Computational Structural Mechanics & Dynamics

Beam Assignment

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❖ Task a : Timoshenko 2-Nodes Beam element with reduce integration for the shear stiffness matrix

For doing this task, two steps shall be completed:

1. Modification of 4 terms in Ks matrix, namely Ks(2,2), Ks(2,4), Ks(4,2) and Ks(4,4)

$$K_s^{(e)} = \left(\frac{GA^*}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \cdots & \frac{(l^{(e)})^2}{4} & -\frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} \\ & \cdots & 1 & -\frac{l^{(e)}}{2} \\ \text{Simetr.} & & \cdots & \frac{(l^{(e)})^2}{4} \end{bmatrix} \quad \text{(Reduced integration)}$$

2. Reduction of gauss point numbers for (Shear) Bs from 2 to 1. The formulas below shows the standard Gauss point numbers for Timoshenko beam element, which are $(\pm\frac{1}{\sqrt{3}})$ but we reduce them to (0).

2 Gauss Points

Shear strain matrix

$$\mathbf{B}_s = \left[\frac{-1}{l^{(e)}} \; , \; \frac{-(1-\xi)}{2} \; , \; \frac{1}{l^{(e)}} \; , \; \frac{-(1+\xi)}{2} \right]$$

$$\gamma_{xz} = \mathbf{B}_s \, \mathbf{a}^{(e)}$$

❖ Task b : pinned beam analysis

A simple beam with both ends pinned would be analyzed using 64 two-noded elements. These elements are:

1. Euler Bernulli element
2. Timoshenko Full Integrate element
3. Timoshenko Reduce Integration element

Main goal is the comparison of these elements for computing maximum displacement, moment and shear for varying a/L ratios:

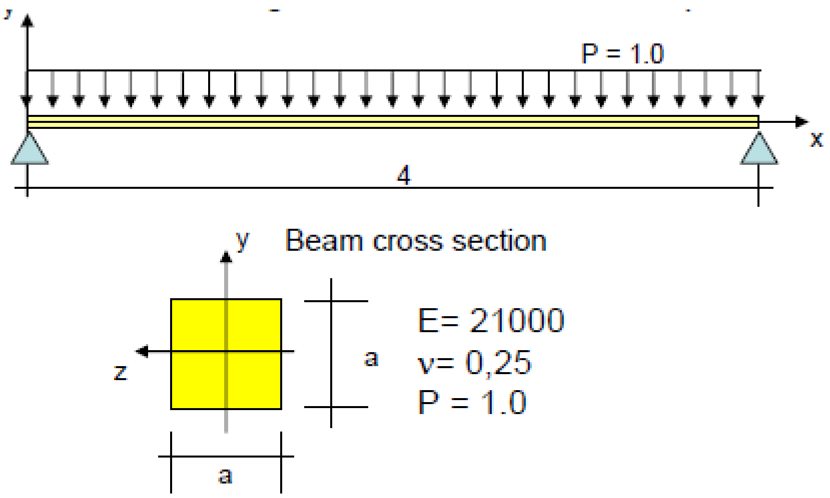


Table1: Definition of Cases

case name	a	area	inertia	L	a/L	q	E	nu	**exact Δ max**
b64q1	0.001	0.000001	8.3E-14	4	0.00025	1	21000	0.25	1.905E+09
b64q2	0.005	0.000025	5.2E-11	4	0.00125	1	21000	0.25	3.048E+06
b64q3	0.01	0.0001	8.3E-10	4	0.0025	1	21000	0.25	1.905E+05
b64q4	0.02	0.0004	1.3E-08	4	0.005	1	21000	0.25	1.190E+04
b64q5	0.05	0.0025	5.2E-07	4	0.0125	1	21000	0.25	3.048E+02
b64q6	0.1	0.01	8.3E-06	4	0.025	1	21000	0.25	1.905E+01
b64q7	0.2	0.04	1.3E-04	4	0.05	1	21000	0.25	1.190E+00
b64q8	0.4	0.16	2.1E-03	4	0.1	1	21000	0.25	7.440E-02
b64q9	1.2	1.44	1.7E-01	4	0.3	1	21000	0.25	9.186E-04

Table3: Maximum Displacement comparison

a	a/L	***Δ max***						
		exact	**Euler**	error %	**Timoshenko**	error %	**Timoshenko-R**	error %
0.001	0.00025	**1.9E+09**	1.9E+09	0	1.5E+06	100	1.9E+09	0
0.005	0.00125	**3.0E+06**	3.0E+06	0	5.7E+04	98	3.0E+06	0
0.01	0.0025	**1.9E+05**	1.9E+05	0	1.4E+04	93	1.9E+05	0
0.02	0.005	**1.2E+04**	1.2E+04	0	2.8E+03	77	1.2E+04	0
0.05	0.0125	**3.0E+02**	3.0E+02	0	2.0E+02	34	3.0E+02	0
0.1	0.025	**1.9E+01**	1.9E+01	1	1.7E+01	12	1.9E+01	1
0.2	0.05	**1.2E+00**	1.1E+00	4	1.1E+00	6	1.1E+00	3
0.4	0.1	**7.4E-02**	6.3E-02	16	6.4E-02	15	6.4E-02	14

Table4: Maximum Moment comparison

a	a/L	*M max*						
		exact	**Euler**	error %	**Timoshenko**	error %	**Timoshenko-R**	error %
0.001	0.00025	**2**	-2.0	0	0.0	100	-2.0	0
0.005	0.00125	**2**	-2.0	0	0.0	98	-2.0	0
0.01	0.0025	**2**	-2.0	0	-0.1	93	-2.0	0
0.02	0.005	**2**	-2.0	0	-0.5	77	-2.0	0
0.05	0.0125	**2**	-2.0	0	-1.3	34	-2.0	0
0.1	0.025	**2**	-2.0	1	-1.8	12	-2.0	1
0.2	0.05	**2**	-1.9	4	-1.9	7	-1.9	4
0.4	0.1	**2**	-1.7	16	-1.7	17	-1.7	16

Table2: Maximum Shear comparison.

a	a/L	*V max*				
		exact	**Timoshenko**	error %	**Timoshenko-R**	error %
0.001	0.00025	**2**	3.9	97	2.0	2
0.005	0.00125	**2**	3.8	91	2.0	2
0.01	0.0025	**2**	3.5	76	2.0	2
0.02	0.005	**2**	2.5	27	2.0	2
0.05	0.0125	**2**	0.1	97	2.0	2
0.1	0.025	**2**	1.3	36	1.9	3
0.2	0.05	**2**	1.7	14	1.9	6
0.4	0.1	**2**	1.6	19	1.7	17

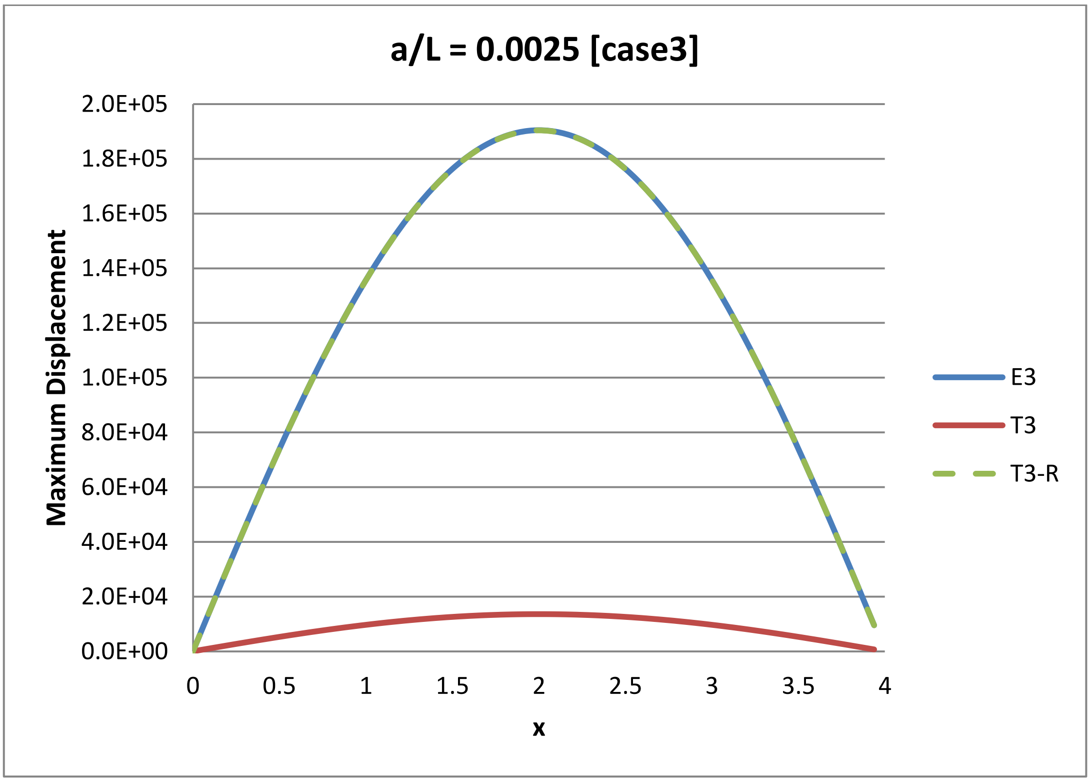
❖ Interpretation

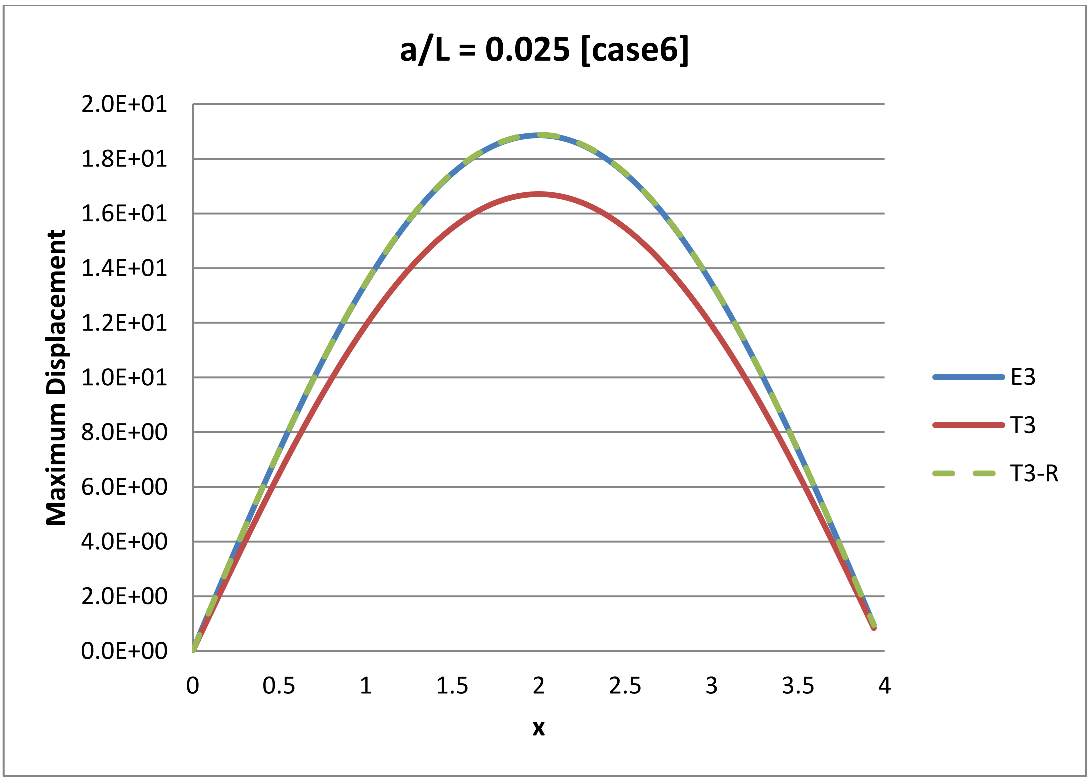
As it is obvious the Reduced Timoshenko elements act almost the same with Euler Bernuli elements.

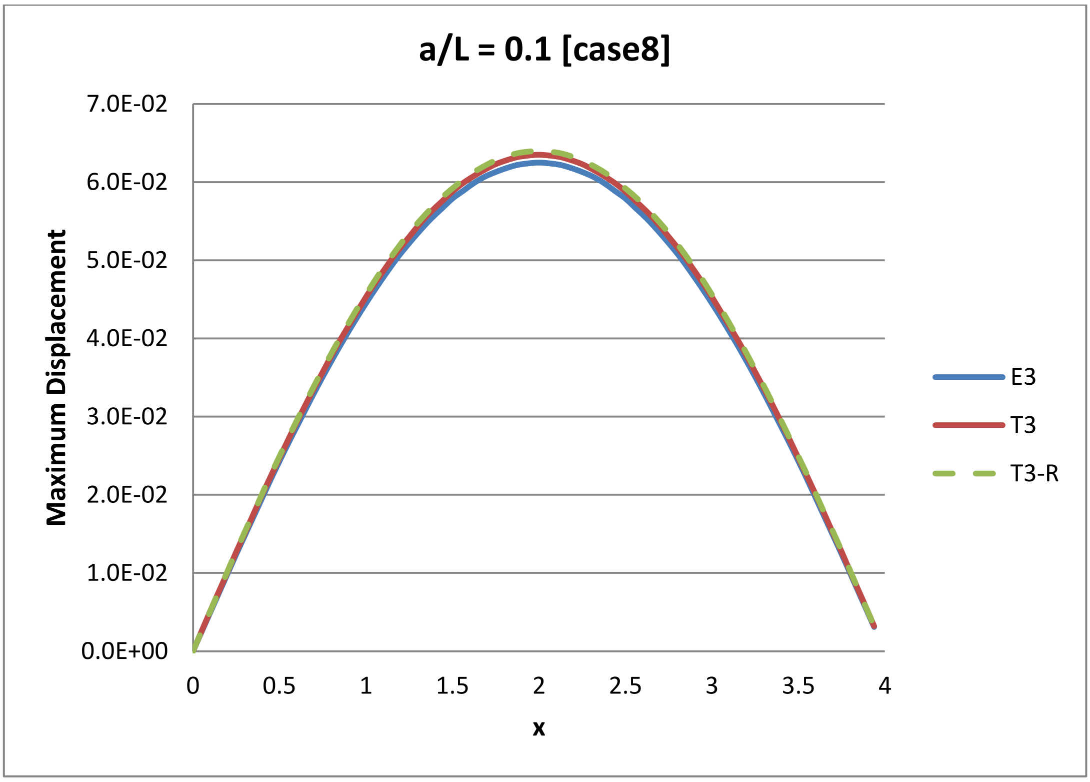
On the other hand for higher a/L ratios the Timoshenko element acts better than Euler Bernuli element as it was expected, because considering the fact that for higher a/L ratios more shear stiffness terms are imposed into the system and Timoshenko method is capable of capturing them.

❖ Graphs

At the end some graphs are provided for further details:







As a result, reduced Timoshenko method is acting always the same with Euler Bernuli element and for higher a/L ratios all methods converge to the same displacement estimations.