In this assignment, the analysis of two different beam elements is to be conducted. These elements are based on two different theories namely the Euler-Bernoulli beam theory and the Timoshenko beam theory. The standard beam theory (Euler-Bernoulli beam theory) assumes no deformation by shear. This can be safely applied to thin beams that are long and slender. For short and deep beams however, displacement due to shear becomes hugely important which is included in the formulation of thick beam element (Timoshenko beam theory). However, there exists a third techniques which combines both of these methods which is the Timoshenko beam element with reduced integration. This method could be applied for both thin and thick beams. In the problem in hand, a simply supported beam from both sides is subjected to a uniform load along its axis. A 64-element mesh is used for this analysis in order to insure the accuracy of the solution so the solution would be convergent and not mesh dependent. The element type is specified using a Matlab code that operates on a file generated using the Mat-Fem extension on GID. The parameters taken into consideration for this comparison are the maximum deflection in the beam which occurs at the center node, the maximum bending moment which occurs at the supports, and the maximum shear which also occurs at the support, but it is only considered for the case of the Timoshenko elements. In order to conduct the analysis, the same beam span was considered; however, the square cross-section's dimension would change thus changing the slenderness of the beam (a/l ratio: where a is the high of the square an l is the span of the beam). Thus, the geometrical parameters of the problem are given in the following Table 1 :

a	a/l	Inertia	Area
0.001	0.00025	8.33333E-14	0.000001
0.005	0.00125	5.20833E-11	0.000025
0.01	0.0025	8.33333E-10	0.0001
0.02	0.005	1.33333E-08	0.0004
0.05	0.0125	5.20833E-07	0.0025
0.1	0.025	8.33333E-06	0.01
0.2	0.05	0.000133333	0.04
0.4	0.1	0.002133333	0.16

Table 1 Geometrical parameters of the problem

The material parameters are presented in Table 2:

E=	21000
v=	0.25
G=	8400

Table 2 Material parameters

The Euler-Bernoulli and the Timoshenko beam elements were applied using the given codes. However, in order to apply the Timoshenko element with reduced integration several modifications to the code had to be conducted. Th first modification is to modify the shear stiffness matrix to the following form (Figure 1):

$$\mathbf{K}_s^{(e)} = \left(\frac{GA^*}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \cdots & \frac{(l^{(e)})^2}{4} & -\frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} \\ & \cdots & 1 & -\frac{l^{(e)}}{2} \\ \text{Simetr.} & & \cdots & \frac{(l^{(e)})^2}{4} \end{bmatrix}$$

Figure 1 Shear stiffness matrix for the Timoshenko beam element with reduced integration

A typical result obtained from the GID postprocessor is shown in the following Figure 2:

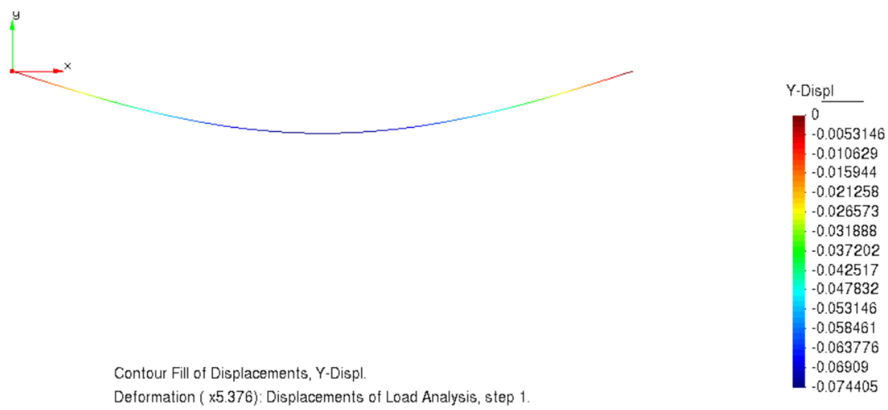

Figure 2 Typical solution shape for the problem

The following results are obtained for the Euler-Bernoulli, Timoshenko and reduced Timoshenko elements.

Max Disp.	Max Moment
-1.90E+09	1.99837
-3047620.00	200.03%
-190476.00	200.03%
-11904.80	200.03%
-304.76	200.03%
-19.05	200.03%
-1.19	200.03%
-0.07	200.03%

Table 3 Euler-Bernoulli element results

Max Disp.	Max Moment	Max Shear
-1461450	0.00153407	1.44E+00
-57401	0.0376582	1.38E+00
-13582.8	0.142575	1.20E+00
-2797.31	0.469782	6.40E-01
-200.427	1.31443	8.01E-01
-16.8752	1.76872	1.58E+00
-1.15964	1.936	1.86E+00
-0.075561	1.98289	1.94E+00

Table 4 Timoshenko element results

Max Disp.	Max Moment	Max Shear
-1.90E+09	1.999	1.97E+00
-3.05E+06	1.999	1.97E+00
-1.90E+05	1.999	1.97E+00
-1.19E+04	1.999	1.97E+00
-3.05E+02	1.999	1.97E+00
-1.91E+01	1.999	1.97E+00
-1.20E+00	1.999	1.97E+00
-7.62E-02	1.999	1.97E+00

Figure 3 Timoshenko with reduced integration element results

Plotting the above results yields the following figures which are useful for comparison purposes:

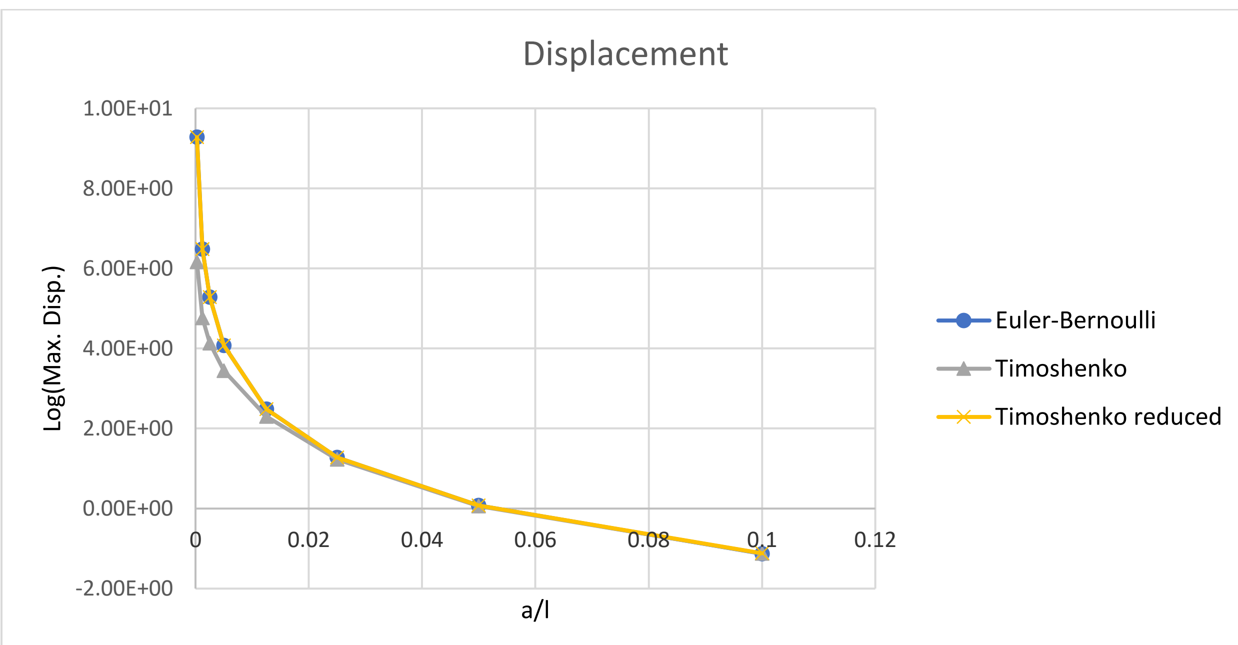

Figure 4 Maximum displacement in the beam for the three types of elements for different cross-section ratios

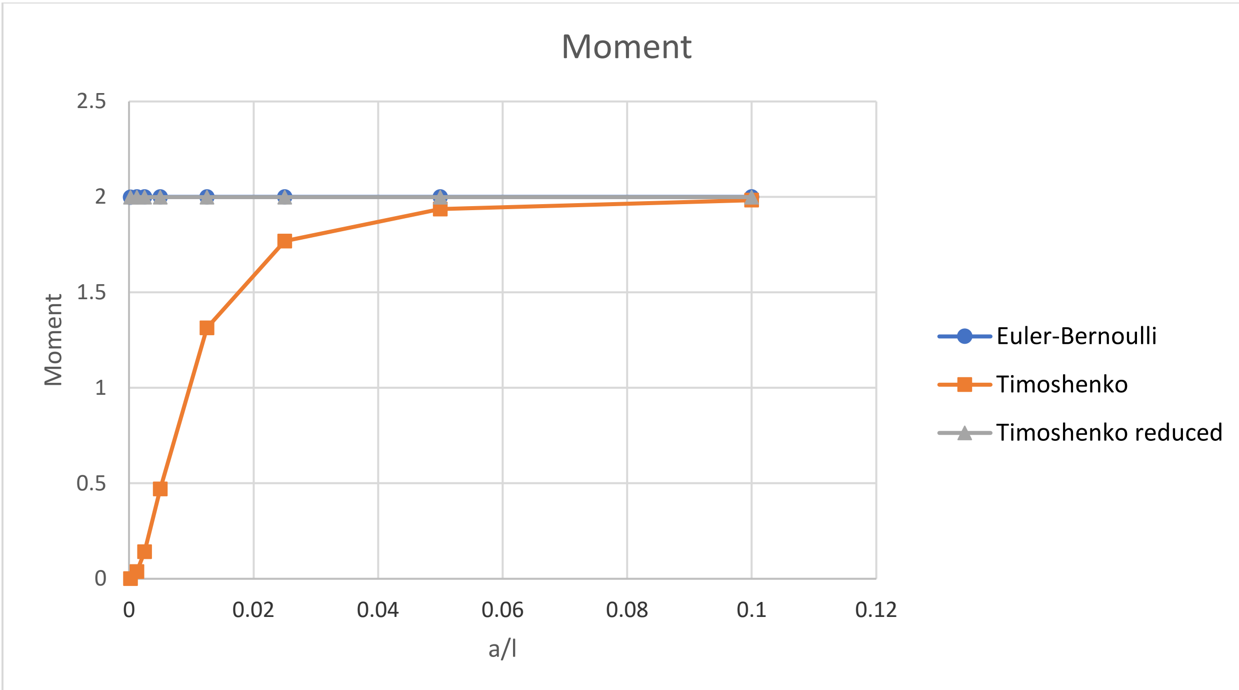

Figure 5 Maximum bending moment in the beam for the three types of elements for different cross-section ratios

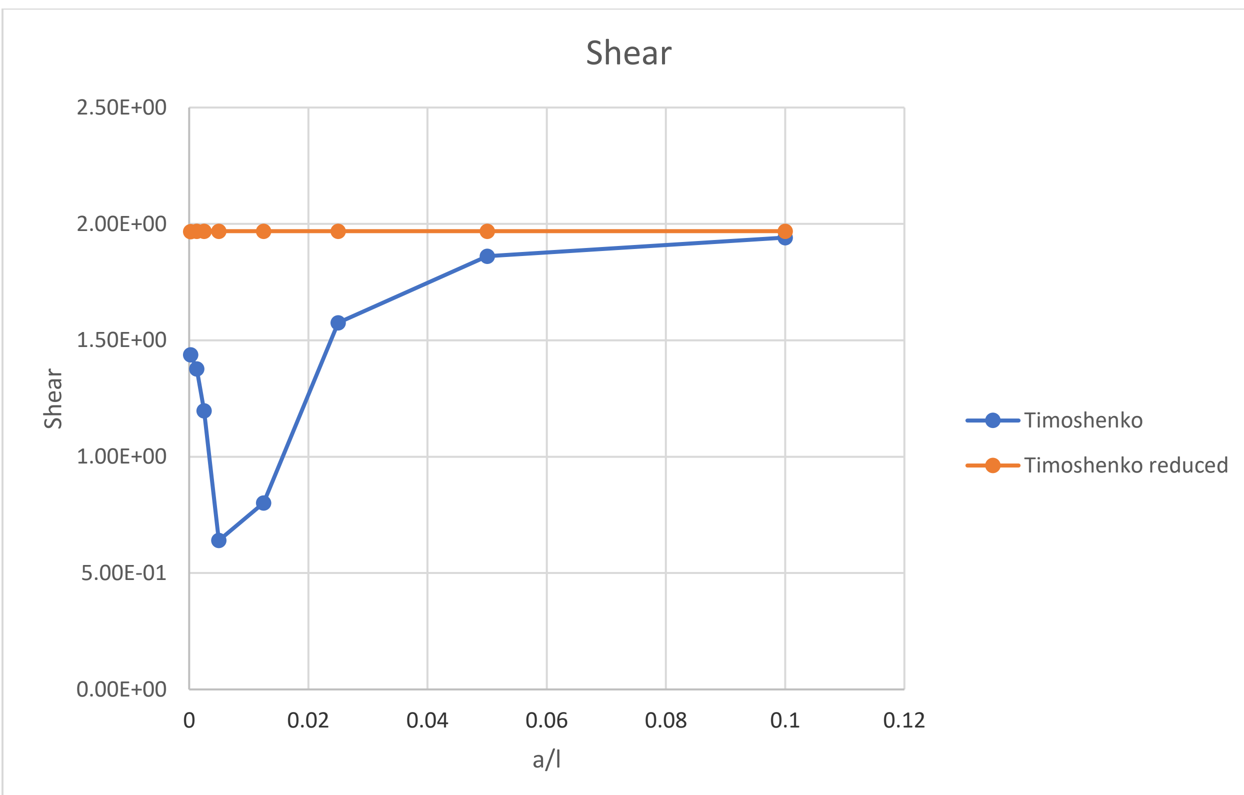

Figure 6 Maximum shear stress in the beam for the Timoshenko normal and reduced elements for different cross-section ratios

The above results highlight the main difference between the Euler-Bernoulli element and the Timoshenko element. It could be observed that for small ratios of a/l typical of a thin beam, the solution of both elements is comparable with regards to the displacement; however, for the moment a significant difference could be observed. This due to the fact that the Timoshenko element does not behave well for the case of thin beams. The more interesting finding is that the Timoshenko element with reduced integration yields almost the exact same results as the Euler-Bernoulli element for both displacement and moment in the case of small a/l ratios. Thus, it could be concluded that the Timoshenko element with reduced integration is able to capture the behavior of thin beams with high accuracy. Moreover, the Timoshenko element with reduced integration is capable of solving for the shear in the element which is not possible to do using the Euler-Bernoulli element. And as previously established, the standard Timoshenko element does not yield accurate results for small a/l ratios thus the results of the shear in this case are expected to be false. For larger a/l ratios, the Timoshenko element with reduced integration behaves similar to the standard Timoshenko element. Thus it could be concluded that the behavior of the Timoshenko element with reduced integration is superior to both the Euler-Bernoulli and the standard Timoshenko elements as it is able to capture the behavior of both thin and thick beams.