SOLID AND STRUCTURAL DYNAMICS ASSIGNMENT.

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1.

r(t) is a constant force F, let's rename r(t)=F,

F=ku + mu''

u, the displacement:

u=A sin(wt)

A is the amplitude of the motion, w is the natural frequency of vibration of the system.

kA sin(wt) - mw^2A sin(wt) –F=0
mw^2A sin(wt) = kA sin(wt) - F

$$w = \sqrt{\frac{\mathrm{kA\,sin(wt)-F}}{\mathrm{mA\,sin(wt)}}} = \sqrt{\frac{k}{m} - \frac{F}{\mathrm{m}\,As\quad\mathrm{(wt)}}}$$

if F=0, $w = \sqrt{\frac{k}{m}}$

But if F≠0 there is a factor which makes the natural frequency of vibration of the system be smaller:

$$\frac{F}{\mathrm{m}\,u}$$

The time-dependent displacement u(t) is now:
u(t)=u + (F/k)
it is displaced F/k, where u is the amplitude of motion.

2.

w = $\sqrt{\frac{k}{m}}$

(m+M) g – ku = (m + M) u''

k= $\frac{(m + M)(g - u'')}{u} = \frac{(m + \rho AL)(g - u'')}{u}$

u is the displacement in the normal direction of the bar
m = mass of the weight at the middle of the uniform bar
M= mass of the bar, M = ρAL

If I suppose M<<m, I can write:

k= $\frac{m\,(g - u'')}{u}$

and
w = $\sqrt{\frac{g - u''}{u}}$

otherwise, w = $\sqrt{\frac{(m + \rho A\ \ )(g - u'')}{mu}}$

3.

$$m = \int N^T N\,\rho dV = \int_0^L N^T N\,\rho\,A dx = N^T N\,\rho\,A\,L$$

$$N^T N = (1/3)\begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$$

$$m = \frac{\rho\,A\,L}{3}\begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\rho\,A\,L}{3} & \frac{\rho\,A\,L}{6} \\ \frac{\rho\,A\,L}{6} & \frac{\rho\,A\,L}{3} \end{pmatrix}$$

4.

A, now depends on x, A(x)
A(0)= A_1
A(L)= A_2

A(x)= A_1 + x

$A_2 = A_1$ +L

m=$\int N^T N\ \rho dV$=$\int_0^L N^T N\ \rho\ A(x)dx$= $\int_0^L N^T N\ \rho\ (A_1\ +\ \text{x})dx = N^T N\ \rho\ A_1\ \ L + N^T N\ \rho \frac{L^2}{2}$
m= $N^T N\ \rho\ A_1\ \ L$ (2 + L)/2

We obtain the same, such as in before exercise, but multiplied by A_1 (2 + L)/2 instead of A

m=$\frac{\rho\ L}{3} A_1\ (2\ +\ \text{L})/2\ \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$=$\frac{\rho\ L}{6} A_1$(2+L)$\begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$

5.

The diagonal mass matrix of the element will also be $m_{11} = 1\ and\ \ m_{22} = 1$
Translation will not affect.