Computational Structural Mechanics & Dynamics

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Assignment 1 + 2

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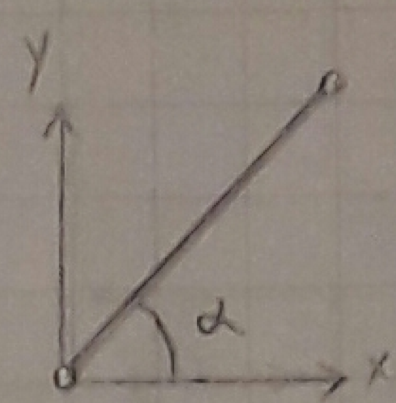
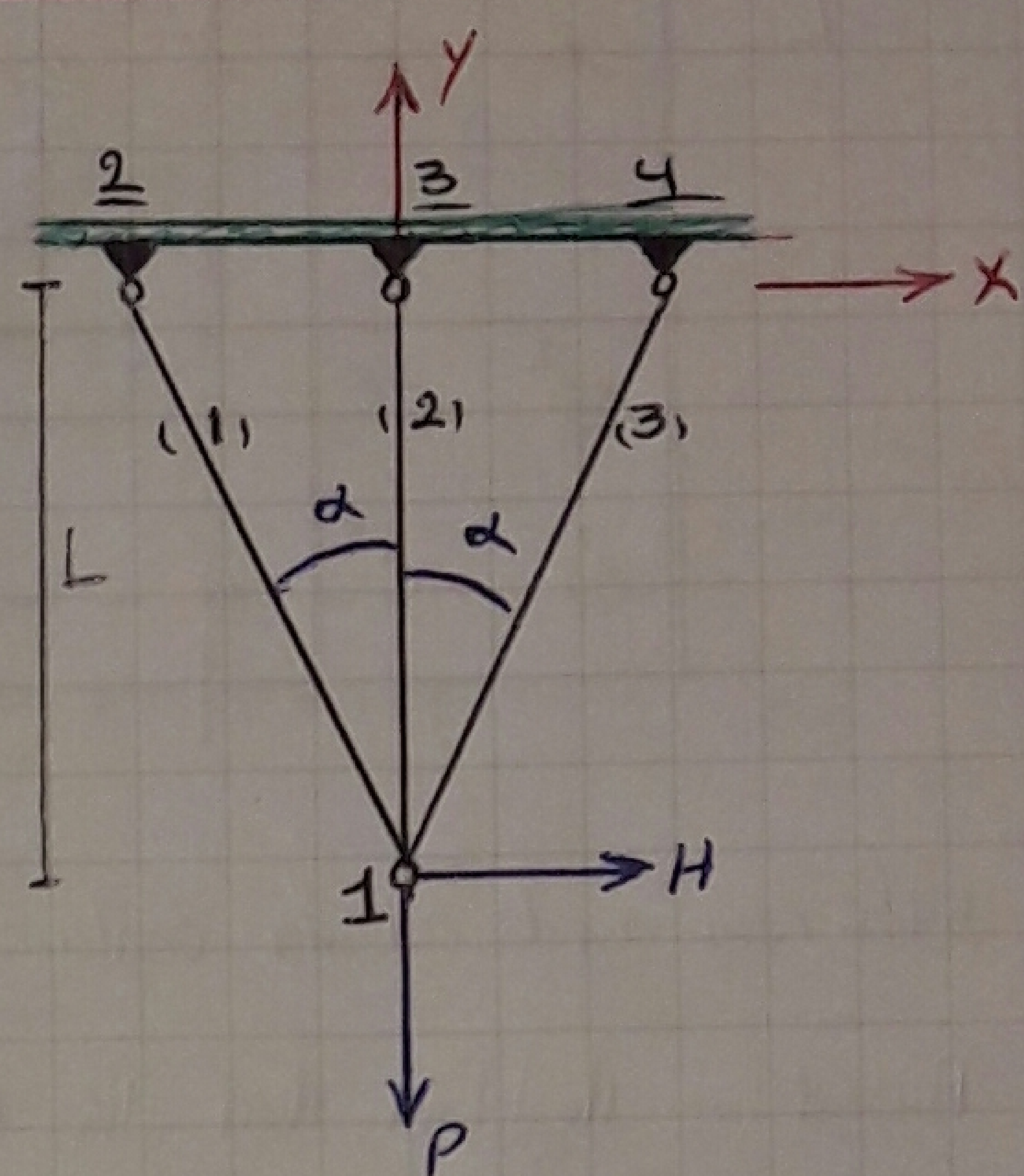
Assignment 1

$$K^e = \frac{E^e A^e}{L^e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$



C = Cos α

S = Sin α



Element	First Node	Last Node	Degree (θ)	Cos(θ)	Sin(θ)
1	1	2	$\frac{\pi}{2}+\alpha$	$-S$	C
2	1	3	$\frac{\pi}{2}$	0	1
3	1	4	$\frac{\pi}{2}-\alpha$	S	C

$$K^1 = \frac{EA}{L/\cos\alpha}\;\begin{array}{c} \begin{array}{cc} \quad① \quad & \quad ② \quad \end{array} \\ \begin{bmatrix} s^2 & -sc & -s^2 & sc \\ -sc & c^2 & sc & -c^2 \\ -s^2 & sc & s^2 & -sc \\ sc & -c^2 & -sc & c^2 \end{bmatrix} \end{array} \begin{array}{c} ① \\ \\ ② \end{array}$$

K^1_{12}

$$K^2 = \frac{EA}{L}\;\begin{array}{c} \begin{array}{cc} \quad① \quad & \quad ③ \quad \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} ① \\ \\ ③ \end{array}$$

$$K^3 = \frac{EA}{L/\cos\alpha}\;\begin{array}{c} \begin{array}{cc} \quad① \quad & \quad ④ \quad \end{array} \\ \begin{bmatrix} s^2 & sc & -s^2 & -sc \\ sc & c^2 & -sc & -c^2 \\ -s^2 & -sc & s^2 & sc \\ -sc & -c^2 & sc & c^2 \end{bmatrix} \end{array} \begin{array}{c} ① \\ \\ ④ \end{array}$$

K^3_{44}

$$K^G = \begin{array}{c} \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\ \begin{bmatrix} K^1_{11}+K^2_{11}+K^3_{11} & K^1_{12} & K^2_{13} & K^3_{14} \\ K^1_{21} & K^1_{22} & 0 & 0 \\ K^2_{31} & 0 & K^2_{33} & 0 \\ K^3_{41} & 0 & 0 & K^3_{44} \end{bmatrix} \end{array} \begin{array}{c} ① \\ ② \\ ③ \\ ④ \end{array}$$

$$K^G U^G = F^G$$

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & -c^2s & cs^2 & c^2s & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix} \cdot \begin{bmatrix} u_{x1} \\ U_{y1} \\ \cancel{U_{x2}} \to 0 \\ \cancel{U_{y2}} \to 0 \\ \cancel{U_{x3}} \to 0 \\ \cancel{U_{y3}} \to 0 \\ \cancel{U_{x4}} \to 0 \\ \cancel{U_{y4}} \to 0 \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

* 5th row and column contains only Zeros and it is rational due to the geometry. Based on the fact that the degree of Bar #2 is 90° and it's located exactly in y-direction So it has No stiffnen in x-direction. Considering this result and noting that Node #3 is merely included in bar #2, we could conclude that all members of global stiffnen Matrix related to the displacement of Node 3 in x-direction would be Zero.

b), c) By applying B.C. which is U_x and U_y for Nodes 2, 3, 4 $= 0$ we could eliminate all rows and Columns of stiffnen Matrix except 1st and 2nd one.

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix}\begin{bmatrix} U_{x1} \\ U_{y1} \end{bmatrix} = \begin{bmatrix} H \\ -P \end{bmatrix} \Rightarrow \begin{cases} U_{x1} = H \times \dfrac{L}{EA} \times \dfrac{1}{2cs^2} \\ U_{y1} = -P \times \dfrac{L}{EA} \times \dfrac{1}{1+2c^3} \end{cases}$$

if $\alpha \to 0$ $\begin{cases} U_{x_1} = \dfrac{H}{0} \to \infty \\ U_{y_1} = \dfrac{-PL}{3EA} \end{cases}$ $\longrightarrow$ it means that each bar would take $\frac{1}{3}$ of force

if $\alpha \to \frac{\pi}{2}$ $\begin{cases} U_{x_1} = \dfrac{H}{0} \to \infty \\ U_{y_1} = \dfrac{-PL}{EA} \end{cases}$ $\longrightarrow$ it means that all of force F would be tolerated by bar d)

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & \text{sym.} & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ & & & & & & & c^3 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} H \\ -P \\ F_{x2} \\ F_{y2} \\ F_{x3} \\ F_{y3} \\ F_{x4} \\ F_{y4} \end{bmatrix}$$

$$u_{x1} = \frac{HL}{EA\,2cs^2}$$

$$u_{y1} = \frac{-PL}{EA\,(1+2c^3)}$$

* row 3 expand $\rightarrow$ $\frac{\cancel{EA}}{\cancel{L}}(-\cancel{c}s^{\cancel{2}})\frac{H\cancel{L}}{\cancel{EA}\,2\cancel{cs^2}} + \frac{\cancel{EA}}{\cancel{L}}(c^2s)\frac{(-P\cancel{L})}{\cancel{EA}(1+2c^3)} + 0+0+0+0+0+0 = F_{x2}$

$$F_{x2} = F^{(1)}\sin\alpha \Rightarrow \left(F^{(1)} = -\frac{H}{2s} - \frac{Pc^2}{1+2c^3} \right)$$

row 6 expand $\rightarrow \frac{EA}{L}(-1)\frac{-PL}{EA(1+2c^3)} = F_{y3} \Rightarrow \left(F_{y3} = F^{(2)} = \frac{P}{1+2c^3} \right)$

row 5 expand $\rightarrow F_{x3} = 0$

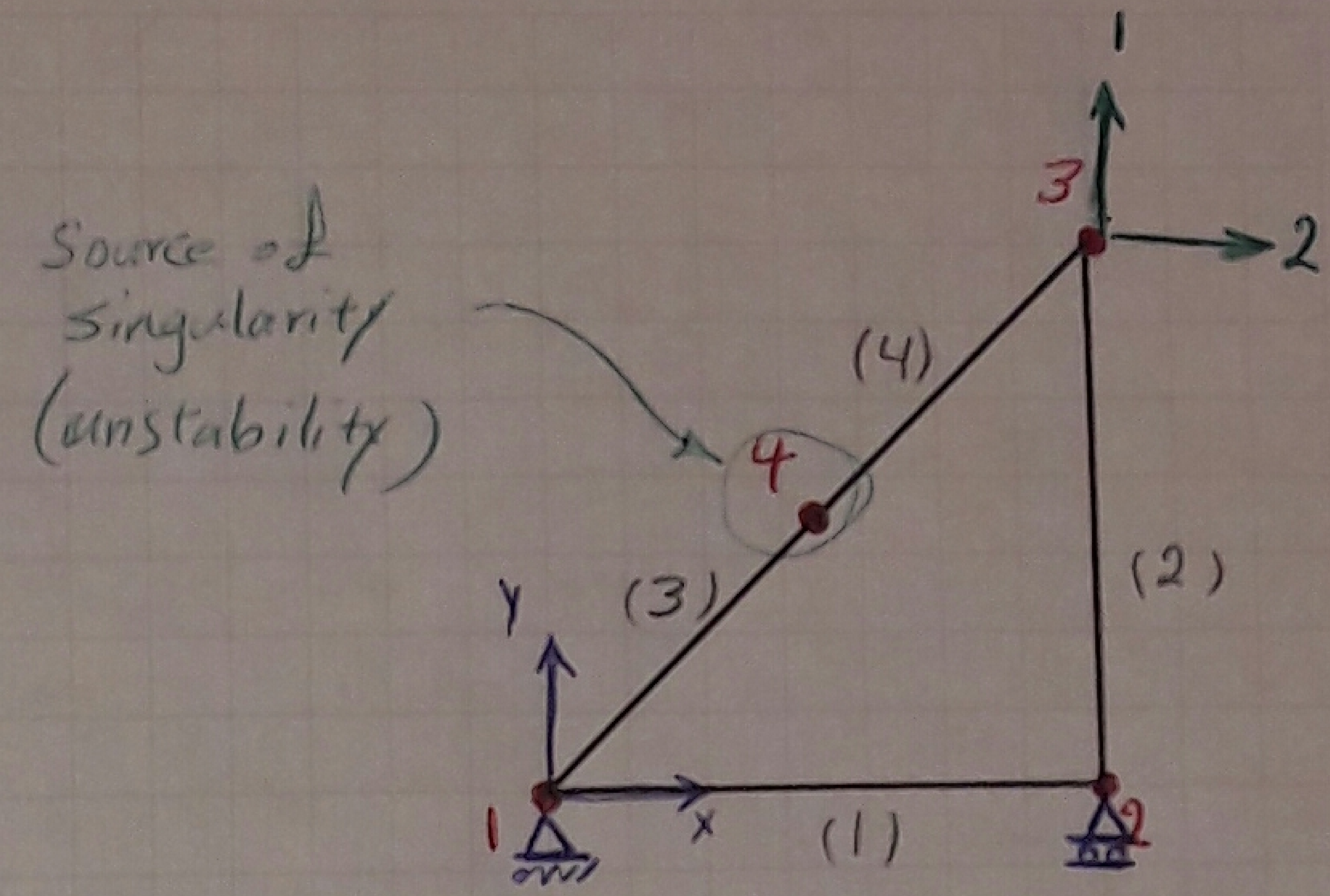
row 7 expand $\rightarrow \frac{\cancel{EA}}{\cancel{L}}(-\cancel{c}s^{\cancel{2}})\frac{H\cancel{L}}{EA\,2cs^2} + \frac{\cancel{EA}}{\cancel{L}}(-c^2s)\frac{-P\cancel{L}}{\cancel{EA}(1+2c^3)} = F_{x4}$

$$F_{x4} = F^{(3)}\sin\alpha \Rightarrow \left(F^{(3)} = -\frac{H}{2s} + \frac{Pc^2}{1+2c^3} \right)$$

* if $\alpha \rightarrow 0$ then $F^{(1)}$ and $F^{(3)} \rightarrow \infty$
$H \neq 0$

it is because from statical point of view, if $\alpha \rightarrow 0$ then the structure is not stable for Horizontal Loadings and if a horizontal Load (H) is applyed to the unstable structure the condition of unstability would appear in the form of infinity forces in element (1) and (3) which have the responsibility to tolerate horizontal forces and transfer them to the boundaries. On the other hand $F^{(2)}$ is not sensitive in the case where $\alpha \rightarrow 0$ because ($F_{x3} = 0$) and rod #2 has no role in careying horizontal Forces.

Assignment 2



element i	$EA^{(i)}$	$L^{(i)}$
1	100	10
2	50	10
3	$200\sqrt{2}$	$5\sqrt{2}$
4	$200\sqrt{2}$	$5\sqrt{2}$

$$K^{e} = \frac{EA^{(e)}}{L}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$4 \rightarrow 3$ in ele # 4

$1 \rightarrow 4$ in ele # 3

$$K^{(1)} = 10 \overbrace{\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}^{1 \rightarrow 2} \qquad K^{2} = 5 \overbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}}^{2 \rightarrow 3} \qquad K^{3} = K^{4} = 20 \overbrace{\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}}^{1 \rightarrow 4}$$

$$K^{(G)} = \begin{array}{c} \\ 1x \\ 1y \\ 2x \\ 2y \\ 3x \\ 3y \\ 4x \\ 4y \end{array} \begin{array}{c} \begin{array}{cccccccc} 1x & 1y & 2x & 2y & 3x & 3y & 4x & 4y \end{array} \\ \begin{bmatrix} 10+20 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & -5 & 20 & 20+5 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 20+20 & 20+20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 20+20 & 20+20 \end{bmatrix} \end{array}$$

$\text{Det}(K^{G}) = 0$; Since [row 7 = row 8] (or col 7 = col 8)

* if two rows or colums are the same in a matrix, its determinant would be zero

$|K^{(G)}| = 0 \Rightarrow$ it is singular and there would be No $\underline{K}^{-1}$ and No solution.

* from physical point of view, it is evident that in node 4 one of conditions for unstability (ie. two rods pined in one point and aligned in the same direction.) is satisfied, So the problem is unstable in point 4.