# Computational Structural Mechanics and Dynamics 

Plates HomeWork
GiD + MATLab

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## Assignment (Problem Data-a1)

a) Think first and answer later.

What kind of strategy (theory, elements, integration rule, boundary conditions, etc) will you use for solving the following problems:


## Answer

The eccentricity of the middle plans of both types of plates implies using Reissner-Mindlin theory for thick and thin plates to analyze the problem. The middle plans of 4 attached small plates do not coincide with the middle plan of central bigger plate and this phenomenon introduces moment and shear. So in order to counter this difference it is suggested to analyze with Rsissner-Mindlin theory. Although it satisfies the condition of Kirchoff's thin plate theory ( $\mathrm{t} / \mathrm{w} \leq 0.10$ ) yet the problem as a whole would be well addressed by Reissner-Mindlin Theory. Keeping in mind the geometry of the problem we can use symmetry condition on one quarter of the geometry. At the interface of both types of plates the compatibility for displacement and rotation must be kept into consideration. Reissner-Mindlin plate elements shall be used to analyze the problem with 4 points gaus integration.

## Assignment (Problem Data-a2)

a) Think first and answer later.

What kind of strategy (theory, elements, integration rule, boundary conditions, etc) will you use for solving the following problems:
a. 2


## Answer

Here both types of plates share common middle plan so this problem can be treated as single geometry problem and classical Kirchoff's thin plate theory ( $\mathrm{t} / \mathrm{w} \leq 0.10$ ) shall be used to analyze it. There is no need to consider moment and shear into account at the interface of both plates, due to no presence of eccentricity. Keeping in view the geometry of the problem symmetry can be applied and only one quarter of the geometry is enough to go through simulation to get full response of structure. MCZ element shall be used to analyze the problem with 4 points gaus integration.

## Assignment (Problem Data-b1)

## Assignment

a) Analyze the shear blocking effect on the Reissner Mindlin element and compare with the MZC element. For the Simple Support Uniform Load square plate.

Use the $5 \times 5$ Mesh.

|  | $t=0,001$ |
| :--- | :--- |
| $E=10.92$ | $t=0,010$ |
| $v=0,3$ | $t=0,020$ |
| $Q=1.0$ | $t=0,100$ |
|  | $t=0,400$ |

Discuses the results observed.

## Simulation Results

The problem data given in above problem is used to simulate the provided code for 5 different thickness values and results are compared for MZC and RM elements types. The simulation results are presented below.
a) Max Vertical Displacement ( $\mathrm{U}_{\mathrm{z}}$ ) vs. Thickness ( t )

Displacement vs Thickness


Figure-1 Graph between Vertical Displacement and Thickness
b) Moment $\mathrm{M}_{\mathrm{x}}$ vs. Thickness ( t )


Figure-2 Graph between Moment $\mathrm{M}_{\mathrm{x}}$ and Thickness
c) Moment $\mathrm{M}_{\mathrm{y}}$ vs. Thickness ( t )


Figure-3 Graph between Moment $\mathrm{M}_{\mathrm{y}}$ and Thickness
d) Moment $\mathrm{M}_{\mathrm{xy}}$ vs. Thickness ( t )


Figure-4 Graph between Moment $\mathrm{M}_{\mathrm{xy}}$ and Thickness
e) Rotation $R_{x}$ vs. Thickness ( $t$ )


Figure-5 Graph between Rotation $\mathrm{R}_{\mathrm{x}}$ and Thickness

## f) Rotation $R_{y}$ vs. Thickness ( t )



Figure-6 Graph between Rotation $\mathrm{R}_{\mathrm{y}}$ and Thickness

## Comments

As condition of ( $\mathrm{t} / \mathrm{w} \leq 0.10$ ) for all thickness cases satisfy the classical thick and thin plate theory, the convergence graph for both vertical displacement and rotations in both $x$ and $y$ directions simulate the similar behavior in case of both types of elements. RM and MCZ elements behave more or less similar to each other as shown in figures $1,5 \& 6$. While MCZ elements shows consistent results regardless the variance in thickness of the plate because the moment behavior exhibits by the middle plan of the plate in case of MCZ elements while RM elements shows very small variation of moment with varying the thickness and it is evident in figures $2,3 \& 4$.

While shear in both $x$ and $y$ directions are only be simulated by the RM elements because MZC elements lacks the ability to show shear in plates.

## Assignment (Problem Data-b2)

## Assignment

b) Define and verify a patch test mesh for the MCZ element.

Discuses
the results
observed.

## Answer

4 MCZ elements have been selected to form a patch which will be test to solve the problem. Figure below shows the patch


Figure-7 A Patch of 4 MCZ elements.
This patch is consisted of 4 elements and 9 nodes with each node has 3 DOF, displacement and $x$ \& $Y$ rotations. The central node of the patch must show the same value of displacement as the value show by boundary nodes.

Using the Matlab code, using initial displacement of 1 at each boundary nodes and central node and see the results.

```
% Fixed Nocles
%
fixdesp = [
    1, 1, 1.0;
    1, 2, 0.0;
    1, 3, 0.0;
    2, 1, 1.0;
    2, 2, 0.0;
    2, 3, 0.0;
    3, 1, 1.0;
    3, 2, 0.0;
    3,3, 0.0;
    5, 1, 1.0;
    5,2, 0.0;
    5, 3, 0.0;
    6, 1, 1.0;
    6, 2, 0.0;
    6, 3, 0.0;
    7, 1, 1.0;
    7, 2, 0.0;
    7, 3, 0.0;
    8, 1, 1.0;
    8, 2, 0.0;
    8, 3, 0.0;
    9, 1, 1.0;
    9, 2, 0.0;
    9, 3, 0.01;
```

Figure-8 Displacement at boundary nodes

| $(1,1)$ | 1.0000 |
| ---: | ---: |
| $(4,1)$ | 1.0000 |
| $(7,1)$ | 1.0000 |
| $(10,1)$ | 1.0000 |
| $(11,1)$ | 0.0000 |
| $(12,1)$ | -0.0000 |
| $(13,1)$ | 1.0000 |
| $(16,1)$ | 1.0000 |
| $(19,1)$ | 1.0000 |
| $(22,1)$ | 1.0000 |
| $(25,1)$ | 1.0000 |

Figure-9 Displacement at centre node
It is evident from the simulation results that central node shows displacement which has the same value as the boundary nodes has. Thus the Patch of 4 MCZ elements passed the test.

A linear displacement field can be used to check for the patch test of elements. Like
$A x+B y+C$
is used in this case for the test. The constants are $A=2, B=3, c=1$. Plugging in the coordinates of individual nodes, the displacements are found. And rotations are calculated by taking derivatives of the above equation in terms of $x$ and $y$ respectively for rotation in $x$ and $y$.

Table- 1 shows the displacement and rotations values for the linear field.

Table-1 showing displacement and rotations of each node

| Node | Displacement | Rotation X | Rotation Y |
| :---: | :---: | :---: | :---: |
| 1 | 31 | 2 | 3 |
| 2 | 41 | 2 | 3 |
| 3 | 16 | 2 | 3 |
| 4 | 26 | 2 | 3 |
| 5 | 1 | 2 | 3 |
| 6 | 51 | 2 | 3 |
| 7 | 11 | 2 | 3 |
| 8 | 36 | 2 | 3 |
| 9 | 21 | 2 | 3 |

Now, the displacements and corresponding rotation values are put into the Matlab code for the boundary nodes (Fig 10). Then the displacements for the centre node is calculated (Fig 11) and then compared with the value found in the above table.

```
% Fixed Nodes
%
fixdesp = [
    1, 1, 31.0;
    1, 2, 2.0;
    1, 3, 3.0;
    2, 1, 41.0;
    2, 2, 2.0;
    2, 3, 3.0;
    3, 1, 16.0;
    3, 2, 2.0;
    3, 3, 3.0;
    5, 1, 1.0;
    5, 2, 2.0;
    5, 3, 3.0;
    6, 1, 51.0;
    6, 2, 2.0;
    6, 3, 3.0;
    7, 1, 11.0;
    7, 2, 2.0;
    7, 3, 3.0;
    8, 1, 36.0;
    8, 2, 2.0;
    8, 3, 3.0;
    9, 1, 21.0;
    9, 2, 2.0;
    9, 3, 3.01;
```

Figure-10 Displacement, rotations of boundary nodes

| $(1,1)$ | 31.0000 |
| ---: | ---: |
| $(2,1)$ | 2.0000 |
| $(3,1)$ | 3.0000 |
| $(4,1)$ | 41.0000 |
| $(5,1)$ | 2.0000 |
| $(6,1)$ | 3.0000 |
| $(7,1)$ | 16.0000 |
| $(8,1)$ | 2.0000 |
| $(9,1)$ | 3.0000 |
| $(10,1)$ | 26.0000 |
| $(11,1)$ | 2.0000 |
| $(12,1)$ | 3.0000 |
| $(13,1)$ | 1.0000 |
| $(14,1)$ | 2.0000 |
| $(15,1)$ | 3.0000 |
| $(16,1)$ | 51.0000 |
| $(17,1)$ | 2.0000 |
| $(18,1)$ | 3.0000 |
| $(19,1)$ | 11.0000 |
| $(20,1)$ | 2.0000 |
| $(21,1)$ | 3.0000 |
| $(22,1)$ | 36.0000 |
| $(23,1)$ | 2.0000 |
| $(24,1)$ | 3.0000 |
| $(25,1)$ | 21.0000 |
| $(26,1)$ | 2.0000 |
| $(27,1)$ | 3.0000 |
|  |  |

Figure-11 Displacement at central node

It can be seen that, the centre nodes moves with the same displacement as it was calculated using linear displacement field. Thus the element passes the patch test and good for analysis.

