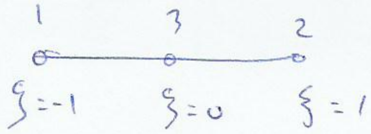
6.1

In meshes a, b and c the node of the interconnected elements do not coincide.

In meshes d, e, f nodes have coincidence, however the number of elements do not fix. ~~this fact is~~ and the element sizes do not match. Thus there is no Continuity $C^0$.

6.2

$$\begin{bmatrix} 1 \\ \bar{x} \\ \bar{\mu} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \bar{x}_1 & \bar{x}_2 & \bar{x}_3 \\ \bar{\mu}_1 & \bar{\mu}_2 & \bar{\mu}_3 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} \qquad \begin{cases} \bar{x}_1 = 0 \\ \bar{x}_2 = L \\ \bar{x}_3 = \frac{1}{2}L + \alpha L \end{cases}$$




$$1 = N_1 + N_2 + N_3$$

$$\bar{x} = \bar{x}_1 N_1 + \bar{x}_2 N_2 + \bar{x}_3 N_3$$

Shape functions $\longrightarrow$

$$\begin{cases} N_1(\xi) = \frac{1}{2}\xi(\xi-1) \to \frac{dN_1}{d\xi} = \xi - \frac{1}{2} \\ N_2(\xi) = \frac{1}{2}\xi(\xi+1) \to \frac{dN_2}{d\xi} = \xi + \frac{1}{2} \\ N_3(\xi) = 1-\xi^2 \to \frac{dN_3}{d\xi} = -2\xi \end{cases}$$

$$\hookrightarrow \bar{x} = \bar{x}_1 N_1(\xi) + \bar{x}_2 N_2(\xi) + x_3 N_3(\xi)$$

$$\bar{x} = 0 \cdot N_1 + \frac{L}{2}\xi(\xi+1) + \left(\frac{1}{2}L + \alpha L\right)(1-\xi^2)$$

$$\bar{x} = \frac{L}{2}(\xi^2 - \xi) + \left(\frac{1}{2}L + \alpha L\right)(1-\xi^2)$$

$$J = \frac{d\bar{x}}{d\xi}$$

$$J = L\xi + \frac{L}{2} - \xi L - 2\alpha L\xi = \frac{L}{2} - 2\alpha L\xi = L\left(\frac{1}{2} - 2\alpha\xi\right)$$

B! If $\alpha = +\frac{1}{4} \rightarrow J = \frac{L}{2}(1-\xi)$

If $\alpha = -\frac{1}{4} \rightarrow J = \frac{L}{2}(1+\xi)$

$$B = \frac{dN}{dx} = J^{-1}\frac{dN}{d\xi} = \frac{1}{L\left(\frac{1}{2} - 2\alpha\xi\right)}\frac{dN}{d\xi}$$

For $\alpha = \frac{1}{4} \rightarrow \xi = 1$

then $\frac{1}{L\left(\frac{1}{2} - 2\alpha\xi\right)}\frac{dN}{d\xi} = \infty$

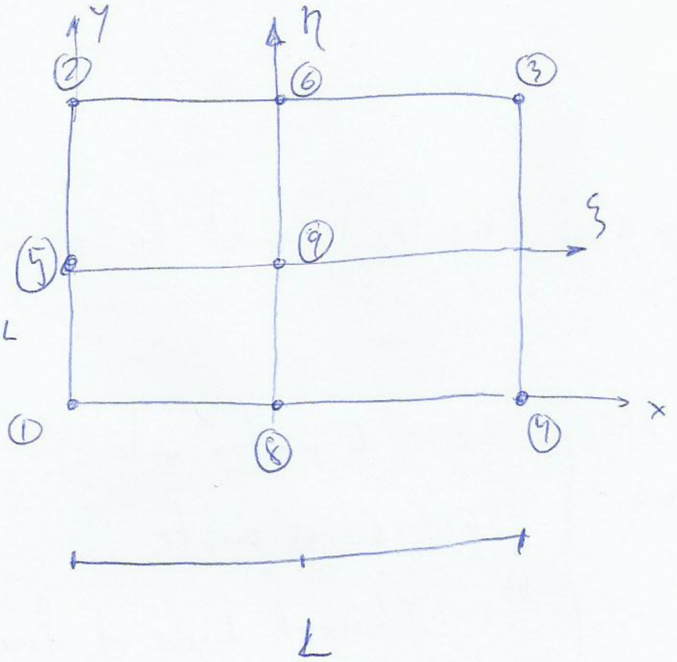
For $\alpha = -\frac{1}{4} \rightarrow \xi = -1$

then $\frac{1}{L\left(\frac{1}{2} - 2\alpha\xi\right)}\frac{dN}{d\xi} = \infty$

6.3)

$x = 0$

$y = \frac{1}{2}L + \alpha L$




Shape functions

$$\left\{\begin{aligned}
N_1 &= \frac{1}{4}(1-\xi)(1-\eta)\xi\eta \\
N_2 &= -\frac{1}{4}(1-\xi)(1+\eta)\xi\eta \\
N_3 &= \frac{1}{4}(1+\xi)(1+\eta)\xi\eta \\
N_4 &= -\frac{1}{4}(1+\xi)(1-\eta)\xi\eta \\
N_5 &= -\frac{1}{2}(1-\xi)(1-\eta^2)\xi \\
N_6 &= \frac{1}{2}(1-\xi^2)(1+\eta)\eta \\
N_7 &= \frac{1}{2}(1+\xi)(1-\eta^2)\xi \\
N_8 &= -\frac{1}{2}(1-\xi^2)(1-\eta)\eta \\
N_9 &= (1-\xi^2)(1-\eta^2)
\end{aligned}\right.$$

$$\bar{x} = x_6 N_6 + x_9 N_9 + x_8 N_8 + x_3 N_3 + x_7 N_7 + x_4 N_4 = \frac{1}{2}L(1+\xi)$$

$$\bar{y} = y_5 N_5 + y_9 N_9 + y_7 N_7 + y_2 N_2 + y_6 N_6 + y_3 N_3 = \frac{-\alpha L}{2}(\xi - \xi^2 - \eta^2\xi + \eta^2\xi^2) + \frac{1}{2}L(1+\eta)$$

$$J = \begin{bmatrix} \frac{dx}{d\xi} & \frac{dy}{d\xi} \\ \frac{dx}{d\eta} & \frac{dy}{d\eta} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}L & -\frac{\alpha L}{2} + \alpha L + \frac{\alpha L \eta^2}{2} - \alpha L \eta^2 \xi \\ 0 & L\left(\frac{1}{2} + \alpha\eta\xi - \alpha\eta\xi^2\right) \end{bmatrix}$$

In node 2 $\left\{\begin{array}{l}\xi = -1\\ \eta = 1\end{array}\right.$

$$J = \begin{bmatrix} 1/2\, L & \frac{5\alpha L}{2} \\ 0 & \frac{1}{2} - 2\alpha \end{bmatrix} \qquad J^{-1} = \begin{bmatrix} 2/L & \frac{-10\alpha}{L(1-4\alpha)} \\ 0 & \frac{1}{L\left(\frac{1}{2}-2\alpha\right)} \end{bmatrix}$$

$J^{-1}$ takes infinite values for:

$L = 0$

$L(1-4\alpha) = 0 \longrightarrow \alpha = \frac{1}{4}$

$L\left(\frac{1}{2} - 2\alpha\right) = 0 \longrightarrow \alpha = \frac{1}{4}$

$\Bigg\}$ then solution is $\left[\alpha = \frac{1}{4}\right]$

6.4)

For a desirable matrix rank, $(n_E . n_G)$ must be equal or higher than $(n_F - n_R)$

$$\left[ n_E . n_G \geq n_F - n_R \right] \quad \text{where} \quad \begin{cases} n_E = \text{E matrix order} \\ n_G = \text{Gauss points} \\ n_F : \text{Element degrees of freedom} \\ n_R = \text{Independent nodes} \end{cases}$$

<u>Plane stress</u>

8 nodes - Hexaedron

$$\left.\begin{array}{l} n_E = 3 \\ n_F = 8.2 = 16 \\ n_R = 3 \end{array}\right\} \; 3.n_G \geq 16-3 \;—\; n_G \geq 4,33 = \underline{\underline{5}} \text{ (minimum)}$$

20 nodes - Hexaedron

$$\left.\begin{array}{l} n_E = 3 \\ n_F = 20.2 = 40 \\ n_R = 3 \end{array}\right\} \; 3.n_G \geq 40-3 \;—\; n_G \geq 12,33 = \underline{\underline{13}} \text{ (minimum)}$$

22 nodes - Hexaedron

$$\left.\begin{array}{l} n_E = 3 \\ n_F = 22.2 = 54 \\ n_R = 3 \end{array}\right\} \; 3.N_G \geq 54-3 \;—\; n_G \geq \underline{\underline{17}} \text{ (Minimum)}$$

64 Nod - Hexaedron

$$N_G \geq \frac{128-3}{3} \geq 41,66 = \underline{\underline{42}} \text{ (minimum)}$$