# Computational Solid Mechanics: Assignment 1

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# 1 Inviscid

## 1.1 Input data

The material parameters used in the three sets of tests can be found in table 1. The material follows a softening law with no Poisson effect considered.

## 1.2 Case 1: Full uniaxial test

For the loading-unloading-loading case, stress increments are set in table 2. We can see that the first increment is set to be outside the damage surface (as  $\Delta \sigma_1$  > Yield stress), so as to start having damage at the end of the first load. The model is behaving as expected. Figures 1 to 4 show the evolution of the damage surfaces and the strain - stress relation. We can see that the full uniaxial test does not show any difference between damage models: as the region in stress space in which the stress path is drawn is the same for both models (the first quadrant), the evolution of said damage surface will be, consequently, the same. Differences can be seen when we consider linear or exponential softening. For this set of hardening parameters, softening develops more rapidly in the exponential case. It can be observed that the exponentially softened damage surface is smaller than in the linear case.

Yield stress	150
Linear hardening H	-1
Exp hardening A	1
Young modulus	200
Poisson ratio $\nu$	0
Ratio comp/trac $n$	1.5
Hardening limit $q_{inf}$	$10^{-6}r_0$

Table 1: Material parameters for inviscid (in consistent units)

$\operatorname{step}$	$\Delta \sigma_1$	$\Delta \sigma_2$
1	160	0
2	-50	0
3	60	0

Table 2: Stress increments in full uniaxial test





Figure 1: Damage surface (above) and strain-stress plot for the only-tension damage model with linear softening, case 1



Figure 2: Damage surface (above) and strain-stress plot for the non-symmetric damage model with linear softening, case 1



Figure 3: Damage surface (above) and strain-stress plot for the only-tension damage model with exponential softening, case 1



Figure 4: Damage surface (above) and strain-stress plot for the non-symmetric damage model with exponential softening, case 1

### 1.3 Case 2: Uniaxial - biaxial test

The initial stress increments can be found in table 3. The path chosen to finish every step outside of the damage surface in the non-symmetric case. The only tension model does not account for any limit in the compression zone, and that difference can be seen in the plots. Comparing figures 5 and 6, for instance, shows that while the biaxial loading and unloading stress paths are along the same line up while inside the elastic region (in red), for the non-symmetric model this is not case. For the latter, there exists a limit for the compression, and it is indeed surpassed (see the path in the stress space). This leads to the degradation of the material, and then, the change in the damage surface. In figure 6 this change is marked in red. Is the beggining of the third path (green line). If we consider exponential softening, the stress-strain trajectories show the corresponding exponential curve during inelastic loading, and (in this case, and given the hardening parameter value) the damage surface is reached earlier.

step	$\Delta \sigma_1$	$\Delta \sigma_2$
1	155	0
2	-230	-230
3	300	300

Table 3: Stress increments in uniaxial - biaxial test



1.3.1 Plots

Figure 5: Damage surface (above) and strain-stress plot for the only-tension damage model with linear softening, case 2



Figure 6: Damage surface (above) and strain-stress plot for the non-symmetric damage model with linear softening, case 2



Figure 7: Damage surface (above) and strain-stress plot for the only-tension damage model with exponential softening, case 2



Figure 8: Damage surface (above) and strain-stress plot for the non-symmetric damage model with exponential softening, case 2

# 1.4 Case 3: Full biaxial test

Stress increments are set in table 4. As with the previous cases, the first stress increment is greater than the yield stress (|120, 120| > 150), so as to have damage since the first step.

step	$\Delta \sigma_1$	$\Delta \sigma_2$
1	120	120
2	-40	-40
3	50	50

Table 4: Stress increments in full biaxial test

The models behave in a completely different way. Both of them experience damage in the first step and during the second step reach the zero stress point only to rise again due to the fact that the situation has changed from tensile unloading to compressive loading. The non-symmetric model (figures 10 and 12) is damaged two times, one during the tensile loading and another one during the compressive loading. In this case, it can be assessed in all the —strain— —stress— graphs that for complete unloading, the model returns to the origin (zero strain for zero stress).



#### 1.4.1 Plots

Figure 9: Damage surface (above) and strain-stress plot for the only-tension damage model with linear softening, case 3



Figure 10: Damage surface (above) and strain-stress plot for the non-symmetric damage model with linear softening, case 3



Figure 11: Damage surface (above) and strain-stress plot for the only-tension damage model with exponential softening, case 3



Figure 12: Damage surface (above) and strain-stress plot for the non-symmetric damage model with exponential softening, case 3

# 2 Viscous

In order to compare how the model's response is affected by viscosity and strain rate, we will first select one particular set of values as the standard (see table 5). When we want to see how the change of one particular parameter affects the behaviour of the solid, we will modify the value of said parameter in both directions (higher and lower). This way we will be able to, at least, see the trend of change in the solid's response: when it's getting stiffer and when it's getting softer. The stress path will be the same as in case 3 (full biaxial loadingunloading-loading).

Yield stress	150
Linear hardening H	-1
Young modulus	200
Poisson ratio $\nu$	0
Viscous coefficient $\eta$	10
Hardening limit $q_{inf}$	$10^{-6}r_0$
Total time	100
$\alpha$ coefficient	0.5

Table 5: Standard configuration for the viscous symmetric model

Results for this standard example can be seen in figure 13. We will refer to this one everytime that we modify the viscosity, the strain rate (or time), and the  $\alpha$  coefficient. We can see the general feature of the viscous solids: our



Figure 13: Standard results for the viscous case

stress path can be outside the damage zone, so we find that the norm of the stresses can be higher than the yield stress (points marked in red). The model can be damaged nonetheless, as the trajectories out of the damage surface and the change of its size suggests and the downwards (not linear due to viscosity) trajectory of the stress - strain path shows.

## 2.1 Effect of different viscosities

In order to asses the influence of the viscosity parameter  $\eta$ , we tried two new values:  $\eta = 1$  and  $\eta = 100$ . For the former, the solid viscous response is almost negligible. We can see that the maximum stress is only slightly higher than the yield stress of 150. For the latter we find the oposite: the model increased viscosity stiffens it. Not only the maximum stresses are higher, but also the material is less damaged





Figure 14: Damage surface (above) and strain-stress plot for viscous test with viscosity  $\eta = 1$ , the other values as in the standard case



Figure 15: Damage surface (above) and strain-stress plot for viscous test with viscosity  $\eta = 100$ , the other values as in the standard case

#### 2.2 Effect of different strain rates

The strain rate can be controlled via the total time used in the computation. High values of time means low strain rates (same strain applied over a longer period of time), and vice versa. For this reason, we used as extra values t = 10 and t = 1000 (one order of magnitude more and one less). As the strain rate increase, the material stiffens: damage appears later and the stress-strain relationship is linear for a longer period of time. We can see that if the relationship between strain rate and viscosity is kept the same, the model's behaviour is also the same. The response for  $\eta = 1$  and total time (indirect measure of strain rate) t = 100 is the same that the response for  $\eta = 10$  and total time t = 1000. We can see that in the previous example the relation viscosity/time was the same  $\frac{\eta}{t} = 0.01$ . If we now reduce the time from the standard value of 100 to 10 (one order of magnitude less) with the rest of the parameters at their standard, we would have a relationship  $\frac{\eta}{t} = 1$ . This is the same as considered in 15, and that is why the model's response is the same. As expected, the model's response depends on the relationship strain rate / viscosity.





Figure 16: Damage surface (above) and strain-stress plot for viscous test with total time t = 10, the other values as in the standard case



Figure 17: Damage surface (above) and strain-stress plot for viscous test with total time t = 1000, the other values as in the standard case

### 2.3 Effect of different alpha (time integration methods)

As the time integration method can lead to instabilities, we will try a combination of parameters such that said isntabilities can appear. The stress path considered differs from the used in the previous tests (see table 6), as in this case we want to be always outside of the damage zone to asses the behaviour during inelastic loading. As we want to use a low  $\Delta t$ , the total time and the time-steps per path used in the computations were t = 1000000 and  $istep_1 = istep_2 = istep_3 = 10$ , all the other parameters (except  $\alpha$ , that we will modify) as standard. Our model then will work under  $\frac{\Delta t}{\eta} = \frac{10000}{3}$ . We can see in figure 18 that for the pure explicit method (Euler time integration) the results are completely spoiled: the solid is experiencing compression when we are inducing tension. For  $\alpha = 0.25$ , results are not as bad as with Euler, but we find instabilities: see that during inelastic loading (green and blue lines) the solution is going up and down. This was expected, as we are using a timeunstable integration coefficient for  $\alpha \in [0 \ 0.5)$ . For  $\alpha > 0.5$  (figures 20 to 22) these instabilities disappear. As we approach  $\alpha = 1$ , and given in this particular case the low viscosity that we are considering, the model behaves more like if it was inviscid. Again, this was expected from the theory.

step	$\Delta \sigma_1$	$\Delta \sigma_2$
1	120	120
2	20	20
3	30	30

Table 6: Stress increments in tests with different  $\alpha$ 



2.3.1 Plots

Figure 18: Damage surface (above) and strain-stress plot for viscous test with  $\alpha = 0$ . The results are spoiled and make no sense



Figure 19: Strain-stress plot for viscous test with  $\alpha = 0.25$ . It shows oscillations (in red) due to instabilities



Figure 20: Strain-stress plot for viscous test with  $\alpha = 0.5$ , Crank-Nicholson method. The method for this  $\alpha$  and higher are unconditionally stable



Figure 21: Strain-stress plot for viscous test with  $\alpha = 0.75$ . The inviscid part plays a higher role than with  $\alpha = 0.5$ 



Figure 22: Strain-stress plot for viscous test with  $\alpha = 1$ . Inviscid case has been recovered

### 2.4 Constitutive operator evolution

Using the same input data from the previous tests we now take a look to the 11 component of the C operator in its analytical and algorithmic versions. We find similar results depending on  $\alpha$ . For values smaller than 0.5, some oscillations due to instabilities are found. For greater than 0.5 values, the results are practicly the same, as it is stable (and, in this case, the time-viscosity relation is not very

important). For the case  $\alpha = 0$ , both results coincide, although in this test we can see that the time integration method fails to compute realistic results.

#### 2.4.1 Plots



Figure 23: Evolution of the component 11 of the analytical tangent constitutive vs time for several values of  $\alpha$ 



Figure 24: Evolution of the component 11 of the algorithmic tangent constitutive vs time for several values of  $\alpha$ 

# 3 Annex: Code

damage\_main

```
1 function [sigma_v, vartoplot, LABELPLOT, TIMEVECTOR] = ...
      damage_main(Eprop, ntype, istep, strain, MDtype, n, TimeTotal)
2
3 global hplotSURF
  4
5 % CONTINUUM DAMAGE MODEL
  <u>e</u>_____
6
  % Given the almansi strain evolution ("strain(totalstep,mstrain)") and a
7
8
  % set of
  % parameters and properties, it returns the evolution of the cauchy stress
9
10
  % and other variables
11 % that are listed below.
12 %
14
  8 -
15 % Eprop(1) = Young's modulus (E)
16 % Eprop(2) = Poisson's coefficient (nu)
17 % Eprop(3) = Hardening(+)/Softening(-) modulus (H)
  % Eprop(4) = Yield stress (sigma_y)
18
  % Eprop(5) = Type of Hardening/Softening law (hard_type)
19
              0 --> LINEAR
20 %
21 %
              1 --> Exponential
22 % Eprop(6) = Rate behavior (viscpr)
              0 --> Rate-independent (inviscid)
23 %
  9
               1 --> Rate-dependent
                                     (viscous)
^{24}
  응
25
  % Eprop(7) = Viscosity coefficient (eta) (dummy if inviscid)
26
  % Eprop(8) = ALPHA coefficient (for time integration), (ALPHA)
27
  9
                0{\leq}\text{ALPHA}{\leq}1 , ALPHA = 1.0 --> Implicit
^{28}
                             ALPHA = 0.0 --> Explicit
^{29}
  응
  8
               (dummy if inviscid)
30
31 %
            = PROBLEM TYPE
32 % ntype
              1 : plane stress
33
  Ŷ
^{34}
  Ŷ
               2 : plane strain
  8
               3 : 3D
35
  응
36
  % istep = steps for each load state (istep1,istep2,istep3)
37
  8
38
39
  % strain(i,j) = j-th component of the linearized strain vector at the i-th
                  step, i = 1:totalstep+1
40 %
41 %
               = Damage surface criterion %
42 % MDtype
  %
               1 : SYMMETRIC
^{43}
               2 : ONLY-TENSION
  8
44
  응
               3 : NON-SYMMETRIC
^{45}
46
  8
47
  8
               = Ratio compression/tension strength (dummy if MDtype
^{48}
  % n
49 % is different from 3)
50 %
51 % TimeTotal = Interval length
52 \ \%
```

```
53
54
   2
      1) sigma_v{itime}(icomp, jcomp) --> Component (icomp, jcomp) of the cauchy
   2
55
                                    stress tensor at step "itime"
56
   ę
                                    REMARK: sigma_v is a type of
   8
57
58
   응
                                    variable called "cell array".
59
   8
   2
60
   ŝ
      2) vartoplot{itime--> Cell array containing variables one wishes to plot
61
62
   응
       vartoplot{itime}(1) = Hardening variable (q)
63
   ÷
       vartoplot{itime}(2) = Internal variable (r)%
64
   8
65
66
   ę
                                   --> Cell array with the label string for
   % 3) LABELPLOT{ivar}
67
                                     variables of "varplot"
   Ŷ
68
69
   8
             LABELPLOT\{1\} \Rightarrow 'hardening variable (q)'
   응
70
71 %
             LABELPLOT{2} => 'internal variable'
   2
72
73
   2
74 % 4) TIME VECTOR ->
76
   % SET LABEL OF "vartoplot" variables
77
   8 --
78
   LABELPLOT = { 'hardening variable (q) ', 'internal variable',...
79
   'Analytic tangent C', 'Algorithmic tangent C'};
80
81
82 E
        = Eprop(1) ; nu = Eprop(2) ;
83 H = Eprop(3);
84 viscpr = Eprop(6);
85 sigma_u = Eprop(4);
se eta = Eprop(7);
      alpha = Eprop(8);
87
88
89
90
   if ntype == 1
91
^{92}
       menu('PLANE STRESS has not been implemented yet', 'STOP');
       error('OPTION NOT AVAILABLE')
93
   elseif ntype == 3
94
       menu('3-DIMENSIONAL PROBLEM has not been implemented yet', 'STOP');
95
       error('OPTION NOT AVAILABLE')
96
   else
97
98
      mstrain = 4
                    ;
      mhist = 6
                  ;
99
100 end
101
   if viscpr == 1
102
       % Comment/delete lines below once you have implemented this case
103
       104
          menu({'Viscous model has not been implemented yet. '; ...
105 % %
             'Modify files "damage_main.m","rmap_dano1" ' ; ...
106 응 응
              'to include this option'},
107 % %
                                       . . .
              'STOP');
108 응 응
109 % %
          error('OPTION NOT AVAILABLE')
```

```
110
111 else
112 end
113
114
115 totalstep = sum(istep) ;
116
117
118 % INITIALIZING GLOBAL CELL ARRAYS
119 % ------
120 sigma_v = cell(totalstep+1,1);
121 TIMEVECTOR = zeros(totalstep+1,1);
123
124
125 % Elastic constitutive tensor
126 % -----
127 [ce] = tensor_elasticol (Eprop, ntype);
128 % Initz.
129 % -----
130 % Strain vector
131 % ------
132 eps_n1 = zeros(mstrain,1);
133 % Historic variables
134 % hvar_n(1:4) --> empty
135 % hvar_n(5) = q --> Hardening variable
136 % hvar_n(6) = r --> Internal variable
137 hvar_n = zeros(mhist,1) ;
138
139 % INITIALIZING (i = 1) !!!!
140 % *********i*
141 i = 1;
142 r0 = sigma_u/sqrt(E);
143 hvar_n(5) = r0; % r_n
144 hvar_n(6) = r0; % q_n
145 eps_n1 = strain(i,:) ;
146 sigma_n1 =ce*eps_n1'; % Elastic
147 sigma_v{i} = [sigma_n1(1) sigma_n1(3) 0;sigma_n1(3) sigma_n1(2) 0 ;
       0 0 sigma_n1(4)];
148
149
150 nplot = 3 ;
151 vartoplot = cell(1,totalstep+1);
152 vartoplot{i}(1) = hvar_n(6) ; % Hardening variable (q)
153 vartoplot{i}(2) = hvar_n(5) ; % Internal variable (r)
   vartoplot{i}(3) = 1-hvar_n(6)/hvar_n(5) ; % Damage variable (d)
154
155 if viscpr
       vartoplot{i}(4) = (hvar_n(6) / hvar_n(5)) * ce(1,1);
156
157
       vartoplot{i}(5) = (hvar_n(6)/hvar_n(5)) *ce(1,1);
   end
158
159
160 % LOOP over states (over the three paths)
161 for iload = 1:length(istep)
162
       % Load states
       for iloc = 1:istep(iload)
163
           i = i + 1 ;
164
           TIMEVECTOR(i) = TIMEVECTOR(i-1) + \Delta_t(iload);
165
166
           % Total strain at step "i"
```

```
167
168
           eps_n1 = strain(i,:);
                                169
           8****************
           %* DAMAGE MODEL
170
           171
172
173
           if (iload*iloc == 1)
              % For inviscid case, 1st rtrial must be done explicitly
174
              Eprop(8) = 0;
175
              rtrial_o = 0;
176
              [sigma_n1, hvar_n, aux_var, rtrial_o] = rmap_dano1...
177
178
                  (eps_n1, hvar_n, Eprop, ce, MDtype, n, △_t, rtrial_o);
          else
179
180
              Eprop(8) = alpha;
              [sigma_n1, hvar_n, aux_var, rtrial_o] = rmap_dano1(eps_n1...
181
                  , hvar_n, Eprop, ce, MDtype, n, A_t, rtrial_0);
182
183
184
           end
185
              % PLOTTING DAMAGE SURFACE
           if(aux_var(1)>0)
186
187
              hplotSURF(i) = dibujar_criterio_dano1(ce, nu, hvar_n(6),...
                  'r:',MDtype,n );
188
              set(hplotSURF(i), 'Color', [0 0 1], 'LineWidth', 1)
189
                                                             ;
           end
190
191
           192
           193
           % GLOBAL VARIABLES
194
195
           8 *********
196
           % Stress
197
           § _____
          m_sigma=[sigma_n1(1) sigma_n1(3) 0;sigma_n1(3) sigma_n1(2) 0;
198
              0 0 sigma_n1(4)];
199
           sigma_v{i} = m_sigma ;
200
201
202
           % VARIABLES TO PLOT (set label on cell array LABELPLOT)
           8 _____
203
204
          vartoplot{i}(1) = hvar_n(6) ; % Hardening variable (q)
           vartoplot{i}(2) = hvar_n(5) ; % Internal variable (r)
205
206
           vartoplot{i}(3) = 1-hvar_n(6)/hvar_n(5) ; % Damage variable (d)
207
           if viscpr
208
              cet = sigma_n1'*sigma_n1;
              vartoplot{i}(4) = (hvar_n(6)/hvar_n(5)) *ce(1,1);
209
              % Tangent analytical operator
210
              vartoplot{i}(5) = \dots
211
212
                  (hvar_n(6)/hvar_n(5)) * ce(1,1) - ((alpha*\Delta_t)/...
           (eta+alpha*△_t))*(1/rtrial_o)*...
213
214
           (hvar_n(6)-hvar_n(5)*H)/(hvar_n(5)^2))...
           *(cet(1,1));
215
216
           end
217
       end
218 end
```

rmap\_dano1

```
1 function [sigma_n1, hvar_n1, aux_var, rtrial] = rmap_dano1 ...
 2
      (eps_n1, hvar_n, Eprop, ce, MDtype, n, dt, rtrial_o)
 3
 4
  5 %*
                                       *
 6 %*
              Integration Algorithm for a isotropic damage model
7 응*
  응*
8
                                                                 *
               [sigma_n1, hvar_n1, aux_var] = ...
 9 %*
10 %* rmap_dano1 (eps_n1, hvar_n, Eprop, ce) *%*
  *
   %∗ INPUTS
                      eps_n1(4) strain (almansi) step n+1
11
  *
                                 vector R4 (exx eyy exy ezz)
12 %*
  *
                       hvar_n(6) internal variables , step n
13
   .
8*
14 %*
                                 hvar_n(1:4) (empty)
  *
15
   응*
                                 hvar_n(5) = r; hvar_n(6) = q
  *
                      Eprop(:) Material parameters
16
   응*
  *
17
   응 *
18 %*
                       ce(4,4)
                                Constitutive elastic tensor
  *
19 8*
                       dt.
                                Time step
20 %*
21 %*
                       rtrial_o previous value of rtrial (inviscid case)
22 %
23 %* OUTPUTS:
                      sigma_n1(4) Cauchy stress , step n+1
  *
                                                                 hvar_n(6) Internal variables , step n+1
^{24}
   응*
  *
                                                                 25 %*
                      aux_var(3) Auxiliar variables for computing
26 \frac{6}{6}
                       const. tangent tensor *
   8******
27
^{28}
29
30 hvar_n1 = hvar_n;
31 r_n = hvar_n(5);
         = hvar_n(6);
32 q_n
33 E
         = Eprop(1);
        = Eprop(2);
34 nu
       = Eprop(3);
35 H
36 sigma_u = Eprop(4);
37 hard_type = Eprop(5) ;
38 visc = Eprop(6);
39 eta = Eprop(7);
40 alpha = Eprop(8);
41
^{42}
  ^{43}
44
```

```
46 응*
      initializing
응*
47 r0 = sigma_u/sqrt(E);
48 zero_q=1.d-6*r0;
49 % if(r_n<0.d0)
50 % r_n=r0;
51 %
      q_n=r0;
52 % end
54
55
************
57 %* Damage surface
응*
                                                      58 [rtrial] = Modelos_de_dano1(MDtype, ce, eps_n1, n, r_n, q_n);
* *
60
61 % rtrial_o = rtrial previous timestep
************
63 %* Ver el Estado de Carga
응*
                                                      응*
      ----> fload=0 : elastic unload
64
 응*
                                                      ----> fload=1 : damage
65
  응*
     (compute algorithmic constitutive tensor)
                                           응*
66 %
67 fload=0;
68 if visc
    ralpha = alpha*rtrial + (1-alpha)*rtrial_o;
69
     if ralpha > r_n
70
71
       %* Loading
       fload = 1;
72
       \Delta_r = (ralpha - r_n) * dt/(eta+alpha*dt);
73
       r_n = r_n + \Delta_r;
74
       if hard_type == 0
75
76
          % Linear
          q_n1= q_n+ H*∆_r;
77
78
        else
          error('EXPONENTIAL LAW not implemented for inviscid case');
79
80
       end
81
82
83
     else
       %* Unloading
84
       fload=0;
85
86
       r_n1= r_n ;
       q_n1= q_n ;
87
88
     end
89
90 else
     if(rtrial > r_n)
91
       %∗ Loading
^{92}
93
       fload=1;
^{94}
95
        ∆_r=rtrial-r_n;
       r_n1= rtrial ;
96
97
       if hard_type == 0
```

```
% Linear
98
99
           q_n 1 = q_n + H \star \Delta_r;
         else
100
            % Comment/delete lines below once you have implemented this case
101
            102
103
      8 8
            menu({'Hardening/Softening exponential law has not
      % been implemented yet. '; ...
104
      8 8
          'Modify file "rmap_dano1" '; ...
105
                 'to include this option'}, ...
106
     응 응
                 'STOP');
107
     8 8
     e e
              error('OPTION NOT AVAILABLE')
108
     %q_rate = 0.05;
109
           q_inf = zero_q; %
110
111
            q_n 1 = q_i n f - (q_i n f - q_n) * exp(H*(1 - (r_n 1/r0)));
         end
112
113
114
        if(q_n1<zero_q)
           q_n1=zero_q;
115
116
         end
117
118
     else
119
120
        %* Elastic load/unload
121
        fload=0;
122
123
         r_n1= r_n
                 ;
        q_n1= q_n ;
124
125
126
127
     end
128 end
129 % Damage variable
130 % -----
131 dano_n1 = 1.d0-(q_n1/r_n1);
  % Computing stress
132
133 % *************
134 sigma_n1 =(1.d0-dano_n1)*ce*eps_n1';
135 %hold on
136 %plot(sigma_n1(1), sigma_n1(2), 'bx')
137
139
140
%* Updating historic variables
142
 응*
                                                         143 % hvar_n1(1:4) = eps_n1p;
144 hvar_n1(5) = r_n1 ;
145 hvar_n1(6)= q_n1 ;
146
  147
148
149
150
152 %* Auxiliar variables
  응*
```

 Modelos\_de\_dano1

```
1 function [rtrial] = Modelos_de_dano1 (MDtype, ce, eps_n1, n, r, q)
 * * * * * * * * * * * * * * * * * * *
           Defining damage criterion surface
 3 %∗
  응*
                                                            8*
 ^{4}
  응*
                                                            5 %*
                         MDtype= 1 : SYMMETRIC
6 %*
  응*
                         MDtype= 2
 \overline{7}
   8*
                                     : ONLY TENSION
  응*
                         MDtype= 3
                                     : NON-SYMMETRIC
 8
   8*
  응*
9
   응*
  응*
10
   응*
  응*
   %∗ OUTPUT:
11
  응*
  .
ક*
                        rtrial
12
  응*
   13
14
15
17 if (MDtype==1) %* Symmetric
18 rtrial= sqrt(eps_n1*ce*eps_n1')
                                            ;
19
20 elseif (MDtype==2) %* Only tension
      s_eff = eps_n1*ce; % Efective stress
^{21}
22
      s_eff_p = 0.5*(s_eff + abs(s_eff)); % Effective stress for only tension
      % As seen in Notes in continuum damage models page 18
23
24 rtrial = sqrt(s_eff_p * eps_n1');
25
26 elseif (MDtype==3) %*Non-symmetric
27
      s = eps_n1*ce; % Effective stress
^{28}
      theta = 0; % Initialize theta = sum <sigma> / sum (|sigma|)
^{29}
        for i=1:length(s)
30
            theta = theta + mac(s(i)); % Numerator
^{31}
         end
32
      theta = theta/sum(abs(s)); %Denominator
33
      rtrial = ((theta + (1-theta)/n)) * sqrt(eps_n1*ce*eps_n1'); % strain r
^{34}
35 % %
       else
36 % %
            if s(1) > 0 \% 4th quadrant
37 8 8
   8 8
           else % 2nd quadrant
38
39 % %
40 응 응
           end
41
42 end
44 return
```

dibujar\_criterio\_dano1

```
1 function hplot = dibujar_criterio_dano1(ce,nu,q,tipo_linea,MDtype,n)
^{2}
3
4
6
  응*
         Inverse ce
 응*
7 ce_inv=inv(ce);
s c11=ce_inv(1,1);
9 c22=ce_inv(2,2);
10 c12=ce_inv(1,2);
11 c21=c12;
12 cl4=ce_inv(1,4);
13 c24=ce_inv(2,4);
15
16
17
18
19
^{20}
^{21}
  ^{22}
23 % POLAR COORDINATES
24 if MDtype==1
^{25}
     tetha=[0:0.01:2*pi];
     26
     %∗ RADIUS
^{27}
    D=size(tetha);
^{28}
                                   %∗ Range
    ml=cos(tetha);
                                   응*
^{29}
30
    m2=sin(tetha);
                                   응*
     Contador=D(1,2);
                                   응*
31
32
33
     radio = zeros(1,Contador) ;
^{34}
35
     s1 = zeros(1,Contador);
         = zeros(1,Contador) ;
     s2
36
37
     for i=1:Contador
38
        % Radius is the tau_sig = q/sqrt(sigma_zeta*C-1*sigma_zeta)
39
        radio(i) = q/sqrt([m1(i) m2(i) 0 nu*(m1(i)+m2(i))]*...
40
          ce_inv*[m1(i) m2(i) 0 ...
41
^{42}
            nu*(m1(i)+m2(i))]');
43
        %Polar projection: r cos, r sin
^{44}
        s1(i)=radio(i)*m1(i);
45
         s2(i)=radio(i)*m2(i);
46
47
     end
48
49
     hplot =plot(s1,s2,tipo_linea);
50
51
52 elseif MDtype==2
53
```

```
54
55
        tetha=[-0.5*pi:0.01:pi]; % Span the angle for onlt tension model
        Implementing McAuley bracket: x*(x>0) = x if x>0, 0 if x =< 0
56
        % sigma+ = <sigma>
57
        58
        %∗ RADIUS
59
60
       D=size(tetha);
                                            %∗ Range
                                           응*
       ml=cos(tetha);
61
       m2=sin(tetha);
                                           응*
62
       Contador=D(1,2);
                                            응*
63
64
65
       radio = zeros(1,Contador) ;
66
67
       s1 = zeros(1,Contador);
       s2
             = zeros(1,Contador) ;
68
69
70
       for i=1:Contador
           radio(i) = q/sqrt([mac(m1(i)) mac(m2(i)) 0 mac(nu*(m1(i)+m2(i)))]*...
71
72
               ce_inv*[m1(i) m2(i) 0 ...
               nu*(m1(i)+m2(i))]');
73
74
           s1(i) = radio(i) * m1(i);
75
           s2(i)=radio(i)*m2(i);
76
77
        end
78
79
       hplot =plot(s1,s2,tipo_linea);
80
81
   elseif MDtype==3
82
        % Comment/delete lines below once you have implemented this case
83
        84
           menu({'Damage surface "NON-SYMMETRIC" has not been implemented yet. '; ...
   8 8
85
   응 응
               'Modify files "Modelos_de_dano1" and "dibujar_criterio_daho1"'; ...
86
               'to include this option'}, ...
87
   8 8
               'STOP');
88
   8 8
   8 8
           error('OPTION NOT AVAILABLE')
 89
   theta= [0:0.01:2*pi]; % Span the angle for non-symmetric model
90
91
   %∗ RADIUS
       D=size(theta);
                                            %∗ Range
92
93
       ml=cos(theta);
                                            응*
       m2=sin(theta);
                                            응*
^{94}
95
       Contador=D(1,2);
                                            8.*
96
97
        radio = zeros(1,Contador) ;
98
       s1 = zeros(1,Contador);
99
             = zeros(1,Contador) ;
100
       s2
101
        for i=1:Contador
           % Radius is the tau_sig = q/sqrt(sigma_zeta*C-1*sigma_zeta)
102
103
           if (theta(i) \geq 0.5*pi) & (theta(i) \leq pi) % If in second quadrant
               radio(i) = (q/sqrt([m1(i) m2(i) 0 nu*(m1(i)+m2(i))]*...
104
                   ce_inv*[m1(i) m2(i) 0 ...
105
106
               nu*(m1(i)+m2(i))]'))/(m2(i) - (m1(i)/n));
107
           elseif (theta(i) > pi) && (theta(i) < 1.5*pi) % If in third quadrant
108
               radio(i) = n*q/sqrt([m1(i) m2(i) 0 nu*(m1(i)+m2(i))]*...
                  ce_inv*[m1(i) m2(i) 0 ...
109
110
               nu*(m1(i)+m2(i))]');
```

```
111
             <code>elseif</code> (theta(i) \geq 1.5*pi) % If in fourth quadrant
112
                  radio(i) = n*(q/sqrt([m1(i) m2(i) 0 nu*(m1(i)+m2(i))]*...
                      ce_inv*[m1(i) m2(i) 0 ...
113
                  nu*(m1(i)+m2(i))]'))/(n*m1(i) - m2(i));
114
             else % If in first quadrant
115
116
                 radio(i) = q/sqrt([m1(i) m2(i) 0 nu*(m1(i)+m2(i))]*...
                      ce_inv*[m1(i) m2(i) 0 ...
117
                 nu*(m1(i)+m2(i))]');
118
119
             \quad \text{end} \quad
             %Polar projection: r cos, r sin
120
121
             s1(i)=radio(i)*m1(i);
             s2(i)=radio(i)*m2(i);
122
123
         end
124
         hplot =plot(s1,s2,tipo_linea);
125
126
127
128
129 end
130
131 return
```