ASSIGNMENT 2.
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ASSIGNMENT 2-1.

1. Three doubts about "FEM Modelling: Introduction"

1. Taking into account the non-continuous nature of matter, what it is the size of the smallest finite element we can take?

2. What differences are between the finite element method in solids and the finite difference or volume method in fluids?

3. What is a spectral element?

2. Three questions that I would ask in an exam (and its answers)

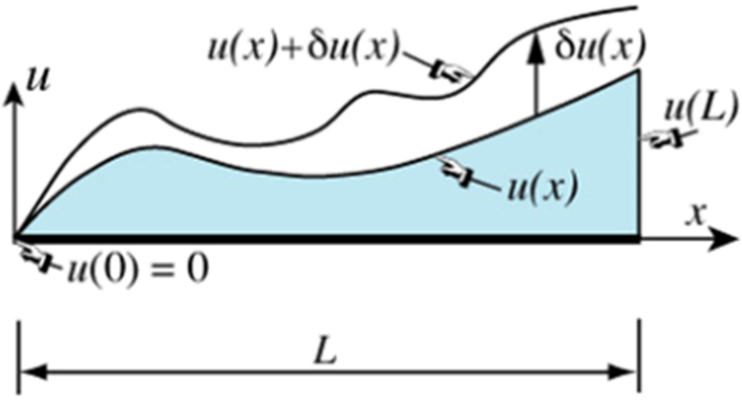
1. What element geometries would you prefer in 2D quadrilaterals or triangles?
Answer: Quadrilaterals

2. And in 3D, what geometries are preferred between bricks, wedges and Tetrahedral?
Answer:
Bricks over wedges.
Wedges over Tetrahedral.

3. What is the correct recipe depending on the DOF involved in the Boundary Conditions?
Answer:
If a Boundary Condition involves one or more DOF in a direct way, it is essential and goes to the Left Hand Side (LHS) of Ku=f
Otherwise it is natural and goes to the Right Hand Side (RHS) of Ku=f

ASSIGNMENT 2-2

1. **Explain the kinematic admissibility requirements stated in slide §6 in terms of physics, namely ruling out the possibility of gaps or interpenetration as the bar material deforms.**



δu(x) is kinematically admissible if u(x) and u(x) + δu(x):

(i) are continuous over bar length i.e. u(x) ϵ C_0 in x ϵ[0,L]
This means that the displacement u at each point x of the bar, u(x), along its length L is continuous. Physically it means no jumps or gaps in the displacements u due to the deformation.
(ii) satisfy exactly displacement BC; in the figure, u(0)=0
u(x) + δu(x) satisfies the boundary conditions. That is, the displacement vectors result of deformation satisfies exactly the boundary conditions, so that this limit is not exceeded and there is no possibility of interpenetration.

2. Dr. Who proposes "improving" the result for the example truss of the 1st lesson by putting one extra node, 4 at the midpoint of member (3) 1-3, so that it is subdivided in two different members: (3) 1-4 and (4) 3-4.
His "reasoning" is that more is better. Try Dr. Who's suggestion by hand computations and verify that the solution "blows up" because the modified master stiffness is singular. Explain physically.

In this case the stiffness matrix of the element in the right and the element in the left are the same:

$$\begin{pmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \end{pmatrix} = \frac{E\,A}{L} \begin{pmatrix} s^2c & -sc^2 & -s^2c & sc^2 & 0 & 0 & 0 & 0 \\ -sc^2 & c^3 & sc^2 & -c^3 & 0 & 0 & 0 & 0 \\ -s^2c & sc^2 & s^2c & -sc^2 & 0 & 0 & 0 & 0 \\ sc^2 & -c^3 & -sc^2 & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{pmatrix}$$

$$\begin{pmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \end{pmatrix} = \frac{E\,A}{L} \begin{pmatrix} s^2c & sc^2 & 0 & 0 & 0 & 0 & -s^2c & -sc^2 \\ sc^2 & c^3 & 0 & 0 & 0 & 0 & s^2c & -c^3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -s^2 & -sc^2 & 0 & 0 & 0 & 0 & s^2c & sc^2 \\ -sc^2 & -c^3 & 0 & 0 & 0 & 0 & sc^2 & c^3 \end{pmatrix} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{pmatrix}$$

But now we have one more node. So node 1, 2 and 3 are still 1, 2 and 3. But we have one more node 4 between 1 and 3. And now node called 4 is node 5. So in the bellow matrix there are 2 more rows between 3 and 4. And 4 will be 5.

2 3 5

* * *

*4

*1

But the stiffness matrix of element 1-3 in the middle is:

Now

$\varphi = \frac{\pi}{2}$, so:

Cos φ = c =0 (for simplicity in the notation)

Sin φ = s= 1 (for simplicity in the notation)

Member (1-4):

f_{x1} =0
f_{y1} =- $u_{y1} + u_{y4}$
f_{x4} = 0
f_{y4} = $u_{y1} - u_{y4}$

$$\begin{pmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \\ f_{x5} \\ f_{y5} \end{pmatrix} = \frac{EA}{L'} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \\ u_{x5} \\ u_{y5} \end{pmatrix}$$

Member (3-4):

f_{x3} =0
f_{y3} =- $u_{y4} + u_{y3}$
f_{x4} = 0
f_{y4} = $u_{y4} - u_{y3}$

$$\begin{pmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \\ f_{x5} \\ f_{y5} \end{pmatrix} = \frac{EA}{L'} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \\ u_{x5} \\ u_{y5} \end{pmatrix}$$

L'=L/2

So member (1-4) will be the sum of (1-4) and (3-4):

$$\begin{pmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \\ f_{x5} \\ f_{y5} \end{pmatrix} = \frac{EA}{L} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & -2 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \\ u_{x5} \\ u_{y5} \end{pmatrix}$$

And the global stiffness matrix will be:

$$\begin{pmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{EA}{L} \begin{pmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s & 0 & 0 \\ 0 & 2+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3-1 & 0 & 0 \\ -cs^2 & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 & 0 & 0 \\ cs^2 & -c^3 & -cs^2 & c^3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & -2 & 0 & 4 & 0 & 0 \\ -cs^2 & -cs^2 & 0 & 0 & 0 & 0 & 0 & 0 & cs^2 & cs^2 \\ -cs^2 & -c^3 & 0 & 0 & 0 & 0 & 0 & 0 & cs^2 & c^3 \end{pmatrix} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \\ u_{x5} \\ u_{y5} \end{pmatrix}$$

which is a singular matrix.

BC tell us zero displacement for the nodes 2, 3 and 5. So:

$$\begin{pmatrix} H \\ -P \\ 0 \\ 0 \end{pmatrix} = \frac{EA}{L} \begin{pmatrix} 2cs^2 & 0 & -cs^2 & -c^2s \\ 0 & 2+2c^3 & -c^2s & -c^3-1 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 4 \end{pmatrix} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x4} \\ u_{y4} \end{pmatrix}$$

$$u_{y1} = 2\,u_{y4}$$

$$u_{y4} = (-\frac{PL}{EA} + c^2s)/3(c^3+1)$$

$$u_{x4} = c/s$$

If $\alpha \rightarrow 0$, $\boldsymbol{u_{x4}} \rightarrow \infty$ **and** $\boldsymbol{u_{x1}} \rightarrow \infty$ which is not possible and:

$$u_{y4} = (-\frac{PL}{6EA})$$

$$u_{y1} = -\frac{PL}{3EA}$$