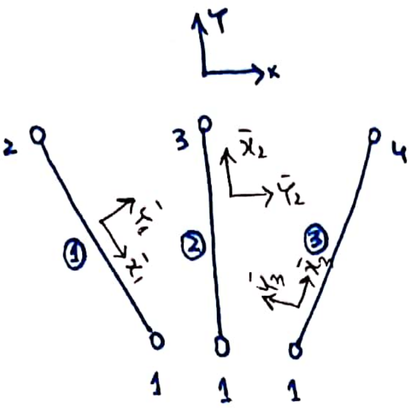
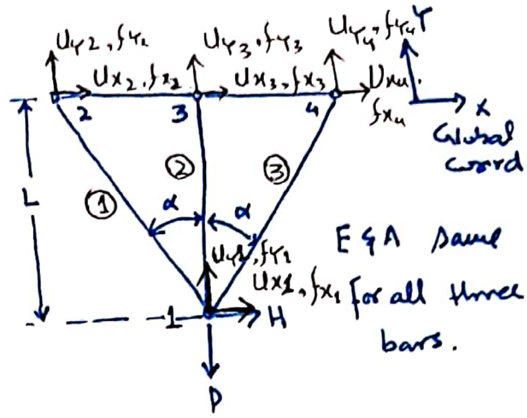
Assignment. No. 1.1 CSMD

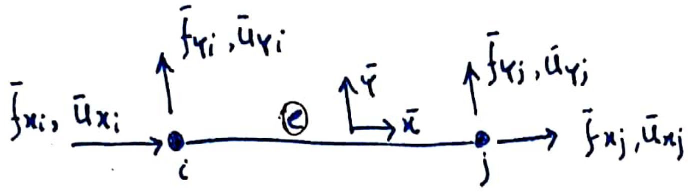
Part 1

Disconnection & localization,





Total DoF = 2 × 4 = 8



Master (Global) Stiffness Equation :-

$$\begin{bmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix} = \begin{bmatrix} K_{x_1x_1} & K_{x_1y_1} & K_{x_1x_2} & \cdots & K_{x_1y_4} \\ K_{y_1x_1} & K_{y_1y_1} & K_{y_1x_2} & \cdots & K_{y_1y_4} \\ K_{x_2x_1} & K_{x_2y_1} & K_{x_2x_2} & \cdots & K_{x_2y_4} \\ \vdots & \vdots & \vdots & & \vdots \\ K_{y_4x_1} & K_{y_4y_1} & K_{y_4x_2} & \cdots & K_{y_4y_4} \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}$$

$$f = Ku$$

Element Stiffness Equations $\Rightarrow \bar{f} = \bar{K}\bar{u} \Rightarrow$

$$\begin{bmatrix} \bar{f}_{xi} \\ \bar{f}_{yi} \\ \bar{f}_{xj} \\ \bar{f}_{yj} \end{bmatrix} = \begin{bmatrix} \bar{K}_{xixi} & \bar{K}_{xiyi} & \bar{K}_{xixj} & \bar{K}_{xiyj} \\ \bar{K}_{yixi} & \bar{K}_{yiyi} & \bar{K}_{yixj} & \bar{K}_{yiyj} \\ \bar{K}_{xjxi} & \bar{K}_{xjyi} & \bar{K}_{xjxj} & \bar{K}_{xjyj} \\ \bar{K}_{yjxi} & \bar{K}_{yjyi} & \bar{K}_{yjxj} & \bar{K}_{yjyj} \end{bmatrix} \begin{bmatrix} \bar{u}_{xi} \\ \bar{u}_{yi} \\ \bar{u}_{xj} \\ \bar{u}_{yj} \end{bmatrix}$$

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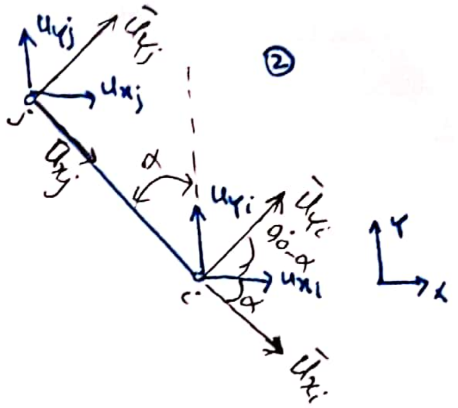
For Element 1 :

$\because \cos(90-\alpha)$
$s = \sin\alpha$

$\because \sin(90-\alpha)$
$c = \cos\alpha$



$$\begin{bmatrix} \bar{u}_{xi} \\ \bar{u}_{yi} \\ \bar{u}_{xj} \\ \bar{u}_{yj} \end{bmatrix} = \begin{bmatrix} s & -c & 0 & 0 \\ s & c & 0 & 0 \\ 0 & 0 & s & -c \\ 0 & 0 & s & c \end{bmatrix} \begin{bmatrix} u_{xi} \\ u_{yi} \\ u_{xj} \\ u_{yj} \end{bmatrix} \Rightarrow \bar{u}^e = T^e u^e$$

$\because L^e = \dfrac{L}{\cos\alpha} = \dfrac{L}{c}$

$$\begin{bmatrix} f_{xi} \\ f_{yi} \\ f_{xj} \\ f_{yj} \end{bmatrix} = \begin{bmatrix} s & s & 0 & 0 \\ -c & c & 0 & 0 \\ 0 & 0 & s & s \\ 0 & 0 & -c & c \end{bmatrix} \begin{bmatrix} \bar{f}_{xi} \\ \bar{f}_{yi} \\ \bar{f}_{xj} \\ \bar{f}_{yj} \end{bmatrix} \Rightarrow f^e = (T^e)^T \bar{f}^e$$

$$\because \bar{K}^e = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{E^e A^e}{L^e}$$

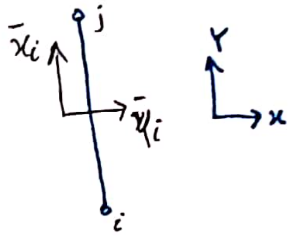
So, $K^e = (T^e)^T \bar{K}^e T^e$

Hence,

$$K^{①} = \frac{EA}{L} \begin{bmatrix} cs^2 & -c^2 s & -cs^2 & c^2 s \\ -c^2 s & c^3 & c^2 s & -c^3 \\ -cs^2 & c^2 s & cs^2 & -c^2 s \\ c^2 s & -c^3 & -c^2 s & c^3 \end{bmatrix}$$

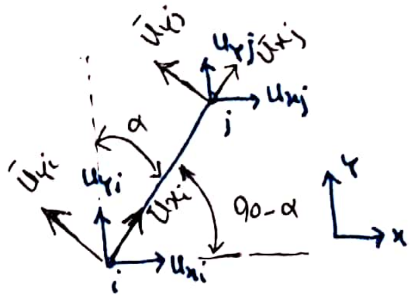
For Element 2 :

$$K^{②} = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$



For Element 3 :

$$K^{③} = \frac{EA}{L} \begin{bmatrix} cs^2 & c^2 s & -cs^2 & -c^2 s \\ c^2 s & c^3 & -c^2 s & -c^3 \\ -cs^2 & -c^2 s & cs^2 & c^2 s \\ -c^2 s & -c^3 & c^2 s & c^3 \end{bmatrix}$$



$\because L^e = \dfrac{L}{\cos\alpha} = \dfrac{L}{c}$

$\because \cos(90-\alpha) = \sin\alpha$

$\because \sin(90-\alpha) = \cos\alpha$

$s = \sin\alpha$

$c = \cos\alpha$

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So, we have.

1 $$\begin{bmatrix} f_{x_1}^1 \\ f_{y_1}^1 \\ f_{x_2}^1 \\ f_{y_2}^1 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} cs^2 & -c^2s & -cs^2 & c^2s \\ -c^2s & c^3 & c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s \\ c^2s & -c^3 & -c^2s & c^3 \end{bmatrix}\begin{bmatrix} u_{x_1}^1 \\ u_{y_1}^1 \\ u_{x_2}^1 \\ u_{y_2}^1 \end{bmatrix}$$

2 $$\begin{bmatrix} f_{x_1}^2 \\ f_{y_1}^2 \\ f_{x_3}^2 \\ f_{y_3}^2 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} u_{x_1}^2 \\ u_{y_1}^2 \\ u_{x_3}^2 \\ u_{y_3}^2 \end{bmatrix}$$

3 $$\begin{bmatrix} f_{x_1}^3 \\ f_{y_1}^3 \\ f_{x_4}^3 \\ f_{y_4}^3 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} cs^2 & c^2s & -cs^2 & -c^2s \\ c^2s & c^3 & -c^2s & -c^3 \\ -cs^2 & -c^2s & cs^2 & c^2s \\ -c^2s & -c^3 & c^2s & c^3 \end{bmatrix}\begin{bmatrix} u_{x_1}^3 \\ u_{y_1}^3 \\ u_{x_4}^3 \\ u_{y_4}^3 \end{bmatrix}$$

Now assemble the local Element Equations to Form Global Equations.

$\cancel{f_{x_1}^1}$ → Global Stiffness Matrix will be of form

$$K = \frac{EA}{L}\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}\begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 2c^2s & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ sc^2 & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix} \end{array}$$

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master (Global) Equations.

after applying Compatibility & Equilibrium.

$$\begin{Bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \frac{EA}{L} \begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ \text{Symm} & & & & & & & c^3 \end{bmatrix} \begin{Bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{Bmatrix}$$

In stiffness matrix rows 5 & columns 5 have zero. Because the force H is at right angle to middle bar (2). So force is shared by element 1 & 3 only. Element 2 experience no force in x-direction. that's why row & columns 5 have zero in stiffness matrix.

Part 2

Boundry conditions:-

At nodes 2,3,4 there are pin support, So displacements are restrained at 2,3 & 4 nodes.

Hence $u_{x_2} = 0$ & $u_{y_2} = 0$ & $u_{x_1} = ?$

$u_{x_3} = 0$ $\quad u_{y_3} = 0$ $\quad u_{y_1} = ?$

$u_{x_4} = 0$ $\quad u_{y_4} = 0$

thus stiffness matrix will be reduced to 2x2

Hence

$$\begin{bmatrix} H \\ -P \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix} \begin{Bmatrix} u_{x_1} \\ u_{y_1} \end{Bmatrix}$$

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Part 3

we have 2- Equations now

$$u_{x_1} = \frac{HL}{EA\,2cs^2} \quad \text{— ①}$$

$$u_{y_1} = -\frac{PL}{EA(1+2c^3)} \quad \text{— ②}$$

Applying Conditions. $\alpha \to 0$ & $\alpha \to \pi/2$

For ① $\alpha \to 0$, $H \neq 0$ & $\alpha \to \pi/2$

$u_{x_1} = \infty$ $\qquad$ $u_{x_1} = \infty$

So u_{x_1} blows up as $\alpha \to 0$, it will behave like vertical bars members with infinite horizontal displacement that's DSM cannot handle this scenerio. Also force H create a moment that causes u_{x_1} to short..

For ② $\alpha \to 0$ & $\alpha \to \pi/2$

$u_{y_1} = -\dfrac{PL}{3EA}$ $\qquad$ $u_{y_1} = +\dfrac{PA}{EA}$

Part 4

Axial Force

For Element ① $\qquad \bar{u}' = T' u'$ $\qquad$ From previous calculation, we know

$$\begin{bmatrix} \bar{u}^1_{x_1} \\ \bar{u}^1_{y_1} \\ \bar{u}^1_{x_2} \\ \bar{u}^1_{y_2} \end{bmatrix} = \begin{bmatrix} s & -c & 0 & 0 \\ c & s & 0 & 0 \\ 0 & 0 & s & -c \\ 0 & 0 & c & s \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} = 0 \\ u_{y_2} = 0 \end{bmatrix}$$

$\because u_{x_1} = \dfrac{HL}{EA\,2cs^2}$

$\because u_{y_1} = -\dfrac{PL}{EA(1+2c^3)}$

As $F^{①} = \dfrac{EA}{L/c} d = \dfrac{EA}{L/c}(\bar{u}_{x_2} - \bar{u}_{x_1})$, And $\bar{u}_{x_1} = u_{x_1}s - u_{y_1}c$

$\bar{u}_{x_2} = 0$

$$d = \bar{u}_{x_2} - \bar{u}_{x_1} = s u_{x_1} + u_{y_1} c$$

$$F^{①} = \frac{EA\,c}{L}\left[\frac{s \cdot HL}{EA\,2cs^2} + \frac{PL \cdot c}{EA(1+2c^3)}\right]$$

$$F^{①} = \frac{H}{2s} + \frac{Pc^2}{1+2c^3}$$

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For Element 2

As $\bar{u}^2 = T^2 u^2$

$$\begin{pmatrix} \bar{u}^2_{y_1} \\ \bar{u}^2_{x_1} \\ \bar{u}^2_{y_3} \\ \bar{u}^2_{x_3} \end{pmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{HL}{2EACS^2} = u_{x_1} \\ -\frac{PL}{EA(1+2C^3)} = u_{y_1} \\ 0 = u_{x_3} \\ 0 = u_{y_3} \end{Bmatrix}$$

$$F^{②} = \frac{EA}{L}\left(\bar{u}^2_{x_3} - \bar{u}^2_{x_1}\right) = \frac{EA}{L}\left[\frac{PL}{EA(1+2C^3)}\right]$$

$$F^{②} = \frac{P}{(1+2C^3)}$$

For Element 3

As $\bar{u}^3 = T^3 u^3$

$$\begin{bmatrix} \bar{u}^3_{x_1} \\ \bar{u}^3_{y_1} \\ \bar{u}^3_{x_4} \\ \bar{u}^3_{y_4} \end{bmatrix} = \begin{bmatrix} S & C & 0 & 0 \\ -C & S & 0 & 0 \\ 0 & 0 & S & C \\ 0 & 0 & -C & S \end{bmatrix} \begin{pmatrix} u_{x_1} = \frac{HL}{2EACS^2} \\ u_{y_1} = -\frac{PL}{EA(1+2C^3)} \\ u_{x_4} = 0 \\ u_{y_4} = 0 \end{pmatrix}$$

$$F^{③} = \frac{EA}{L/C}\left(\bar{u}^3_{x_4} - \bar{u}^3_{x_1}\right) = \frac{CEA}{L}\cdot\left(\frac{L}{EA}\right)\left(-\frac{H}{2CS} + \frac{PC}{1+2C^3}\right)$$

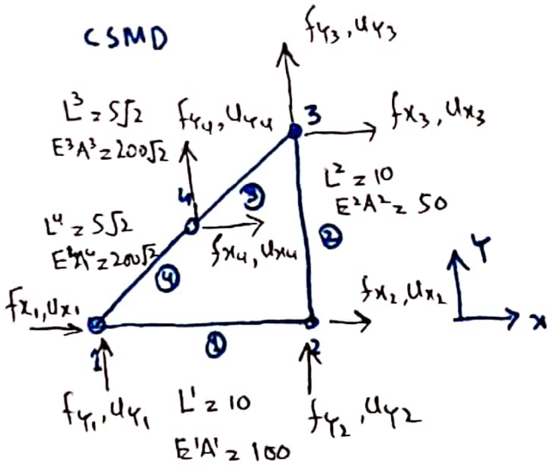
$$F^{③} = -\frac{H}{2S} + \frac{C^2 P}{1+2C^3}$$

$F^{①}$ & $F^{③}$ becomes unstable as $\alpha \to 0$, $H \neq 0$.

ADNAN ALI IMRAN

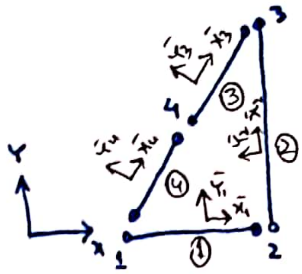
Assignment No. 1.2

CSMD



Improved problem

Disconnection & localization:-



For Element 1

Similar to the previous case, we can write Element stiffness matrix like this. (transmission angle = 0°)

$$\overset{①}{K} = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = 10\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \because \frac{EA}{L} = \frac{100}{10} = 10$$

Equations.

$$\begin{bmatrix} f_{x_1}^1 \\ f_{y_1}^1 \\ f_{x_2}^1 \\ f_{y_2}^1 \end{bmatrix} = \begin{bmatrix} 10 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} u_{x_1}^1 \\ u_{y_1}^1 \\ u_{x_2}^1 \\ u_{y_2}^1 \end{bmatrix}$$

For Element 2

(transmission Angle = 90°)

$$\overset{②}{K} = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} = 5\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \qquad \because \frac{EA}{L} = \frac{50}{10} = 5$$

Equations:-

$$\begin{bmatrix} f_{x_2}^2 \\ f_{y_2}^2 \\ f_{x_3}^2 \\ f_{y_3}^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & 0 & 5 \end{bmatrix}\begin{bmatrix} u_{x_2}^2 \\ u_{y_2}^2 \\ u_{x_3}^2 \\ u_{y_3}^2 \end{bmatrix}$$

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For Element 3

(transmission angle = 45°)

$$K^{③} = \frac{EA}{L}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix} = 40\begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix}$$

$\because \frac{EA}{L} = \frac{200\sqrt{2}}{5\sqrt{2}} = 40$ & $\because c^2 = (\cos 45)^2 = 0.5$; $s^2 = (\sin 45)^2 = 0.5$; $\because cs = \cos 45 \times \sin 45 = 0.5$

Equations :-

$$\begin{Bmatrix} f^3_{x_3} \\ f^3_{y_3} \\ f^3_{x_4} \\ f^3_{y_4} \end{Bmatrix} = \begin{bmatrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{bmatrix}\begin{Bmatrix} u^3_{x_3} \\ u^3_{y_3} \\ u^3_{x_4} \\ u^3_{y_4} \end{Bmatrix}$$

For Element 4

(transmission angle = 45°) , Similar to Element 3

Equations :-

$$\begin{Bmatrix} f^4_{x_2} \\ f^4_{y_2} \\ f^4_{x_4} \\ f^4_{y_4} \end{Bmatrix} = \begin{bmatrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{bmatrix}\begin{Bmatrix} u^4_{x_1} \\ u^4_{y_1} \\ u^4_{x_4} \\ u^4_{y_4} \end{Bmatrix}$$

Now assemble them into Global matrix. and it will become like

$$K \text{ (Global Stiffness Matrix)} = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \\ ⑥ \\ ⑦ \\ ⑧ \end{array}\begin{array}{c} \begin{array}{cccccccc} ① & ② & ③ & ④ & ⑤ & ⑥ & ⑦ & ⑧ \end{array} \\ \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \end{bmatrix} \end{array}$$

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Master Equations after applying Equilibrium and Compatibility $\Rightarrow F = KU$

$$
\begin{bmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix}
=
\begin{bmatrix}
30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\
 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\
 & & 10 & 0 & 0 & 0 & 0 & 0 \\
 & & & 5 & 0 & -5 & 0 & 0 \\
 & & & & 20 & 20 & -20 & -20 \\
 & & & & & 25 & -20 & -20 \\
 & & & & & & 40 & 40 \\
 & \text{Symm} & & & & & & 40
\end{bmatrix}
\begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}
$$

This stiffness matrix is singular because each row and column cancelled out. So in order to solve it, we have to apply B.C (known to us) in order to reduce the matrix.

As we have $u_{x_1} = u_{y_1} = u_{y_2} = 0$ & $f_{x_2} = 0$, $f_{x_3} = 2$, $f_{y_3} = 1$

So,

$$
\begin{bmatrix} 0 \\ 2 \\ 1 \\ f_{x_4} \\ f_{y_4} \end{bmatrix}
=
\begin{bmatrix}
10 & 0 & 0 & 0 & 0 \\
0 & 20 & 20 & -20 & -20 \\
0 & 20 & 25 & -20 & -20 \\
0 & -20 & -20 & 40 & 40 \\
0 & -20 & -20 & 40 & 40
\end{bmatrix}
\begin{bmatrix} u_{x_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}
$$

This stiffness matrix is still singular even after applying the known B.C. In order to solve this we need extra B.C. The structure is still hanging in the air.

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