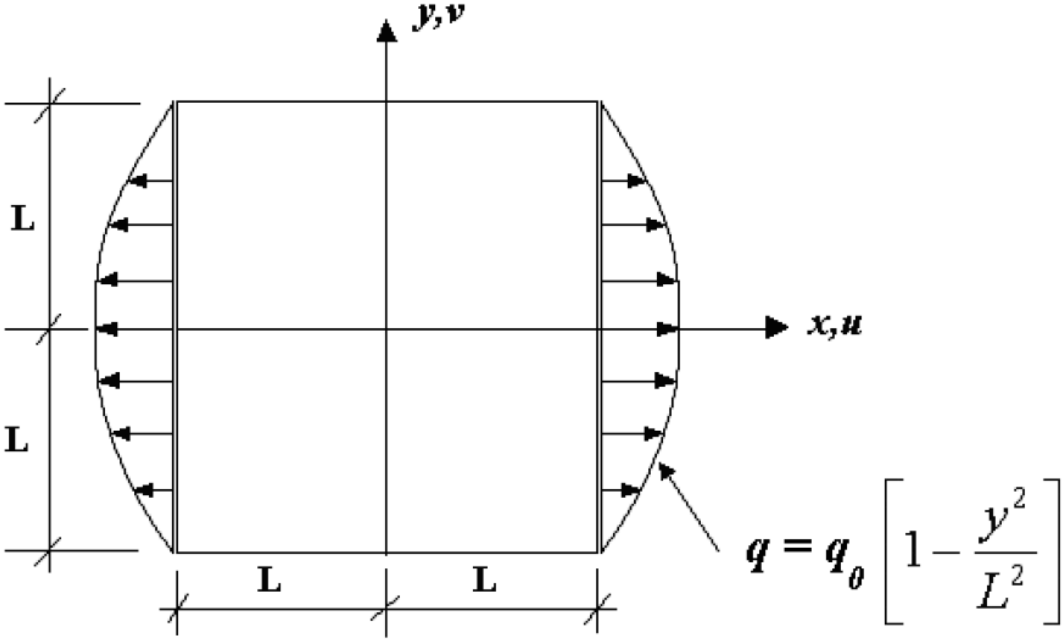
Computational Structural Mechanics and Dynamics

Practice 1

Jose Andino Saint Antonin

Exercise 1: Analysis of a thin plate under parabolic tensile force

Analyze the thin plate shown in the figure, which is submitted to a parabolic load. Compare the obtained results with the solution that is obtained when refining the mesh. Use triangular elements with 3 and 6 nodes and quadrilaterals with 4, 8 and 9 nodes. Use symmetry conditions to simplify the problem.



Data

$$\text{Material} \begin{cases} E = 2.1e5\ \text{MPa} \\ \nu = 0.30 \\ \\ \text{Thickness} = 0.10\ \text{m} \end{cases}$$

$L = 1.0\ \text{m}$

$q_\theta = 100 \dfrac{\text{MN}}{\text{m}}$

Solution A:

The problem was solved by taking advantage of the two axis of symmetry of the plate. Therefore, only ¼ of the whole plate was modelled. For each edge, we fixed number of elements to 5 (6 nodes) and made two grids, one with triangles and another one with squares.

The main difficulty found was the representation of the parabolic load. The software only allows constant or linear loads. Therefore, we had to manually enter a discretized version of the loads into 6 node on the edge where the pulling occurs:

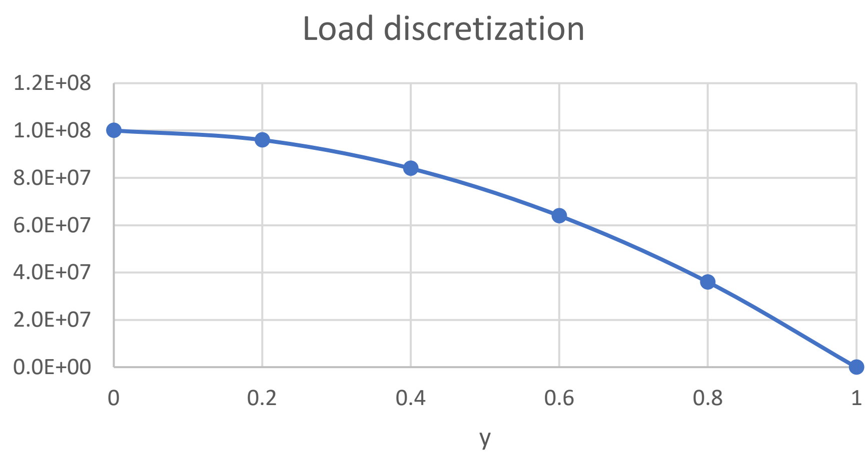


Figure 1 discrete representation of the loads

The edges of symmetry were prescribed with displacement conditions. For the vertical axis of symmetry, zero displacement in the x direction was prescribed (loads on both sides are symmetric). For the horizontal axis of symmetry, zero displacement in the y direction was prescribed.

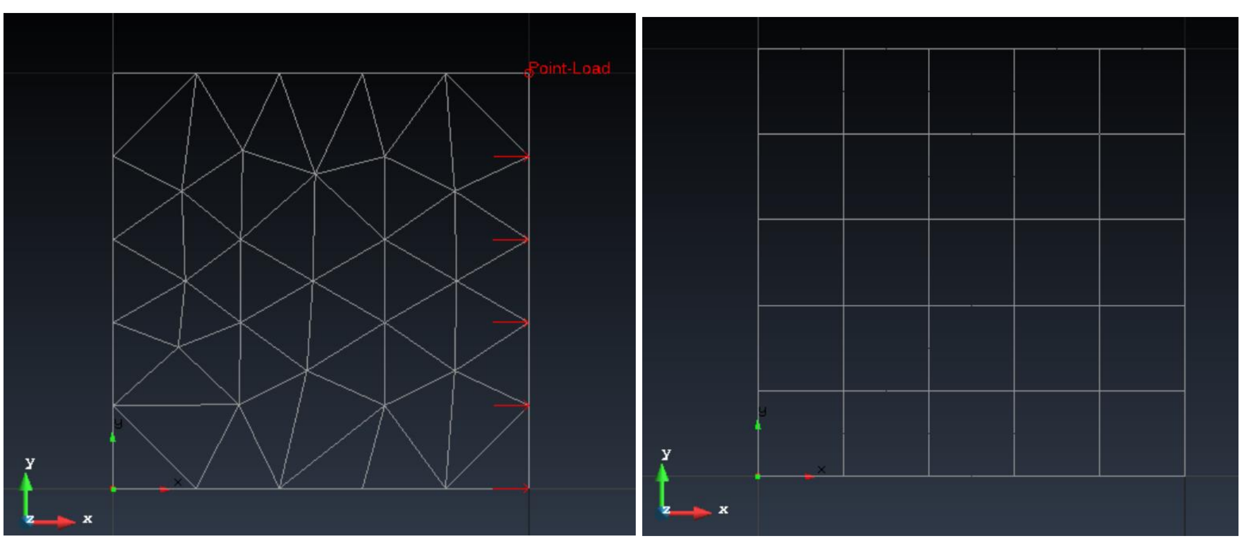


Figure 2 triangular and square meshes

The simulation was carried out using triangular elements with 3 and 6 nodes and quadrilaterals with 4, 8 and 9 nodes. The displacements in the x and y direction are shown in the following figures for all these cases.

First of all, the results are consistent across the range of elements and order of the elements. Higher order elements simply provide more detail & resolution.

It is interesting to note that discretization of the parabolic load seems to work well for 3 node triangles. In the rest of the cases, and especially in the case of high order quads, there are artifacts related to the discretization of the loads into node loads as opposed to correctly representing the load as a linear function or better, quadratic function on the edge of the elements. If this had been possible, the results of the simulation with high order quads would have been much smoother.

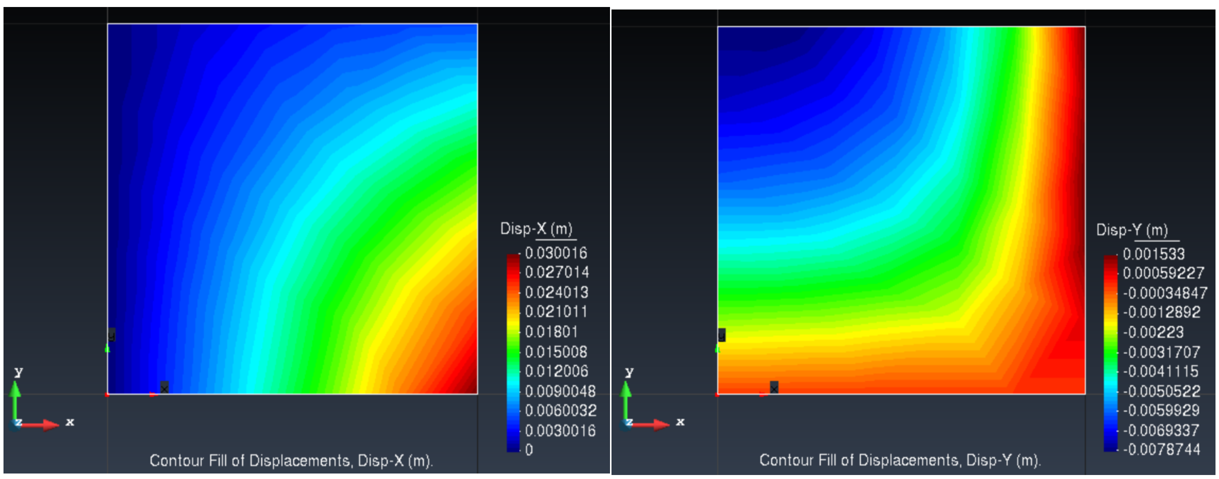


Figure 3 linear shape functions on triangular elements – X and Y displacements

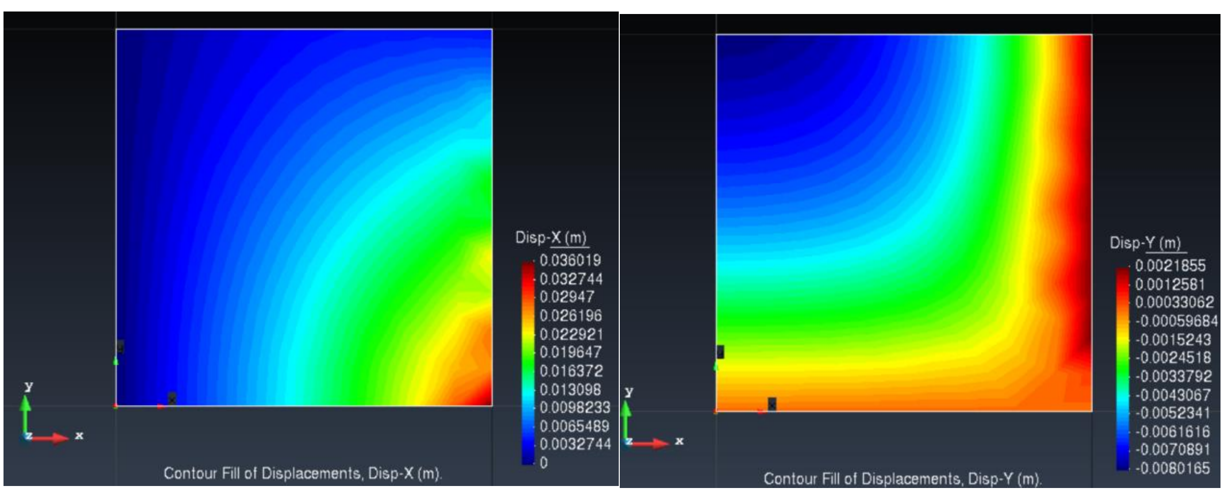


Figure 4 quadratic shape functions on triangular elements– X and Y displacements

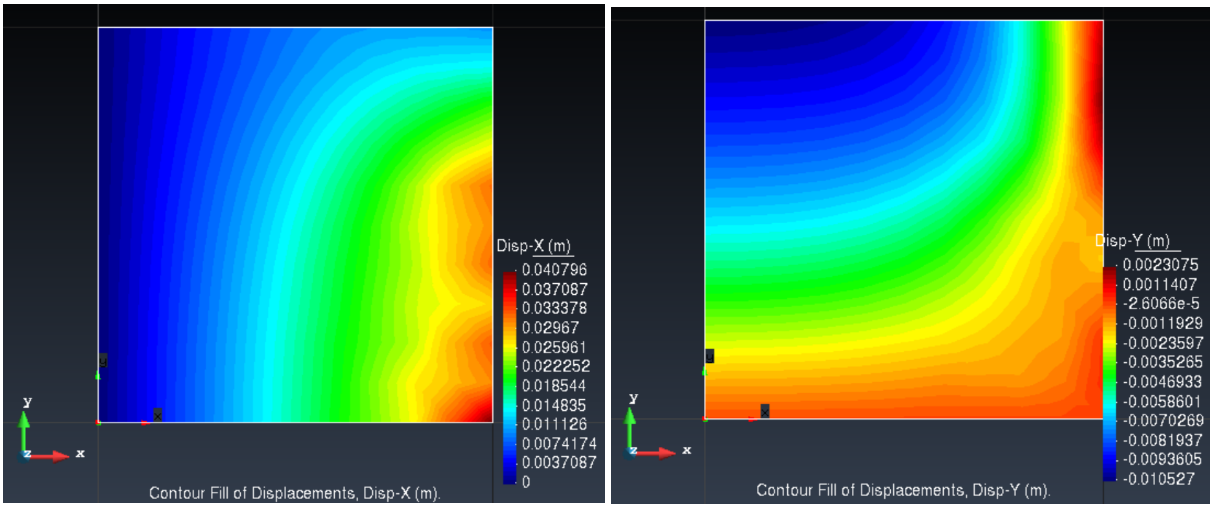


Figure 5 linear shape functions for quadrilaterals (squares in this case)

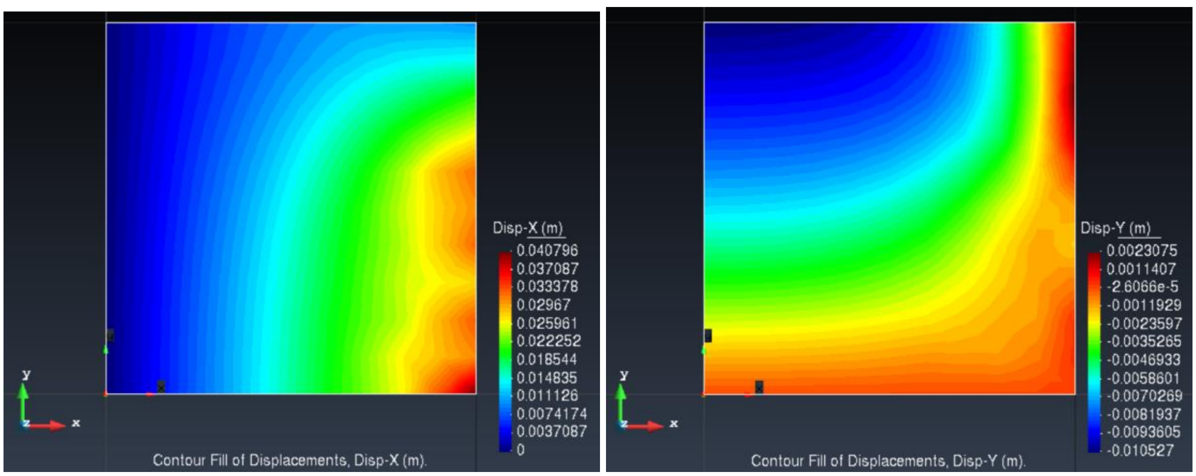


Figure 6 8-point square element results

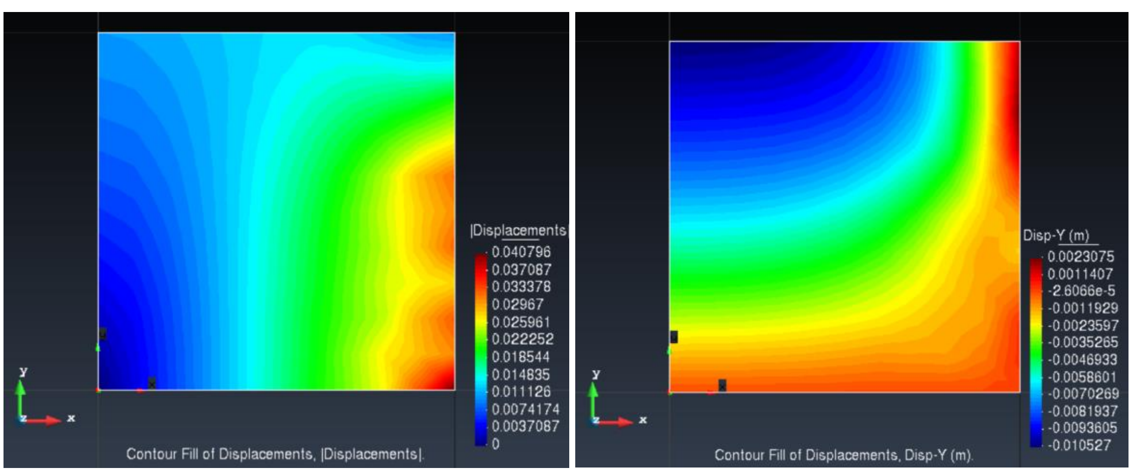


Figure 7 9-point square element results

It is also worth noting that the displacement vectors are pointing in the positive x direction at the bottom of the simulated square. This is consistent with the prescribed zero vertical displacement in this region.

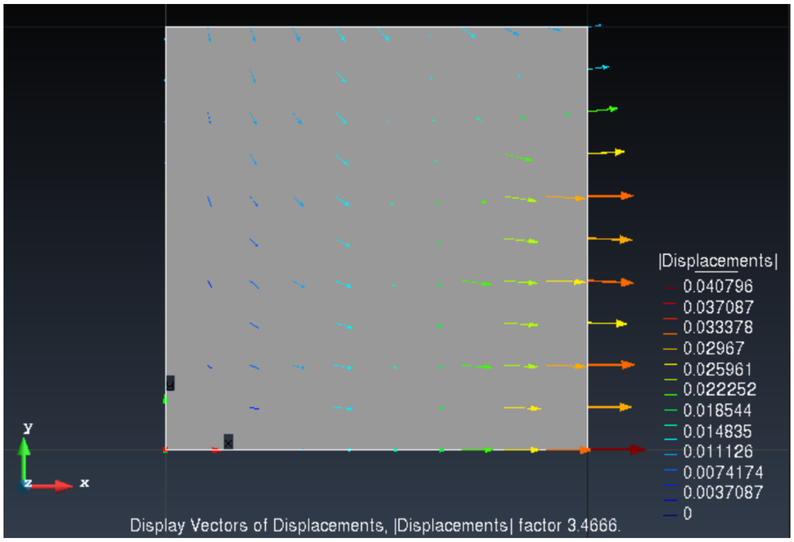


Figure 8 displacement vectors

Exercise 2: Plate with two sections.

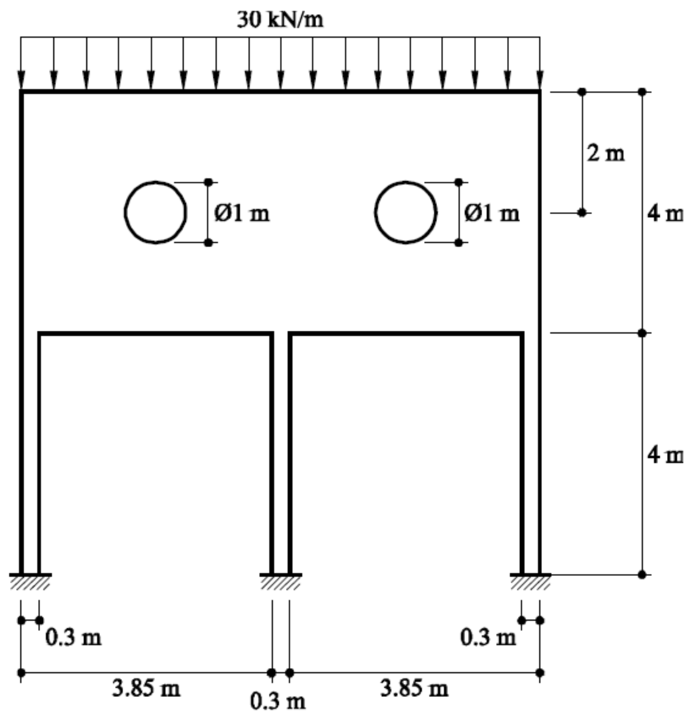
The structure in the figure presents a reinforced concrete plate with two holes, supported by three columns. The central column undergoes a displacement δ due to sag of the foundation caused by a leakage in some pipes nearby.

Analyze the distribution of the stresses that the drop of the central column produces.

-Case I: Dead weight + Uniform load

-Case II: Dead weight + Uniform load + Settlement of the central column

Assume the hypothesis of plane stress. Use triangular elements with 3 nodes for the analysis.



Data

$$E = 3.0e10 \frac{N}{m^2}$$

$$\gamma = 25000 \frac{N}{m^3}$$

$$\nu = 0.2$$

$t = 0.20\ m$ (Thickness of the plate and the columns)

Solution

The problem was gridded as shown in the figure:

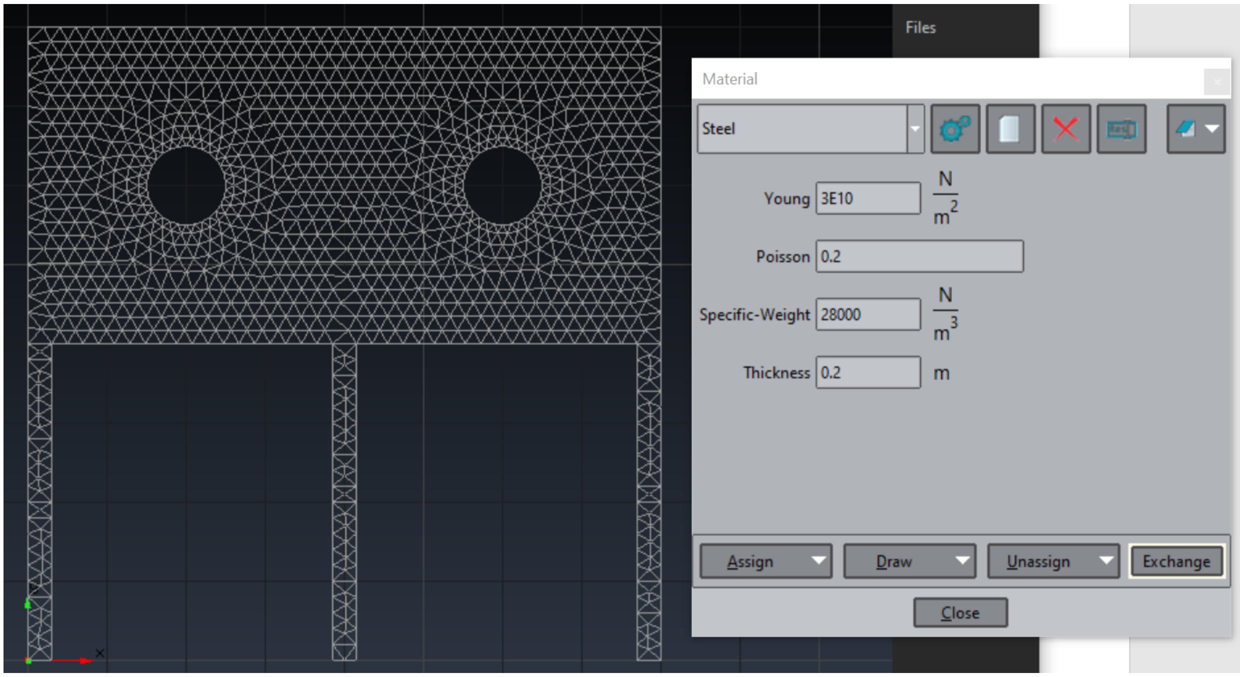


Figure 9   mesh and material properties

To tackle the issue of the central column drop, we assumed a delta of 0.05m. The column is being pulled by its subsiding foundation. The loads on top are 30kN/m. Since the length is 8m therefore and has 41 nodes, the 240kN distributed in 41 nodes, 5.71kN per node (again, the loads are represented at node level, which works well for linear triangles.

Case I: This shows no subduction and it is a 'baseline' to compare the subduction case with. The following figures show x and y displacements and stresses:

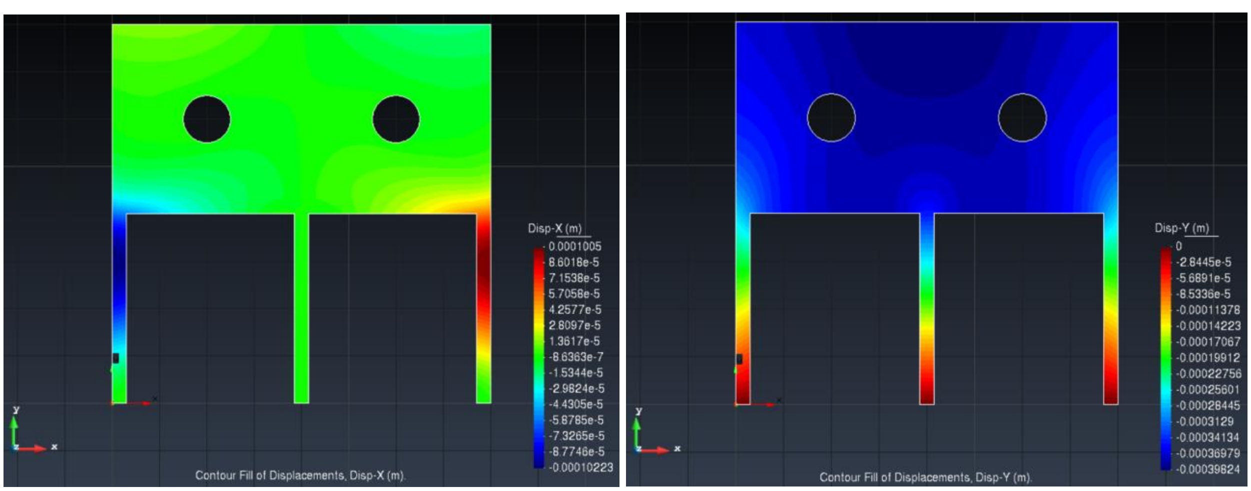


Figure 10 X and Y displacements for the case without subduction

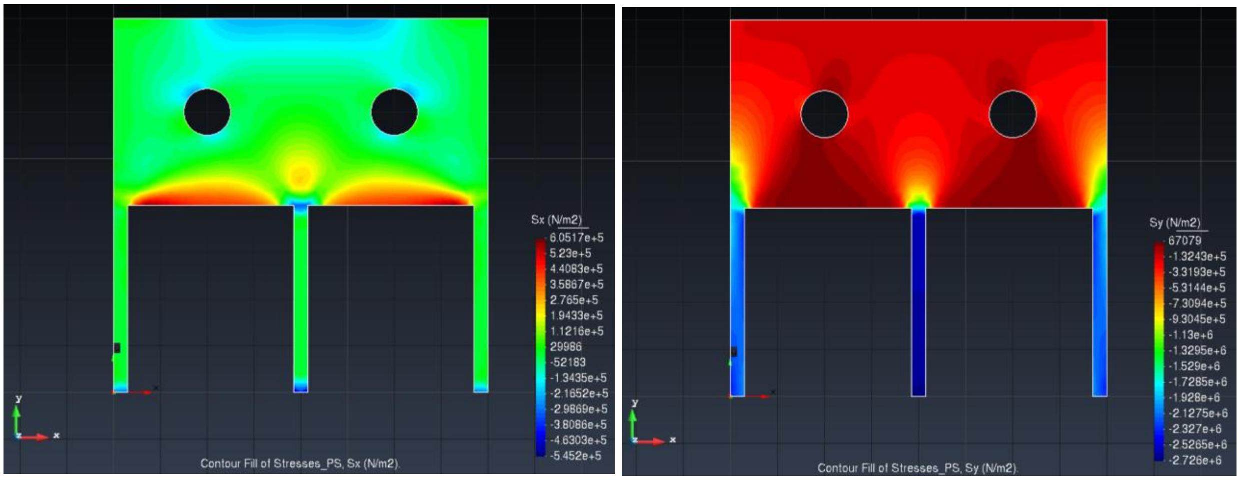


Figure 11 X and Y stresses, no subduction case

Case II: subduction

Now the central colum is pulled down by its foundation 5cm (displacement prescribed in the base of the colum of y=-0.05m)

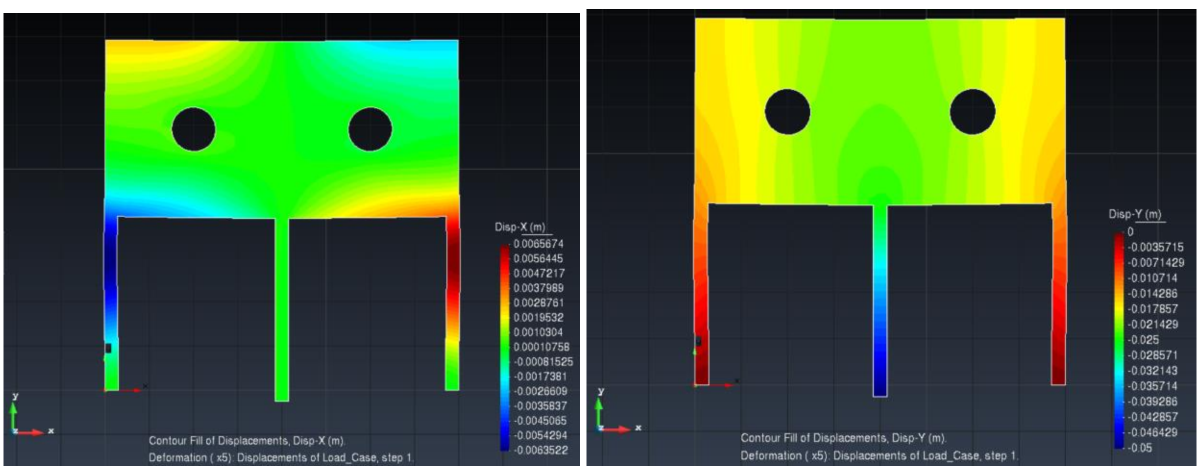


Figure 12 displacements for the subduction case

As we can observe, the column on the left and right experience displacements towards the outside of the structure. Additionally, the lateral columns compress with the central column is getting stretched. This is also reflected in the stressed (see below) where the y stress on the lateral columns is negative (and large) and in the central column it is positive and large.

It is interesting to point out that the 'belly' of the structure where the central column is pulling it experiences tension in the x direction, while the top of the structure experiences compression in the x direction.

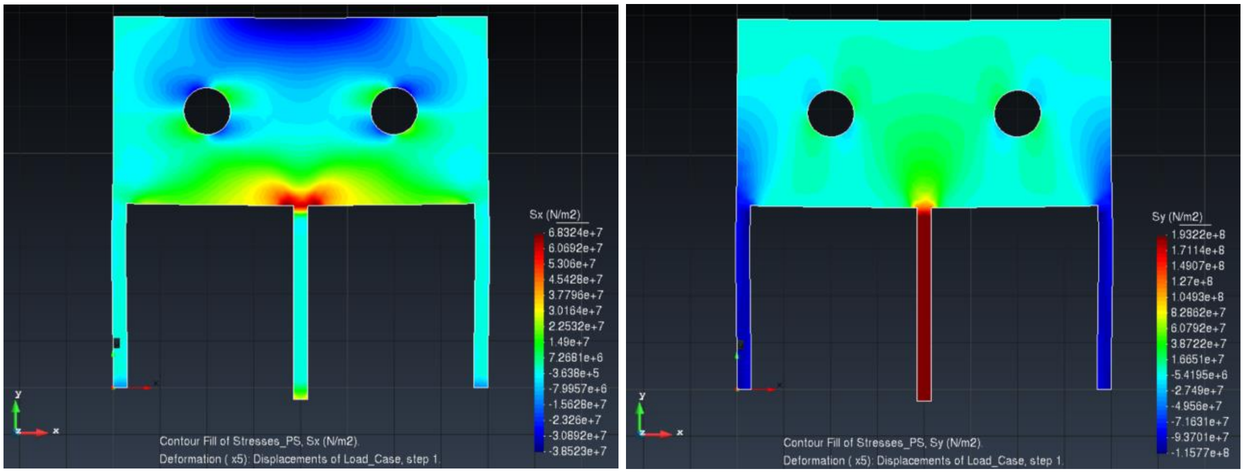
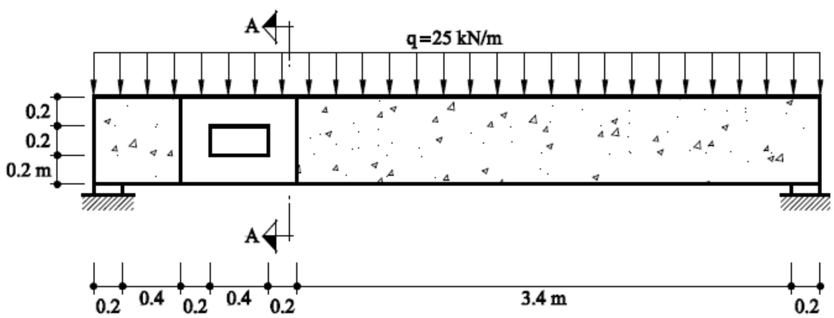


Figure 13 X and Y direction stresses for the subduction case. Notice that the condition set of displacement y=-0.05 on the base causes the central column to be on tension

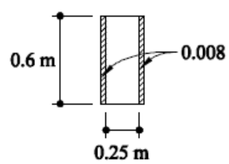
Exercise 3: Plate with ventilation hole.

The structure represents a reinforced concrete plate with simple supports. This plate possesses a hole for a ventilation pipe. Due to a change in the initial project, the design load for which the plate was calculated increased significantly. This motivated the placement of a metal reinforcement sheet on both sides of the plate in the area of the hole.

Analyze the state of stress in the plate and the metal reinforcement sheets. Assume the plane stress hypothesis. Use structured mesh of quadrilateral elements with four nodes.



A-A



Data

$$\text{Concrete}\begin{cases} E = 3.0e10\dfrac{N}{m^2} \\ \nu = 0.2 \\ t = 0.25\,m \\ \gamma = 25000\dfrac{N}{m^3} \end{cases} \qquad \text{Steel}\begin{cases} E = 2.1e11\dfrac{N}{m^2} \\ \nu = 0.3 \\ t = 0.016\,m\ (\text{Two sheets of } 0.008m) \\ \gamma = 78000\dfrac{N}{m^3} \end{cases}$$

Solution

The geometry and gridding were modeled in GID. Adequate properties were used for the concrete and metal portions of the plate. The elements used are linear quads (squares in fact), which provide grid quality excellent (all the cell angles are the same). All this is shown in the three figures below:

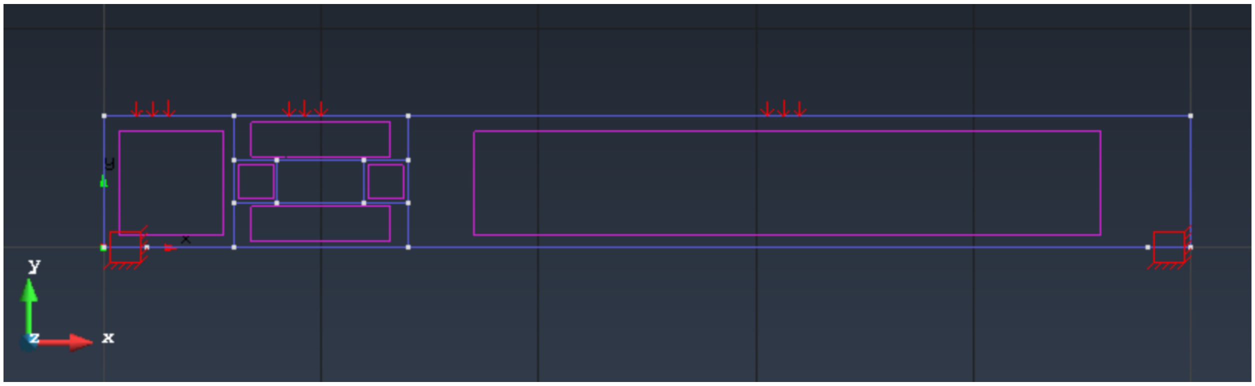


Figure 14 geometrical model with displacement and load conditions

Figure 15 Concrete (blue) and steel (grey) portions of the plate

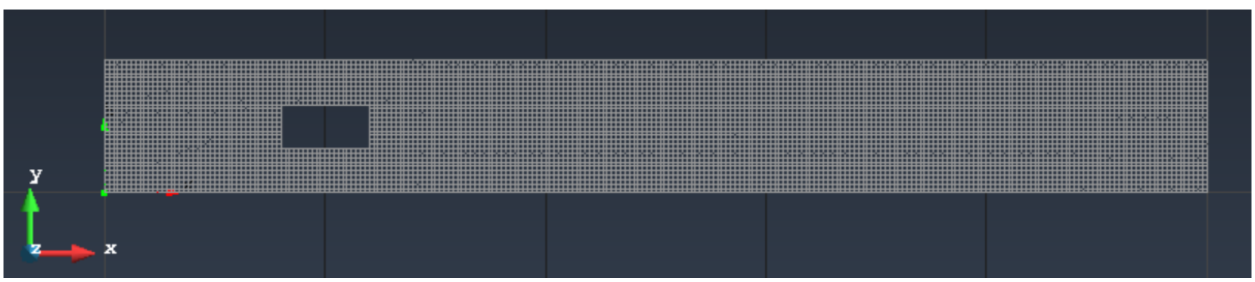


Figure 16 mesh with 25cm squares. Num. of Quadrilateral elements=8.300, Num. of nodes=8.698

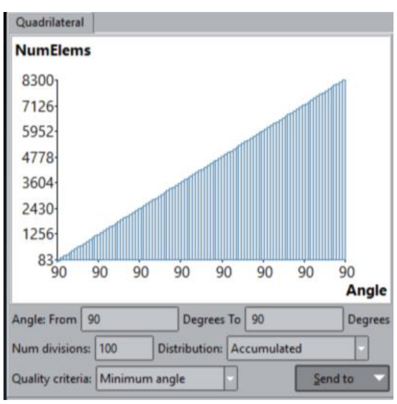


Figure 17 Grid quality is high, as all elements are squares

The simulation was carried using place stress hypothesis as requested. Here we see the deformation and stresses experiences by the plate under the loads prescribed in the problem

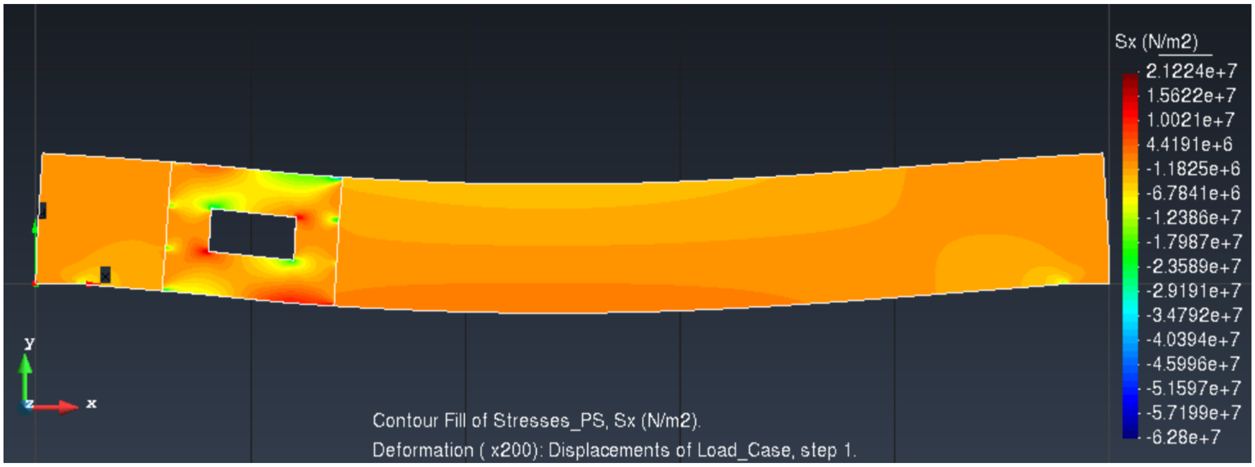


Figure 18 Stresses in the X direction in color code, nodes shown in their location after deformation, exaggerated 200X

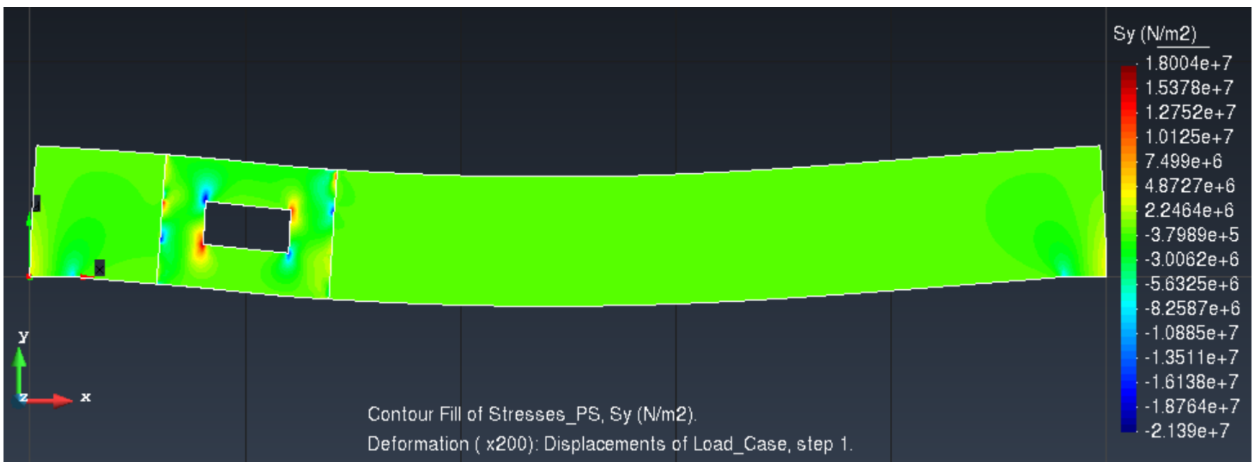


Figure 19 Stresses in the Y direction in color code, nodes shown in their location after deformation, exaggerated 200X

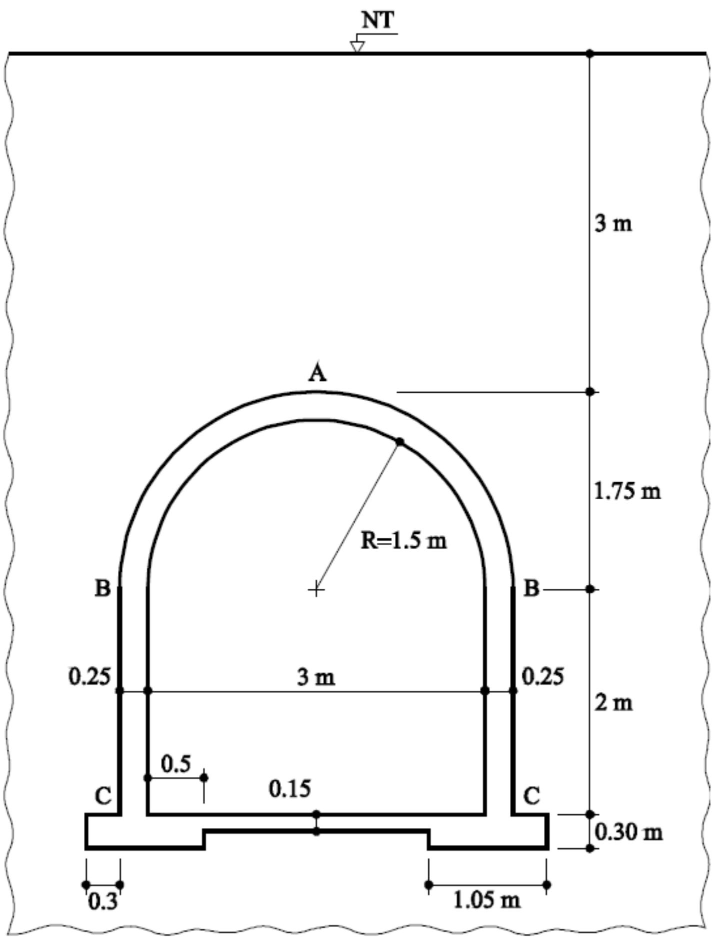
Its worth pointing out that the largest stresses occur on the internal corner of the steel plate. Round shaped corners would decrease these stresses and reduce the risk of failure due to fatigue.

Exercise 4: Tunnel

The structure shown in the figure represents the cross-section of a tunnel made of reinforced concrete. The tunnel is used by the oil industry to transport sunflowers from a warehousing silo to the processing plant.

Analyze the state of stress in the cross-section of the tunnel, considering that the base slab is elastically supported by the ground. Use the hypothesis of planar deformation.

Use quadrilateral elements with four nodes.



Data

$$\text{Concrete}\begin{cases} E = 3.0e10\,\dfrac{N}{m^2} \\ \nu = 0.2 \\ \gamma = 25000\,\dfrac{N}{m^3} \end{cases} \qquad \text{Ground pressure}\begin{cases} P_A = 5.4e4\,\dfrac{N}{m^2} \\ P_{BC} = 2.16e4\,\dfrac{N}{m^2} \\ P_{AB} = \text{linear variation} \end{cases}$$

$$\text{Load coefficient of the ground} = 50\,\frac{N}{cm^3}$$

Solution

The geometry of the tunnel was created in GID and assigned material properties and loads as per the problem requirements. Due to software issues, we could not make quad elements.

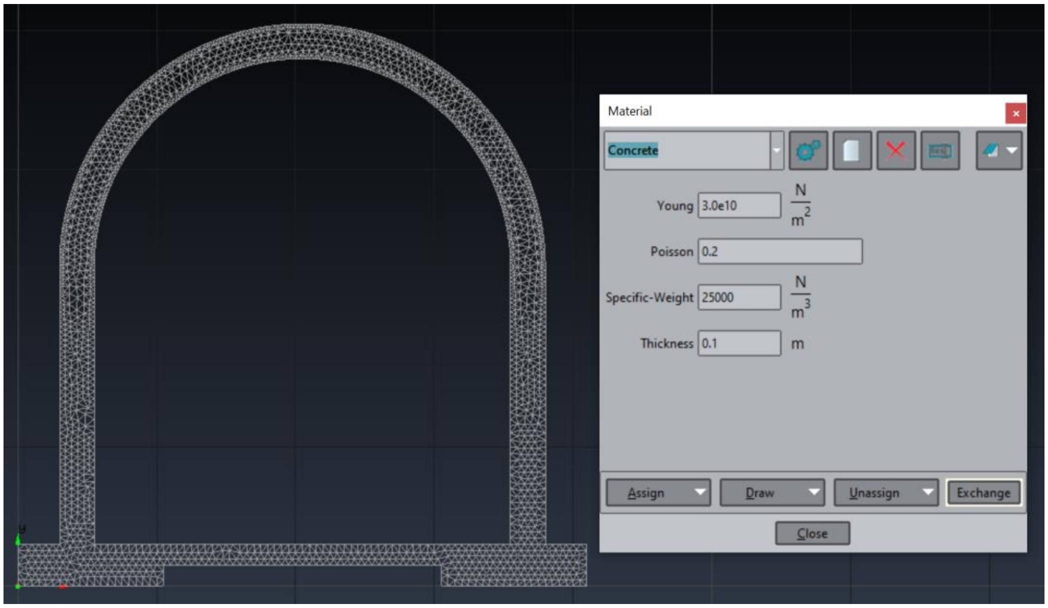


Figure 20 geometry of grid utilized

In particular, the (outside) bottom of the tunnel was prescribed with pressure type of load, like the upper side and the left and right sides. However, it was impossible to get the model to run under these conditions. It is possible that the problems is not well posed in that case, as there may be vertical acceleration if the conditions prescribed are not perfectly accurate.

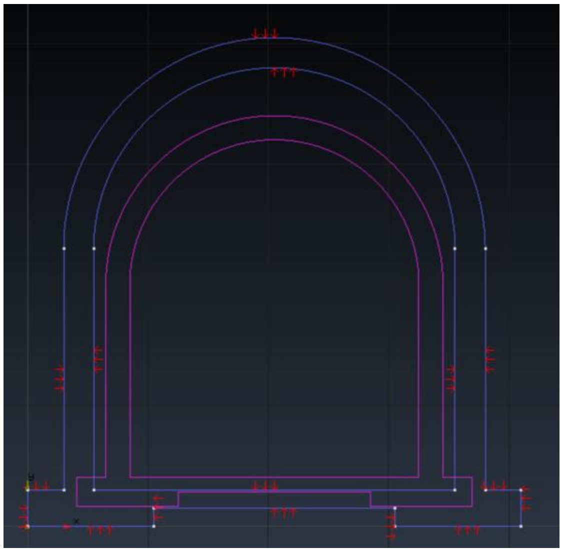


Figure 21 contour conditions based on pressure (internal set to zero gauge pressure).

Therefore, in order to simplify and understand the stresses and deformation, we simply had to assume that the outer bottom of the tunnel had zero vertical displacement.

As shown in the next two figures, the bulk of the displacement occurs on the walls which tend to collapse towards the center of the tunnel.

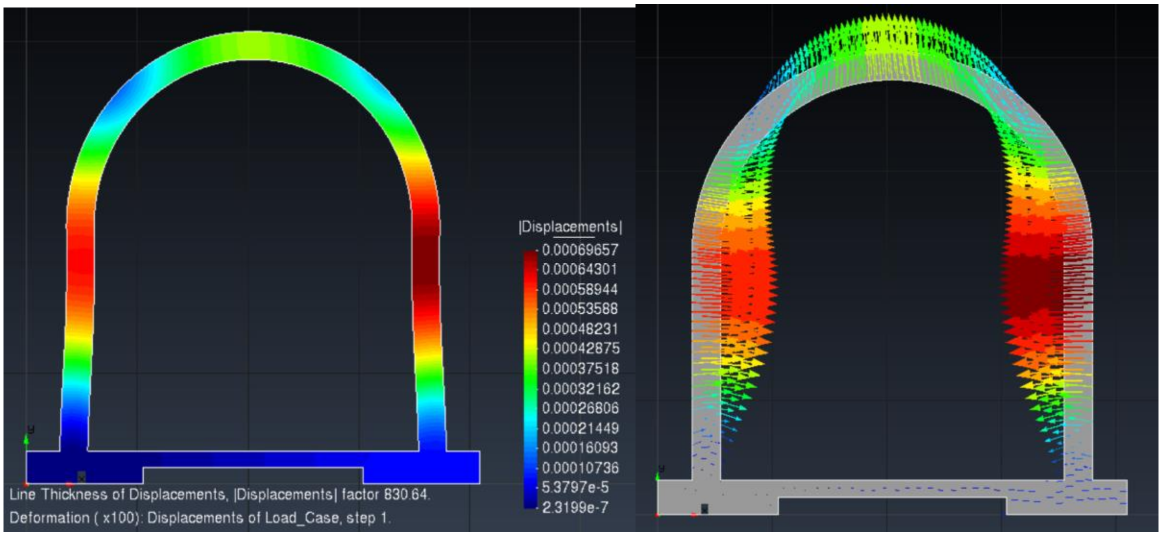


Figure 22 displacements using pressure boundary conditions but with zero vertical displacement in the base prescribed.

Stresses are largest of the outer top of the tunnel (very large compressive stress) while tension is experience in the top of the tunnel (inside wall). The corners where the walls attach to the base also experience significant stress.

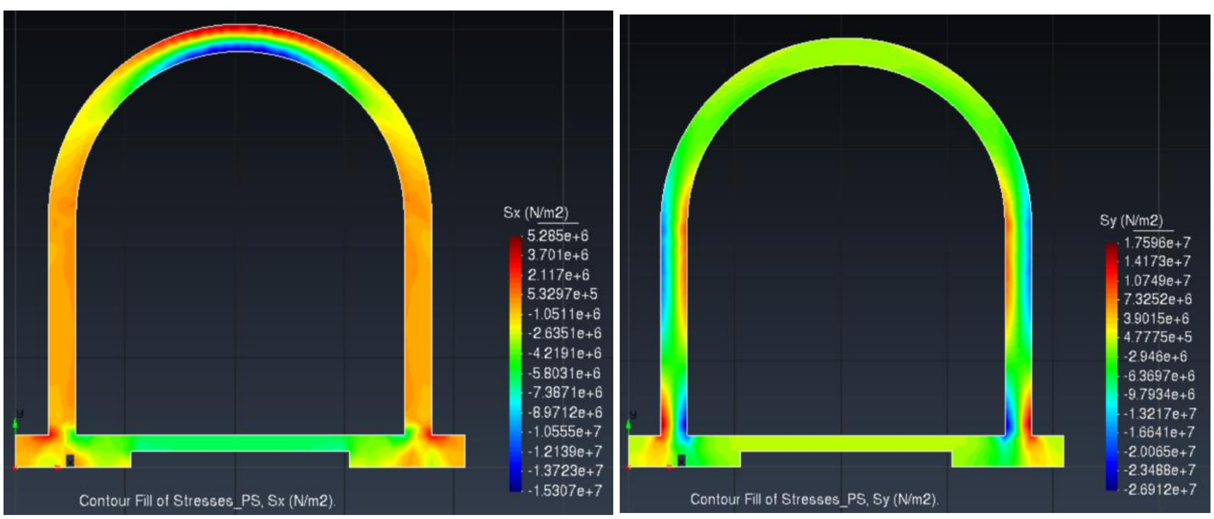


Figure 23 stresses in the x and y direction