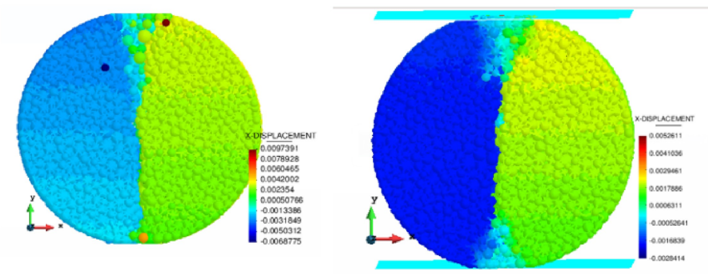

Figure 31: DEM horizontal displacement

Another way to validate our results is checking that the rupture The deformation stress curves that the specimen undergoes during the brazilian test have also been studied. In the following graphs can be seen how the stresses vary.

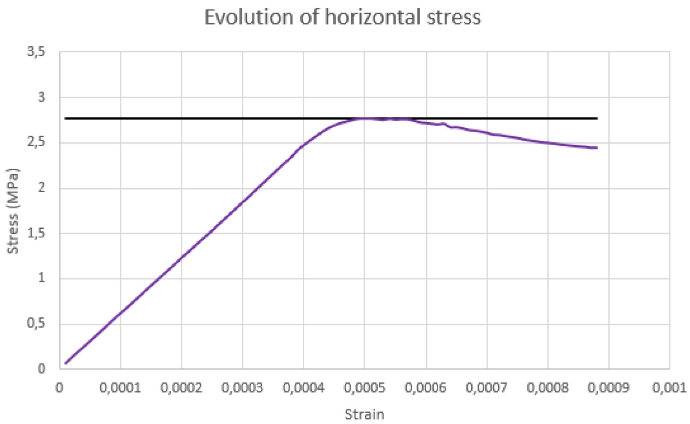

Figure 32: MPM horizontal stress a 3D Brazilian Analysis

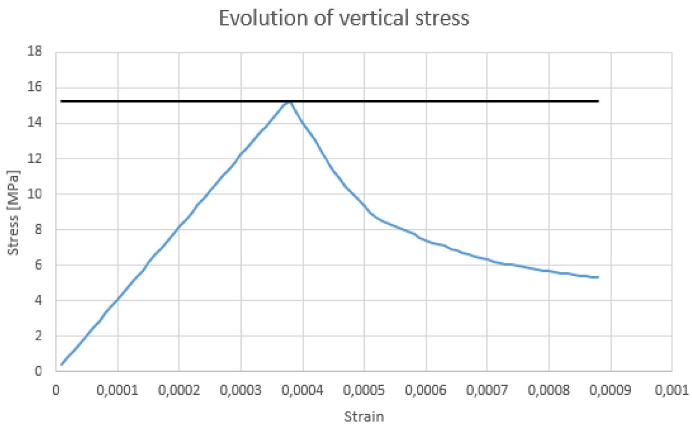

Figure 33: MPM vertical stress for a 3D Brazilian Analysis

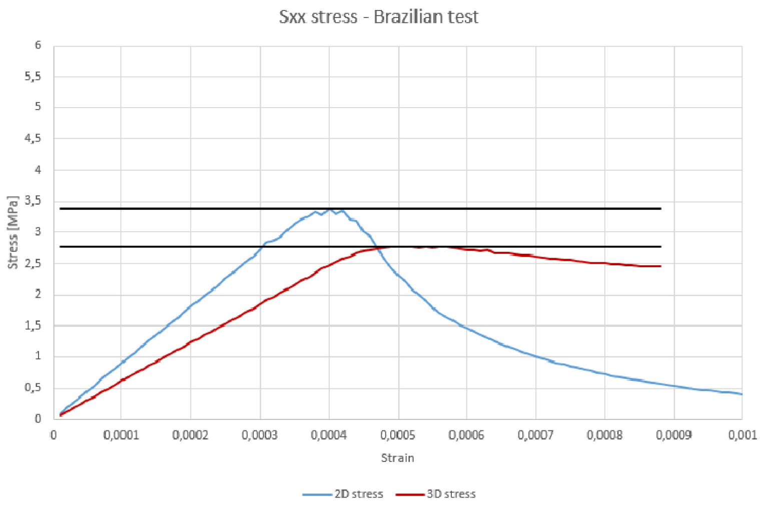

Figure 34: MPM comparison of stresses in 2D and 3D

3.4 Laboratory results for a brazilian test

In this section are shown different pictures of samples fractured after a brazilian test is performed.

Figure 35: Fracture of the samples at Lab with concrete H30

Figure 36: Fracture of the samples at Lab with concrete H50

4 MPM Shear Test

The test procedure for the determination of shear strength in the laboratory consists of a cylindrical sample with concentric notches on its upper and lower faces. The load is applied by compression at one end by the outer crown and in the central zone by the other, the cylindrical section between the notches being subjected to a pure shear state. In the MPM analysis we have a problem in 2D and it is considered a section of equal dimensions that of laboratory a thickness equal to 1, that is to say what we have really is a rectangular section, but the results are obtained in a section of equal dimensions That of the laboratory, so that it serves to see if the method works.

Several tests have been performed simulating a shear test with the Material Point Method (MPM). In order to see how the method works when a shear stress takes place, different meshes and different displacements imposed for calculations have been considered.

In the first place, a test was carried out with meshes of triangular elements and size 0.002. A stepwise displacement equal to -2E-7 was imposed. Taking into account that in the end this displacement gives us a force, which is what really happens in the laboratory.

At the following picture it could be seen the geometry used for the problem.

Figure 37: Configuration of the Shear test in GiD

And the configuration of the test is in the following manner

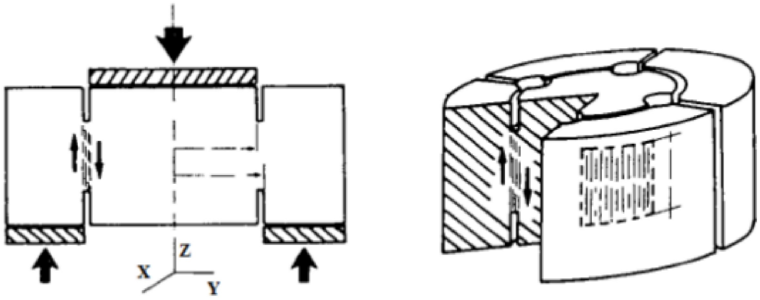

Figure 38: Configuration of the Shear test in lab

As you can see we have an H-30 concrete test piece, this test piece has 4 notches (”holes”). The displacement is imposed by the plates that are shown in dark gray. The central upper plate imposes a downward displacement, whereas the two inferior plates impose that same displacement value upwards. In this way the section between the notches is subjected to pure shear.

The strain-strain curve obtained with DEM has been compared with the one obtained for this case.

Evolution of the vertical stress with time in shear test

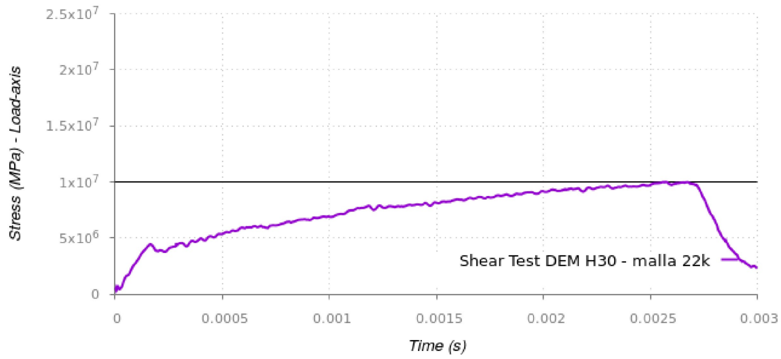

Figure 39: DEM evolution of the vertical stress with time in shear test

In the graph it can be seen that the maximum stress that the shear test piece supports is 1E7 Pa, that is to say 10 Mpa.
In the case of the MPM we have considered an average of the stresses at the loading surface of the top of the specimen. With this average voltage as a function of the number of steps we obtain the following curve

4.1 Analysis of 2D shear test

The results that will be shown in this section are the ones get with triangular meshes.

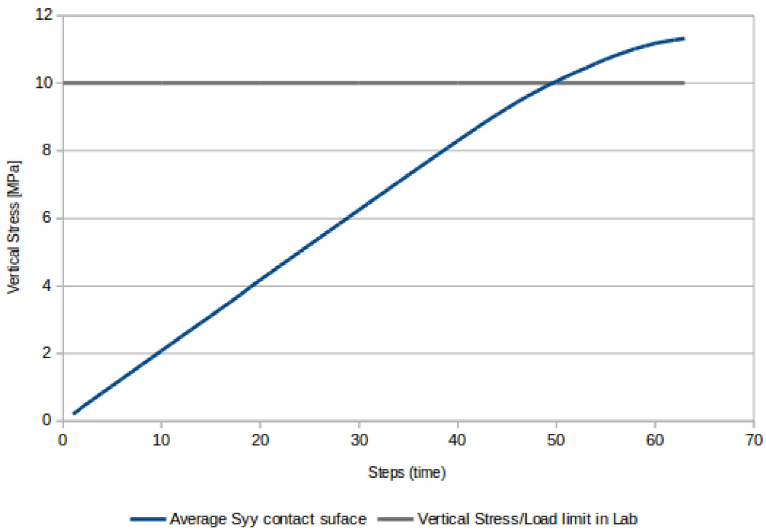

Figure 40: MPM evolution of vertical stress overtime in a shear test

In the case of MPM, the resistance obtained with shear is 11.3237, which is quite close to the value of the laboratory and to that obtained with DEM, it is verified that the model works correctly.

In the previous image the sample with an increased deformation is shown (coefficient of increase of 234), clearly it is possible to be observed that due to the imposed displacements the probe in the upper part moves towards the center.

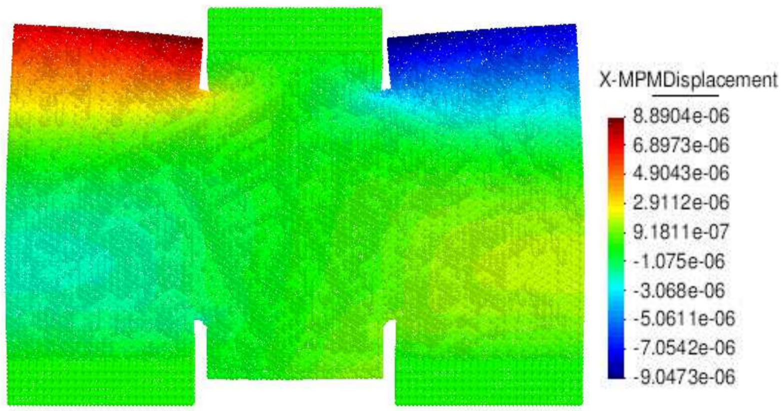

Figure 41: MPM evolution of vertical stress overtime in a shear test

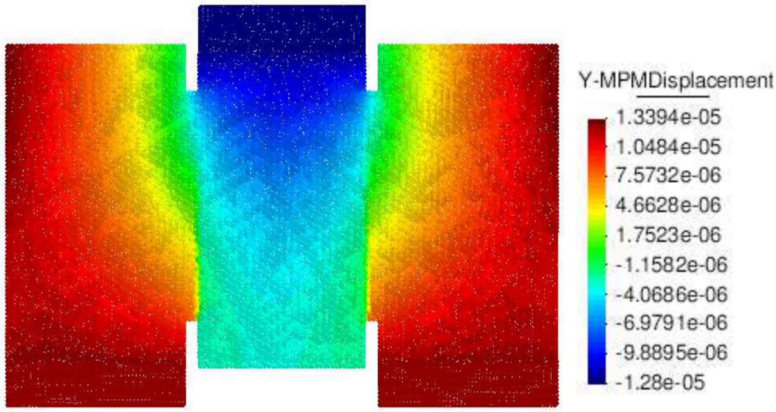

Figure 42: MPM evolution of vertical stress overtime in a shear test

In this figure it can be clearly seen that there is a discontinuity of vertical displacements just in the section between the notch and the central part of the specimen. This discontinuity makes a lot of sense, and must be so because the central part of the specimen moves down while the two sides move upwards. It is just in the sections of the notches where the probe breaks. This observation can be clearly seen in the image of the damage distribution shown below.

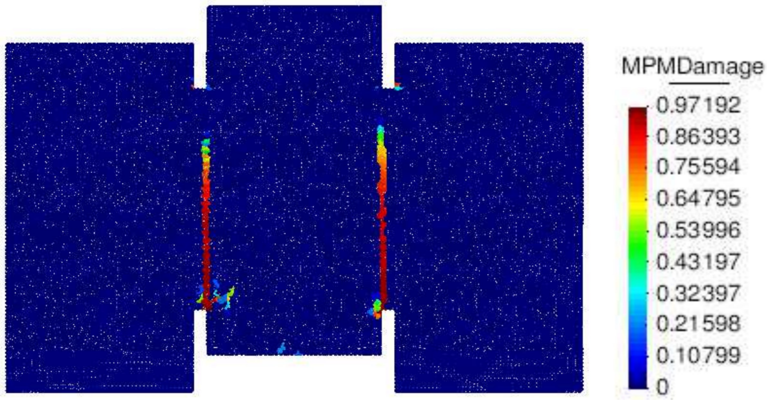

Figure 43: MPM evolution of vertical stress overtime in a shear test

As mentioned before, the specimen breaks right into the sections of the notches, you see a line with damage almost equal to 1 in that section which means that the specimen breaks, we have no resistance on that surface.

MPM results for quadrilateral meshes in a shear test

On the other hand the simulation has been done using meshes with square elements to see with which better results are obtained. In this case meshes with square elements of size 0.001 have been used, and a displacement of 1E-7 m has been imposed.

MPM stress evolution overtime in a shear test

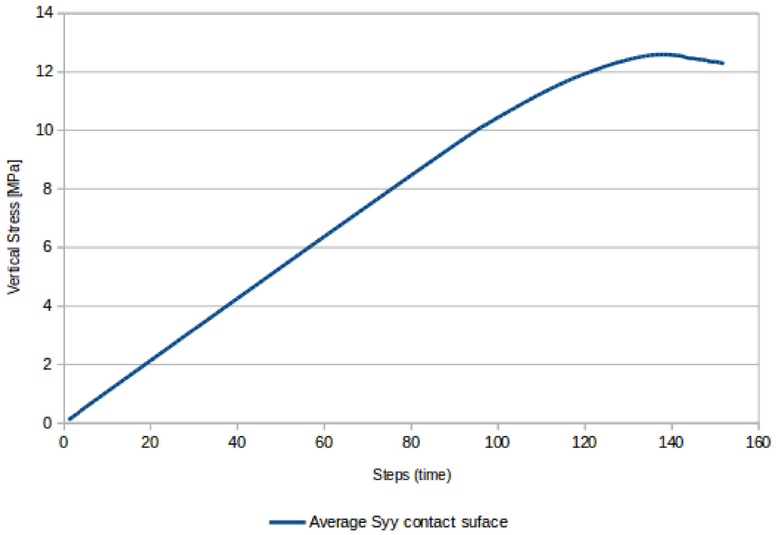

Figure 44: MPM evolution of stress overtime in shear test

It could be seen again that we get a maximum shear stress with a value of 12.29 MPa that is very similar to the value of lab, so it could be conclude that it is a good approximation in order to know how is going to be the material behaviour under shear stress.

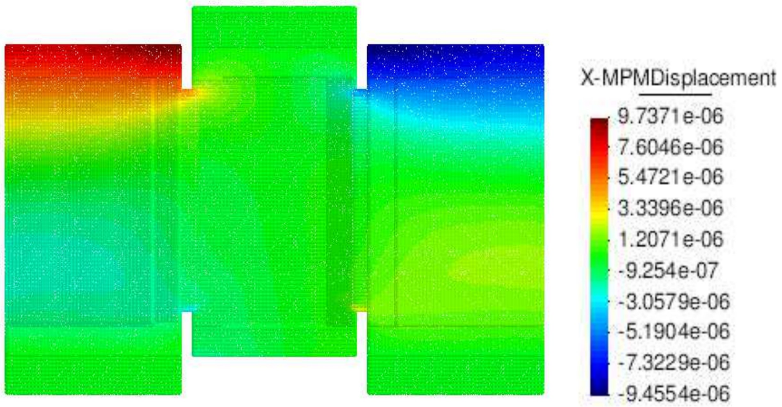

Figure 45: MPM horizontal displacement in the sample

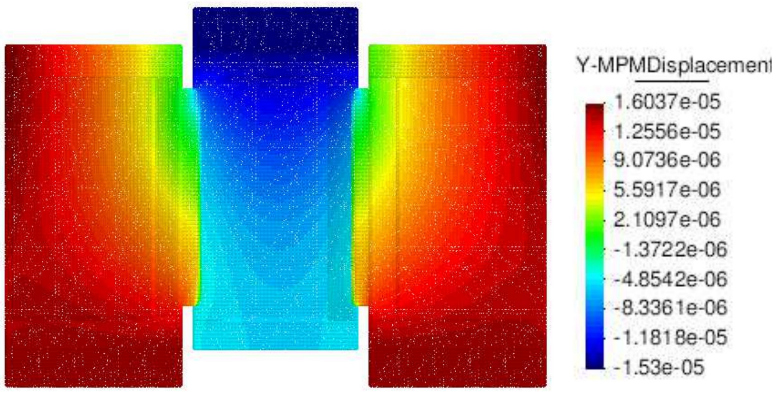

Figure 46: MPM vertical displacement in the sample

In this case the displacements are much more pronounced than with the other mesh. A more advanced damage state is also reached and it makes sense for the specimen to break and the parts to separate. In the damage figure shown below we can see that a damage equal to 0.99297 is reached while the mesh of triangles damage was 0.97192.

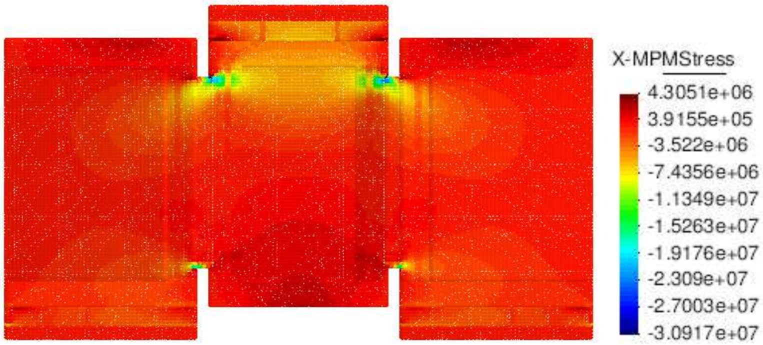

Figure 47: MPM Horizontal stress

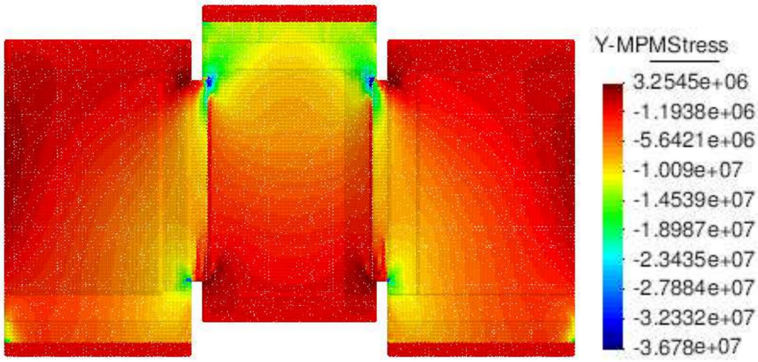

Figure 48: MPM Vertical stress

As for the vertical stresses it can be seen that the higher and concentrated stresses are just in the section between the central part of the test tube and the notch. On the other hand the tensions in the final state show a discontinuity because the probe breaks.
MPM damage distribution on the concrete sample

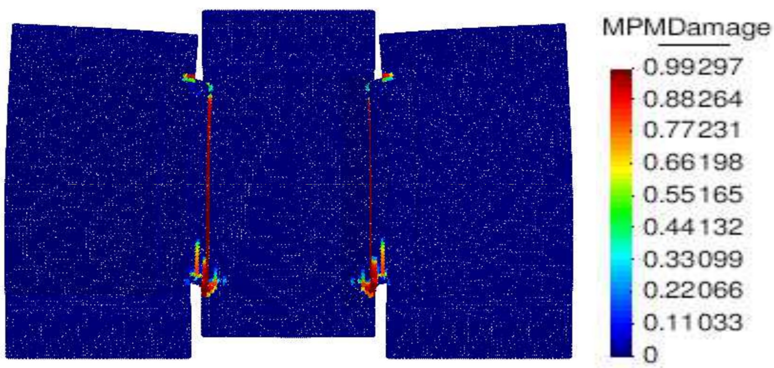

Figure 49: MPM Vertical stress

It can be observed that the square meshes give better results than the triangular ones. The figure above shows the damage to the specimen, it can clearly be seen that this distribution has a much finer and longer break line than in the case of a triangular mesh. The result we get is the expected and the test at the end of the test would end up breaking into 3 pieces.

Here we can see the total displacement of the sample for this case using MPM and using DEM

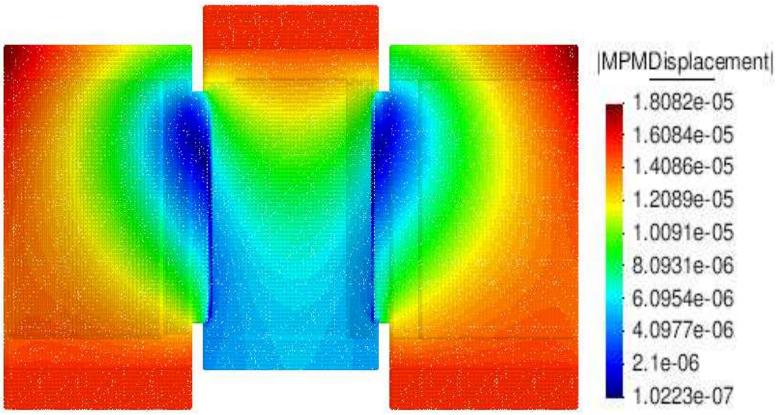

Figure 50: MPM Vertical stress

DEM total displacement in a shear test

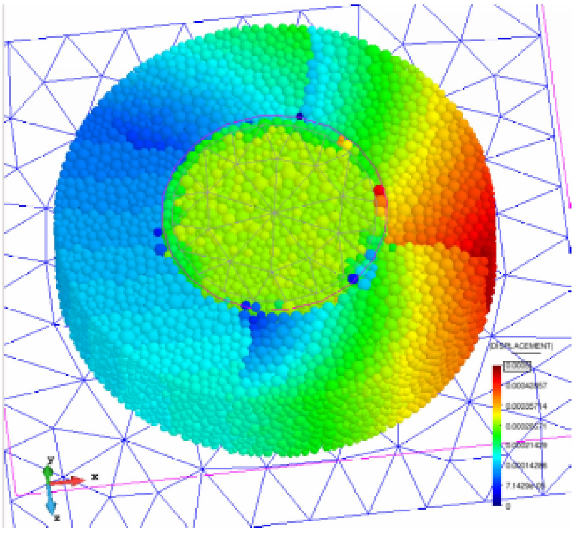

Figure 51: Total displacement in a DEM analysis

4.2 Analysis of 3D shear test

As in the other 2 cases, a 3D analysis has been done for a shear test. Below are shown the results obtained. Note: For this we do not plot any stress graphs because a cutting section could not be done in this analysis.

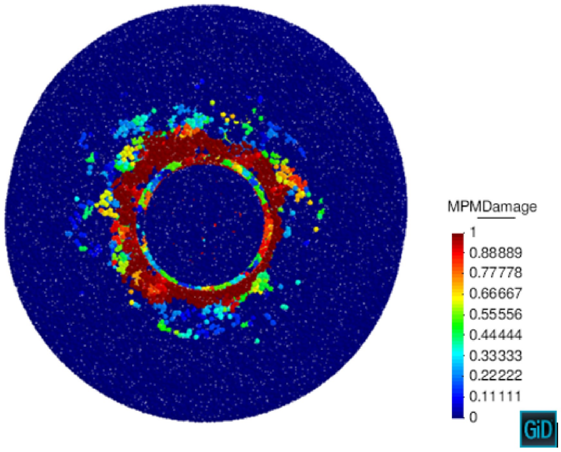

Figure 52: MPM damage distribution in a 3D shear test

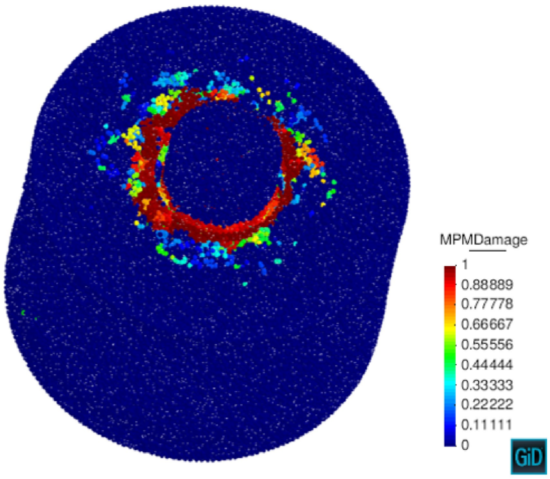

Figure 53: MPM damage distribution in a 3D shear test

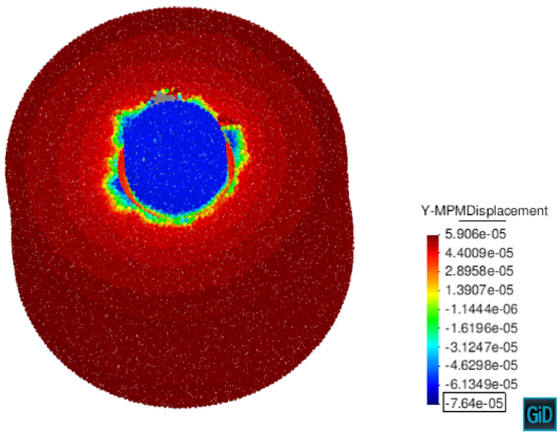

Figure 54: MPM vertical displacement in a 3D shear test

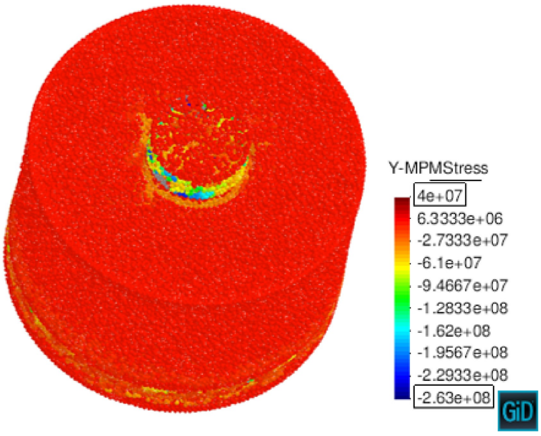

Figure 55: MPM vertical stress in a 3D shear test

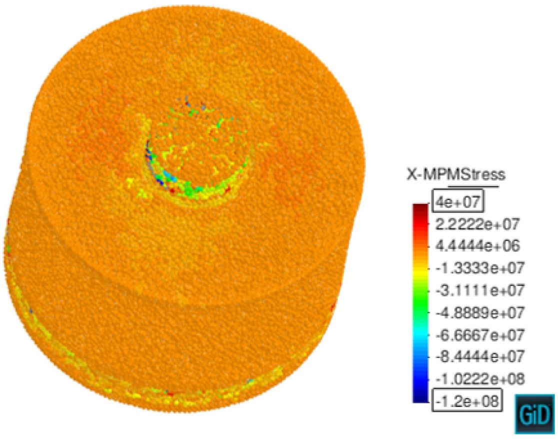

Figure 56: MPM horizontal stress in a 3D shear test

4.3 Laboratory results for a shear test

As can be seen the way o fracture is similar, so the results with MPM are good aproximations Laboratory results for a shear test

Figure 57: Fracture of H30 sample in a shear test

Figure 58: Fracture of H50 sample in a shear test

5 Conclusions

Throughout the industrial training the results that have been obtained are quite good and similar to those obtained with other methods. So our code works although there are aspects that should be improved. For example, as we have seen, the distribution of tensions in some cases is not what it should be. In spite of this, information can be obtained on how the breakage occurs, the displacements and the deformation tension curves.

It would therefore be necessary to look for some way to improve the weak points of the method. As a first option to improve the results, we think of a UP formulation, which depends as much on the displacements as on the pressures. Despite this, this UP formulation can not be used for the MPM case and new alternatives have been sought.

One way to improve the results obtained with MPM could be using the Improved Material Point Method (IMPM). But even though IMPM provides a general framework for improving MPM, it has limitations in terms of how to apply boundary conditions in complicated geometry. An interesting situation is how to make the conditions of the contour in a situation where the body boundary does not match the grid. In MPM and IMPM we apply the boundary conditions at the outer grid points closest to the body instead of imposing boundary conditions at the exact boundary. As a result, errors are introduced. As a future line of research it would be interesting to find out how to improve the handling of the contour conditions in complicated geometry.

One possible approach is to use ideas from the submerged boundary method, the submerged interface method, the dummy domain method.For example, in the method of immersed limits, the body is immersed in the background grid, where the body boundary also does not match the background grid. In such a situation, It may be useful to use ghost cells or lagrangian multipliers.

Another line of interesting research would be to improve IMPM for an arbitrary order. To achieve this, have to be combined ideas from non-mesh particle methods and finite element methods. So in conclusion there is a long road of research ahead to improve the MPM and IMPM, and it would be interesting to study the different behaviors that can be obtained.