

**COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS**  
**Master of Science in Computational Mechanics/Numerical Methods**  
**Spring Semester 2019**

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Assignment 8: Shells

Analyse the following concrete hyperbolic shell under self-weight. Explain the behaviour of all the stresses presented.

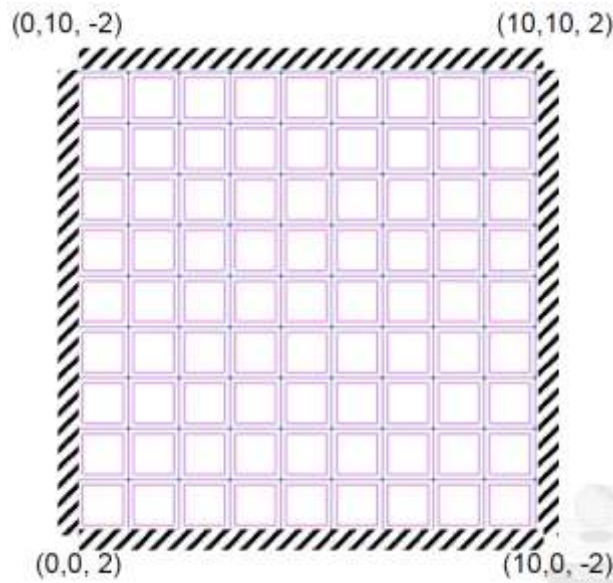


Figure 1: Hyperbolic shell

To simulate the shell, the input file has been written. The variables declared are the following:

- **Material properties:** As the material is concrete, the Young modulus has been chosen as  $E = 3 \cdot 10^{10} \text{Pa}$ , the Poisson ratio is  $\nu = 0.2$ . The material density has been estimated of  $\rho = 2500 \text{ kg/m}^3$ . The thickness is of 0.1m

```
% Material Properties
```

```
young = 3e10 ;  
poiss = 0.2 ;  
thick = 0.1 ;  
denss = 2.5e3 ;
```

Figure 2: Material properties

- **Coordinates matrix:** As the shape is bilinear, the geometry has been defined using Lagrange interpolation functions:

```

% Coordinates

global coordinates
coordinates = zeros(100, 3);

for i = 1.0:100.0
    x = mod(i-1, 10) * 10.0 / 9.0;
    y = floor((i-1)/10) * 10.0 / 9.0;
    coordinates(i, 1) = x;
    coordinates(i, 2) = y;

    coordinates(i, 3) = 2*((10-x)*(10-y))/((10-00)*(10-00))-...
        2*((10-x)*(00-y))/((10-00)*(00-10))-...
        2*((00-x)*(10-y))/((00-10)*(10-00))+...
        2*((00-x)*(00-y))/((00-10)*(00-10));
end

```

Figure 3: Coordinates matrix

- **Connectivity matrix:** The elements used are triangular so the connectivity matrix has been written as:

```

% Elements

global elements
elements = zeros(162, 3);

for i = 1:81
    el = i + floor((i-1)/9);
    elements(1 + (i-1)*2, :) = [el, el + 1, el + 10];
    elements(2 + (i-1)*2, :) = [el + 1, el + 10, el + 11];
end

```

Figure 4: Connectivity matrix

- **Prescribed displacements:** All nodal displacements have been prescribed in all boundary nodes:

```

% Fixed Nodes

fixdesp = zeros(108, 3);
for i = 1 : 10
    fixdesp(1 + (i-1)*3, :) = [i, 1, 0.0];
    fixdesp(2 + (i-1)*3, :) = [i, 2, 0.0];
    fixdesp(3 + (i-1)*3, :) = [i, 3, 0.0];

    fixdesp(1 + (i-1)*3 + 78, :) = [i + 90, 1, 0.0];
    fixdesp(2 + (i-1)*3 + 78, :) = [i + 90, 2, 0.0];
    fixdesp(3 + (i-1)*3 + 78, :) = [i + 90, 3, 0.0];
end
for i = 1 : 8
    fixdesp(1 + (i-1)*3 + 30, :) = [i*10 + 1, 1, 0.0];
    fixdesp(2 + (i-1)*3 + 30, :) = [i*10 + 1, 2, 0.0];
    fixdesp(3 + (i-1)*3 + 30, :) = [i*10 + 1, 3, 0.0];

    fixdesp(1 + (i-1)*3 + 54, :) = [i*10 + 10, 1, 0.0];
    fixdesp(2 + (i-1)*3 + 54, :) = [i*10 + 10, 2, 0.0];
    fixdesp(3 + (i-1)*3 + 54, :) = [i*10 + 10, 3, 0.0];
end

```

Figure 5: Prescribed displacements

- **Loads:** As in the problem there are no external loads apart of the self-weight the vectors have been left empty:

```

% Point loads

pointload = [ ];

% Side loads

sideload = [ ];

```

Figure 6: Loads

## Results

After performing the simulation, the results are visualised with GiD:

### Displacements:

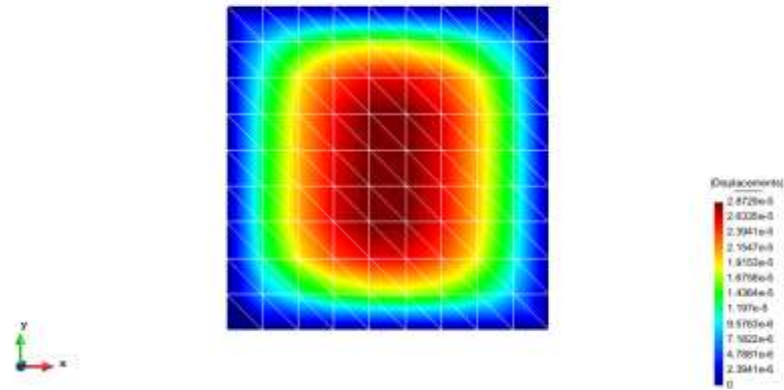


Figure 7: Displacements

As expected, the displacements are larger in the central part of the structure. They are mainly in the z-direction.

### Membrane stresses

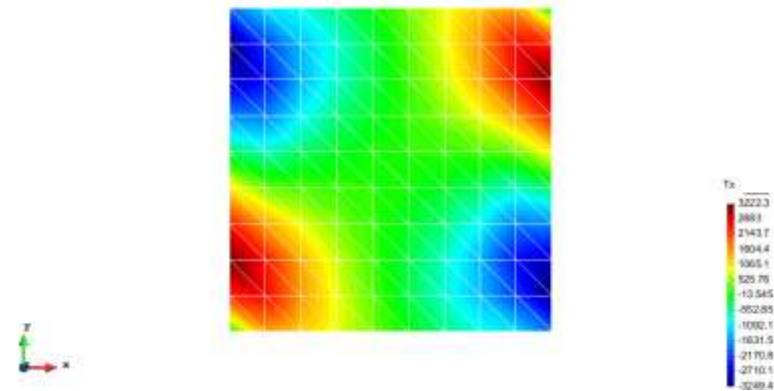


Figure 8: Tx

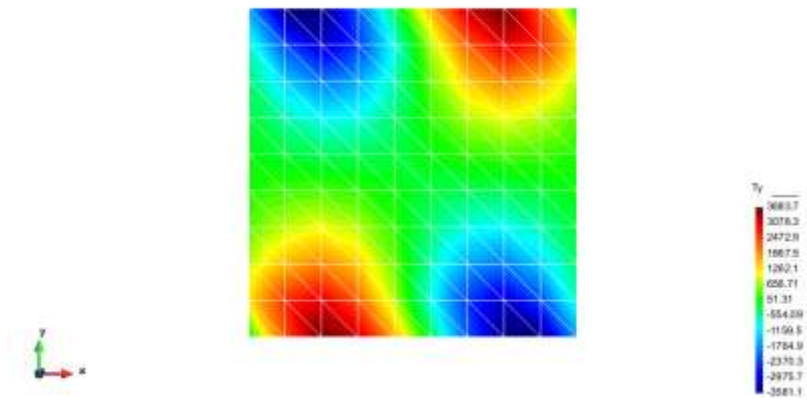


Figure 9:  $T_y$

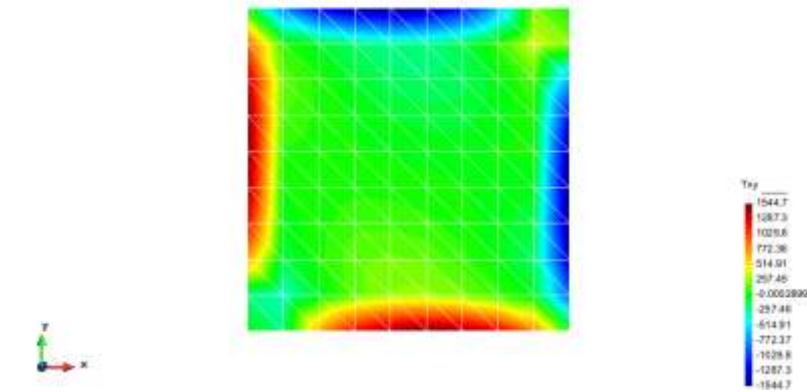


Figure 10:  $T_{xy}$

The membrane stresses are concentrated on the boundary of the domain and preserve symmetry around of the diagonal.

## Moments

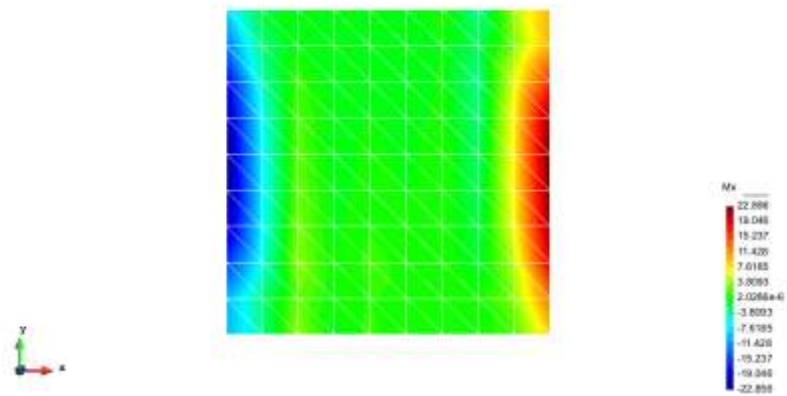


Figure 11:  $M_x$

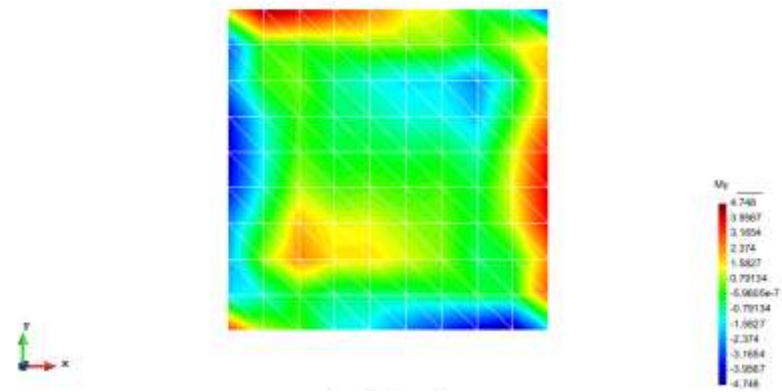


Figure 12:  $M_y$

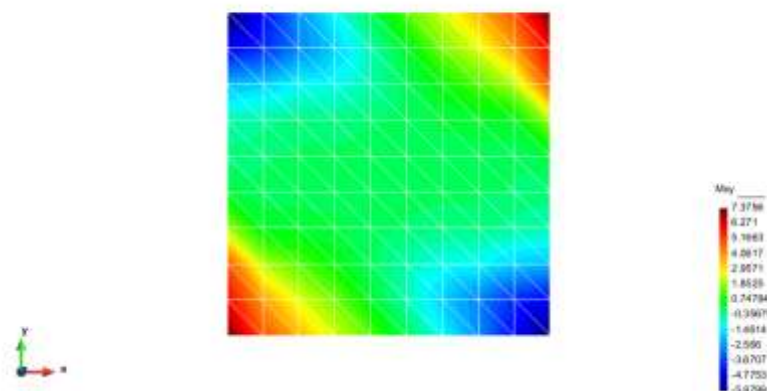
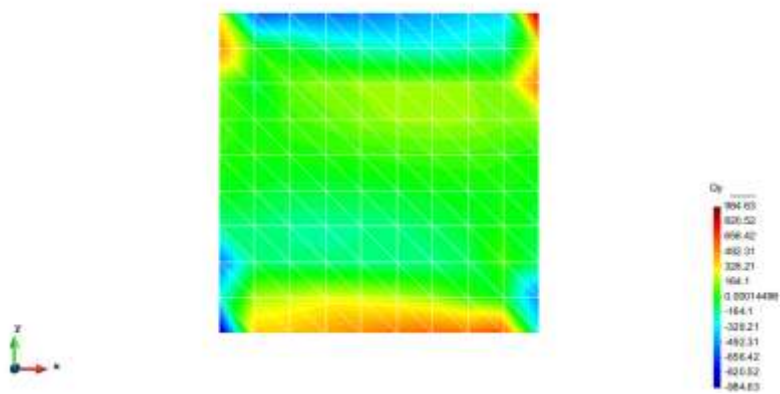
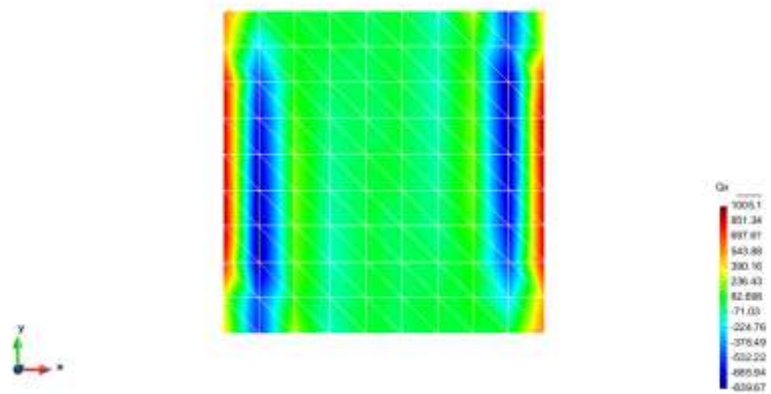


Figure 13:  $M_{xy}$

As in the membrane stresses, the moments are concentrated on the edges of the geometry and each component preserves some kind of symmetry or anti-symmetry. It is interesting to note that  $M_x$  and  $M_y$  present very different behaviour.

## Shear



Again, the  $Q_x$  and  $Q_y$  shear forces are concentrated around the boundary and present anti-symmetry around the  $y$  and  $x$  axis respectively. In this case, although the behaviour is not equal is similar in magnitude.