## Universitat Politècnica de Catalunya

#### MASTER OF SCIENCE IN COMPUTATIONAL MECHANICS

Computational Structural Mechanics and Dynamics

# Practice 3 Plates

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# **1** Clamped Plate

A squared plate is clamped on all edges as shown on Figure 1.1 and it's subject to a uniform distributed load of  $q = 10^4 \text{ N/m^2}$ . To analyse the state of stress 3 types of discretization were considered, namely the triangular plate elements DKT, triangular Reissner-Midlin elements with 6 nodes with reduced integration and quadrilateral elements QLLL. Examples of the domain discretization can be seen in Figure 1.3.



Figure 1.1: Problem geometry and boundary conditions

Material properties: Concrete 
$$\begin{cases} E = 3.0e10 \frac{N}{m^2} \\ \nu = 0.2 \\ t = 0.1m \end{cases}$$
 Load  $\begin{cases} q = 10^4 \frac{N}{m^2} \end{cases}$ 



Figure 1.2: Element type

Before evaluating the state of stresses of the plate under the specified loading through Finite Elements (GiD), a convergence study is performed to guarantee the used mesh is not responsible for adding computational errors. This problem in particular allows the convergence analysis to be compared to the analytical solution, providing a parameter for verification of the results. The results of the convergence of the maximum displacement analysis are presented on Figure 1.4 in terms of the degrees of freedom present on each discretization.

Analytical solution:

$$D = \frac{Et^3}{12(1-\nu)} = 2604166.667$$
$$f = \frac{\alpha q l^4}{D} = 0.0012353864$$

Discretization:



Figure 1.3: Example of domain discretizations using Quadrilateral (a) and Triangular (b) elements



Figure 1.4: Convergence of the maximum displacement (plate center) for different element types

As we can see, all element types converged to the correct solution eventually. However, it's worth noticing that the DKT element type, alonside the Quadrilateral QLLL converged with less elements (and hence, degrees of freedom) than the Triangular Reissner-Midlin.

The state of the shear stresses are presented on Figure 1.5. As expected, the shear is really high on the vicinity of the clamped edges, given their displacement constraints. The displacement (Figure 1.6), on the other hand, is highest on the center, where the lowest stiffness and, thus, stress, is present.



Figure 1.5: Shear stress on the x (a) and y (b) directions



Figure 1.6: Displacement field countour and deformation with a x500 factor

## 2 Thin plate with internal hole

A steel plate is supported by four columns and it's subject to a uniform distributed load of  $q = 10^4 \text{ N/m}^2$  as shown on Figure 2.1. To analyse the state of stress triangular plate elements DKT were used. The domain discretization can be seen in Figure 2.2. The discretization was performed with the objective of refining the regions where the largest gradients are expected to be present, hence around the places in contact with the columns and around the corners.



Figure 2.1: Problem geometry and boundary conditions



Figure 2.2: Domain discretization

As expected, the highest stresses are found on the surroundings of the points in contact with the supports, where the movement constraints are present. The principal stresses are depicted on Figures 2.3 and 2.4. Similarly to what happened on the problem seen on Chapter 1, the displacements are larger on the middle section, as evident on Figure 2.5.



Figure 2.3: Resulting principal stresses I



Figure 2.4: Resulting principal stresses II



Figure 2.5: Displacement field contour and deformation with x500 factor

# **3** Thick plate with internal hole

A concrete plate is supported by four columns and it's subject to a uniform distributed load of  $q = 10^4 \text{ N/m}^2$  and self-weight as shown on Figure 3.1. To analyse the state of stress triangular Reissner-Midlin elements with 6 nodes with reduced integration were used. The domain discretization can be seen in Figure 3.2. The discretization was performed with the objective of refining the regions where the largest gradients are expected to be present, hence around the places in contact with the columns and around the corners.



Figure 3.1: Problem geometry and boundary conditions



Figure 3.2: Domain discretization

As expected, the highest stresses are found on the surroundings of the points in contact with the supports, where the movement constraints are present. The principal stresses are depicted on Figure 3.3. The displacements are larger on the sections between columns, creating the deformation pattern found on Figure 3.4.



Figure 3.3: Principal stresses I (a) and II (b)



Figure 3.4: Displacement field contour and deformation with x500 factor