Beams Assignment

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1 Problem 1

The shear stiffness matrix is modified in the original Timoshenko code to get the reduced integration form.

$$K_s = \frac{GA}{l}\begin{bmatrix} 1 & l/2 & -1 & l/2 \\ .. & l^2/4 & -l/2 & l^2/4 \\ .. & .. & 1 & -l/2 \\ .. & .. & .. & l^2/4 \end{bmatrix}$$

The stresses are calculated using only one Gauss point at zero instead of two points. This is modified in the stress calculation code.

2 Problem 2

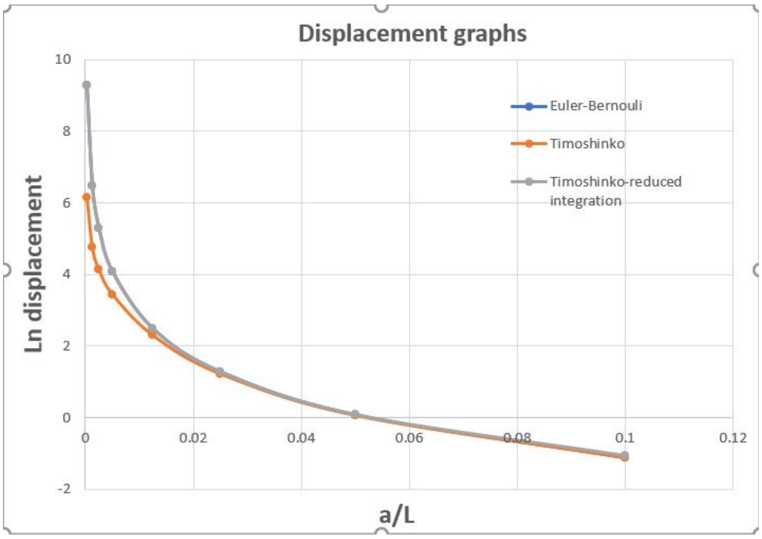

Figure 1: Displacement graphs for the 3 elements

As shown in the above graph, it should be noted that the displacements decrease as the a/L ratio decreases. That is because the area increases so the material withstands the deformation. At low a/L (thin beams), Timoshenko theory provides inaccurate results, and the other 2 elements coincide. At high values of a/L, the 3 types of elements coincide with each other.

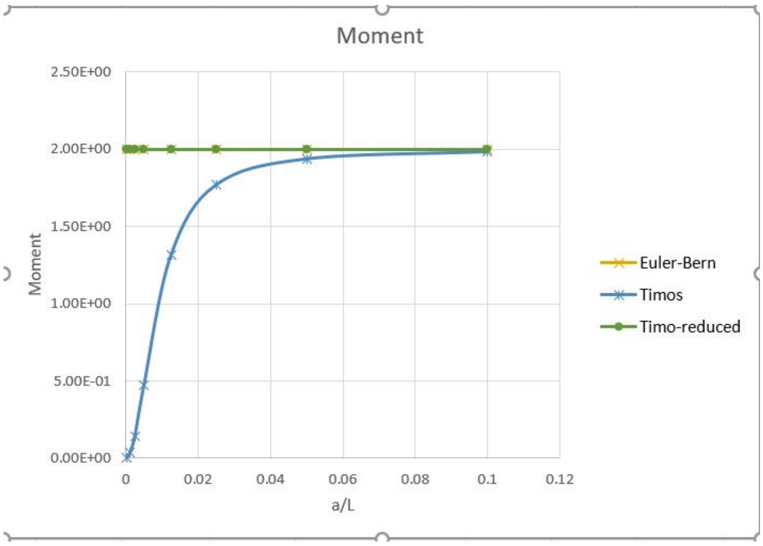

Figure 2: Moment graphs for the 3 elements

As shown in the above graph, the Euler-Bernolli and Timoshenko elements provide similar results for the moment which are accurate for both thin and wide beams. However, the solution using Timoshenko full integration elements provide poor results for thin beams and as the ratio a/L increases, the results become better.

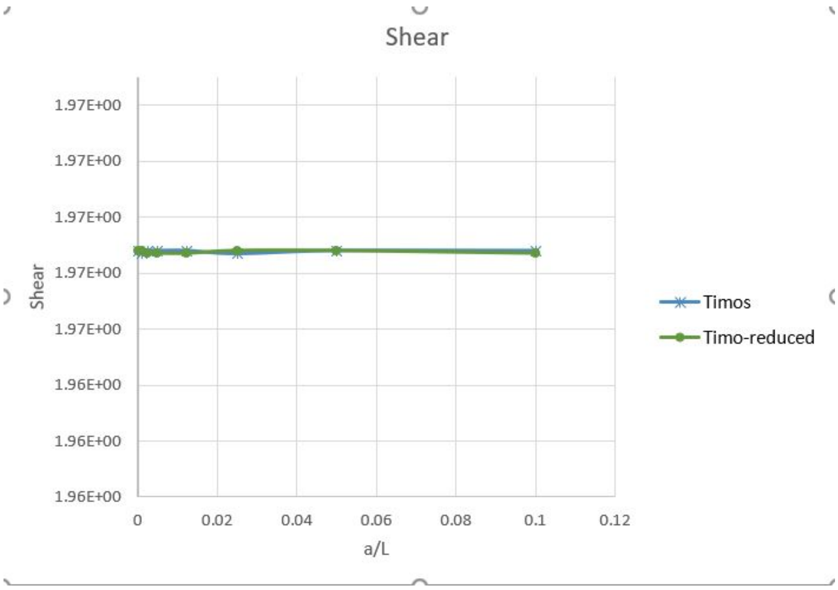

Figure 3: Explicit, Picard, and Newton-Rhapson methods ($\epsilon = 0.0001, dt = 0.005$)

The above graph is only for Timoshenko and Timoshenko reduced elements. That's because Euler-Bernolli's theory assumes no shear is induced in the beam. The results for both methods almost coincide with each other.