Assignment 3.1

Given

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \; ; \; \mu = G = \frac{E}{2(1+\nu)}$$

$$\frac{\lambda}{\mu} = \frac{E\nu}{(1+\nu)(1-2\nu)} \times \frac{2(1+\nu)}{E} = \frac{2\nu}{1-2\nu}$$

$$\Rightarrow \frac{\lambda}{\mu}(1-2\nu) = 2\nu \Rightarrow 2\nu\left(1+\frac{\lambda}{\mu}\right) = \frac{\lambda}{\mu}$$

$$\Rightarrow \nu = \frac{\lambda}{2(\lambda+\mu)}$$

$$1+\nu = 1+\frac{\lambda}{2\lambda+2\mu} = \frac{3\lambda+2\mu}{2(\lambda+\mu)}$$

$$E = 2\mu(1+\nu) = \frac{2\mu(3\lambda+2\mu)}{2(\lambda+\mu)} = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$$

1 The inverse relationship:-

$$E = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$$

$$\nu = \frac{\lambda}{2(\lambda+\mu)} \quad \text{(Ans)}$$

(2)

(ii) Elasticity matrix for plane stress:-

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{(1+\nu)(1-\nu)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix}$$

$$1-\nu = 1-\frac{\lambda}{2(\lambda+\mu)} = \frac{\lambda+2\mu}{2\lambda+2\mu} \quad ; \quad \nu = \frac{\lambda}{2(\lambda+\mu)}$$

$$1+\nu = \frac{3\lambda+2\mu}{2\lambda+2\mu} \quad ; \quad E = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$$

The elasticity matrix:-

$$\frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$$= \frac{\mu(\cancel{3\lambda+2\mu})}{\lambda+\mu} \times \frac{(2\lambda+2\mu)}{(\cancel{3\lambda+2\mu})} \times \frac{(2\lambda+2\mu)}{\lambda+2\mu} \begin{bmatrix} 1 & \frac{\lambda}{2(\lambda+\mu)} & 0 \\ \frac{\lambda}{2(\lambda+\mu)} & 1 & 0 \\ 0 & 0 & \frac{\lambda+2\mu}{4\lambda+4\mu} \end{bmatrix}$$

$$= \frac{4\mu(\lambda+\mu)}{\lambda+2\mu} \times \frac{1}{2(\lambda+\mu)} \begin{bmatrix} 2\lambda+2\mu & \lambda & 0 \\ \lambda & 2\lambda+2\mu & 0 \\ 0 & 0 & \frac{\lambda+2\mu}{2} \end{bmatrix}$$

$$= \frac{2\mu}{\lambda+2\mu} \begin{bmatrix} 2\lambda+2\mu & \lambda & 0 \\ \lambda & 2\lambda+2\mu & 0 \\ 0 & 0 & \frac{\lambda+2\mu}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4\lambda\mu + 4\mu^2}{\lambda+2\mu} & \frac{2\mu\lambda}{\lambda+2\mu} & 0 \\ \frac{2\mu\lambda}{\lambda+2\mu} & \frac{4\lambda\mu+4\mu^2}{\lambda+2\mu} & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

(Ans)

$\boxed{\text{plain strain}}$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-2\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix}$$

$$1-2\nu = 1 - \frac{\lambda}{\lambda+\mu} = \frac{\mu}{\lambda+\mu}$$

The Elasticity matrix:-

$$\frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

$$= \frac{(3\lambda+2\mu)\mu}{(\lambda+\mu)} \times \frac{2(\lambda+\mu)}{(3\lambda+2\mu)} \times \left(\frac{\lambda+\mu}{\mu}\right) \times \frac{1}{2(\lambda+\mu)}$$

$$\times \begin{bmatrix} \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda+2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

$$= \begin{bmatrix} \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda+2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix} \quad (AM)$$

3 For plain strain:-

$$E = \begin{bmatrix} \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda+2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

$$= \begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

$$= E_\lambda + E_\mu \quad (AM)$$

$$E_\lambda = \begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$E_\mu = \begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

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(4)

$$E_\lambda = \lambda \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \frac{E\nu}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (AM)$$

$$E_\mu = \mu \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \frac{E}{2(1+\nu)} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (AM)$$

Assignment :- 3.2

① $K^{(e)}$:-

We know that

$$u_x = u_{x1}\psi_1 + u_{x2}\psi_2 + u_{x3}\psi_3$$
$$u_y = u_{y1}\psi_1 + u_{y2}\psi_2 + u_{y3}\psi_3$$

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = N U^e \; ; \; N = \begin{bmatrix} \psi_1 & 0 & \psi_2 & 0 & \psi_3 & 0 \\ 0 & \psi_1 & 0 & \psi_2 & 0 & \psi_3 \end{bmatrix}$$

Kinematic equations:-

$$e = \partial N U^e = \frac{1}{2A}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix}$$

$$= \underline{B} U^e$$

For the plane stress:-

$$\sigma = \underline{E}\underline{e} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix} = \underline{E}\,\underline{e}$$

Element stiffness matrix:-

$$K^{(e)} = \int_{\Omega^e} h B^T E B \, d\Omega$$

($\underline{B}$ and $\underline{E}$ are constant over Ω^e)

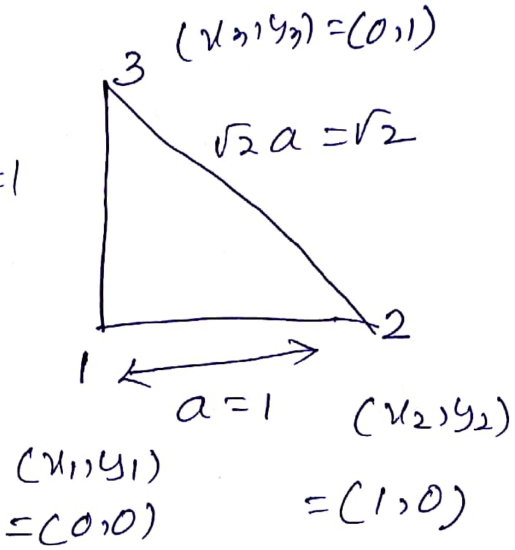
$$K^e = \frac{1}{4A^2}\begin{bmatrix} y_{23} & 0 & x_{32} \\ 0 & x_{32} & y_{23} \\ y_{31} & 0 & x_{13} \\ 0 & x_{13} & y_{31} \\ y_{12} & 0 & x_{21} \\ 0 & x_{21} & y_{12} \end{bmatrix} \underline{E} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \times \int_{\Omega^e} h \, d\Omega$$

Here h is uniform (h = constant)

$$\therefore \int_{\Omega^e} h \, d\Omega = hA \qquad \left(\begin{matrix} y_{ij} = y_i - y_j \\ x_{ij} = x_i - x_j \end{matrix} \right)$$

$$K^e = \frac{h}{4A} \begin{bmatrix} y_{23} & 0 & x_{32} \\ 0 & x_{32} & y_{23} \\ y_{31} & 0 & x_{13} \\ 0 & x_{13} & y_{31} \\ y_{12} & 0 & x_{21} \\ 0 & x_{21} & y_{12} \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{12} & E_{22} & E_{23} \\ E_{13} & E_{23} & E_{33} \end{bmatrix} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

So



$y_{23} = -1;\ x_{32} = -1;\ y_{12} = 0$

$y_{23} = -1;\ y_{31} = 1;\ x_{21} = 1$

$y_{23} = -1;\ y_{31} = 1;\ y_{12} = 0$

$x_{13} = 0;\ y_{31} = 1;$

$A = \frac{1}{2} \times 1 \times 1 = 1/2;\ h = 1;\ (\nu = 0)$

$$K^e_{mi} = \frac{1}{4 \times \frac{1}{2}} \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \times \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

For $\nu = 0$

$$K^e_{mi} = \frac{1}{2} \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \times E \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

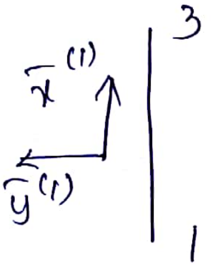
$$= E \begin{bmatrix} 0.75 & 0.25 & -0.5 & -0.25 & -0.25 & 0 \\ 0.25 & 0.75 & 0 & -0.25 & -0.25 & -0.5 \\ -0.5 & 0 & 0.5 & 0 & 0 & 0 \\ -0.25 & -0.25 & 0 & 0.25 & 0.25 & 0 \\ -0.25 & -0.25 & 0 & 0.25 & 0.25 & 0 \\ 0 & -0.5 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

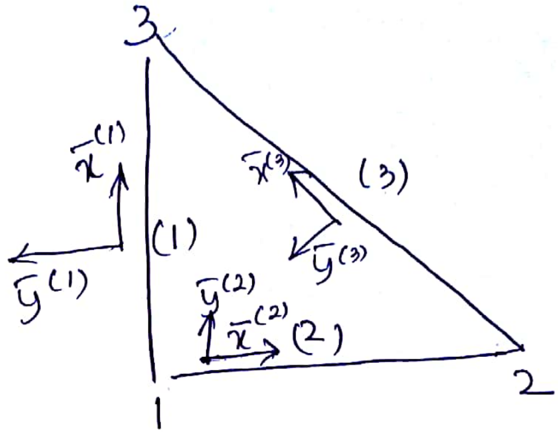
For bar Elements $\boxed{K^{bar}}$

$L_{13} = L_{12} = 1 = a$

$L_{23} = \sqrt{2}a = \sqrt{2}$

$\boxed{\text{Element:-1}}$





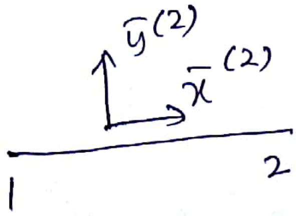
Angle of orientation of axes $= 90^0$

$$K^{(1)} = \frac{EA_1}{L_{13}} \begin{bmatrix} \cos^2 90 & s90\,c90 & -c^2 90 & -s90\,c90 \\ s90\,c90 & s^2 90 & -s90\,c90 & -s^2 90 \\ -c^2 90 & -s90\,c90 & c^2 90 & s90\,c90 \\ -s90\,c90 & -s^2 90 & s90\,c90 & s^2 90 \end{bmatrix}$$

$$= EA_1 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$\boxed{\text{Element:-2}}$ $\theta = 0$ (Angle of orientation of axes)

$A_1 = A_2$;

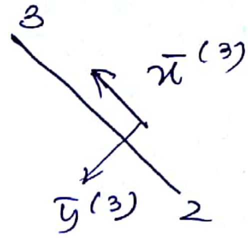


$$K^{(2)} = \frac{EA_1}{L_{12}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= EA_1 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Element-3 θ = Angle of orientation $= 135^0$;

$L_{23} = \sqrt{2}$



$$K^{(3)} = \frac{EA_3}{L_{23}} \begin{bmatrix} c^2 135 & s135\,c135 & -c^2 135 & -s135\,c135 \\ s135\,c135 & s^2 135 & -s135\,c135 & -s^2 135 \\ -c^2 135 & -s135\,c135 & c^2 135 & s135\,c135 \\ -s135\,c135 & -s^2 135 & s135\,c135 & s^2 135 \end{bmatrix}$$

$$= \frac{EA_3}{\sqrt{2}} \begin{bmatrix} 1/2 & -\frac{1}{2} & -\frac{1}{2} & 1/2 \\ -\frac{1}{2} & \frac{1}{2} & 1/2 & -\frac{1}{2} \\ -\frac{1}{2} & 1/2 & 1/2 & -\frac{1}{2} \\ 1/2 & -\frac{1}{2} & -\frac{1}{2} & 1/2 \end{bmatrix}$$

$$= \frac{EA_3}{2\sqrt{2}} \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Globalized Element Stiffness Matrix :-

$$\begin{bmatrix} F_{x1}^{(1)} \\ F_{y1}^{(1)} \\ F_{x3}^{(1)} \\ F_{y3}^{(1)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & EA_1 & 0 & -EA_1 \\ 0 & 0 & 0 & 0 \\ 0 & -EA_1 & 0 & EA_1 \end{bmatrix} \begin{bmatrix} U_{x1}^{(1)} \\ U_{y1}^{(1)} \\ U_{x3}^{(1)} \\ U_{y3}^{(1)} \end{bmatrix}$$

$$\begin{bmatrix} F_{x1}^{(2)} \\ F_{y1}^{(2)} \\ F_{x2}^{(2)} \\ F_{y2}^{(2)} \end{bmatrix} = \begin{bmatrix} EA_1 & 0 & -EA_1 & 0 \\ 0 & 0 & 0 & 0 \\ -EA_1 & 0 & EA_1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_{x1}^{(2)} \\ U_{y1}^{(2)} \\ U_{x2}^{(2)} \\ U_{y2}^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} f_{x2}^{(3)} \\ f_{y2}^{(3)} \\ f_{x3}^{(3)} \\ f_{y3}^{(3)} \end{bmatrix} = \frac{EA_3}{2L_2} \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{x2}^{(3)} \\ u_{y2}^{(3)} \\ u_{x3}^{(3)} \\ u_{y3}^{(3)} \end{bmatrix}$$

Expanded Element Stiffness Equations By Enforcing Compatibility Rule

$$\begin{bmatrix} f_{x1}^{(1)} \\ f_{y1}^{(1)} \\ f_{x2}^{(1)} \\ f_{y2}^{(1)} \\ f_{x3}^{(1)} \\ f_{y3}^{(1)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & EA_1 & 0 & 0 & 0 & -EA_1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -EA_1 & 0 & 0 & 0 & EA_1 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix}$$

$$\begin{bmatrix} f_{x1}^{(2)} \\ f_{y1}^{(2)} \\ f_{x2}^{(2)} \\ f_{y2}^{(2)} \\ f_{x3}^{(2)} \\ f_{y3}^{(2)} \end{bmatrix} = \begin{bmatrix} EA_1 & 0 & -EA_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -EA_1 & 0 & EA_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix}$$

$$\begin{bmatrix} f_{x1}^{(3)} \\ f_{y1}^{(3)} \\ f_{x2}^{(3)} \\ f_{y2}^{(3)} \\ f_{x3}^{(3)} \\ f_{y3}^{(3)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{EA_3}{2L_2} & \frac{-EA_3}{2L_2} & \frac{-EA_3}{2L_2} & \frac{EA_3}{2L_2} \\ 0 & 0 & \frac{-EA_3}{2L_2} & \frac{EA_3}{2L_2} & \frac{EA_3}{2L_2} & \frac{-EA_3}{2L_2} \\ 0 & 0 & \frac{-EA_3}{2L_2} & +\frac{EA_3}{2L_2} & \frac{EA_3}{2L_2} & \frac{-EA_3}{2L_2} \\ 0 & 0 & \frac{EA_3}{2L_2} & \frac{-EA_3}{2L_2} & \frac{-EA_3}{2L_2} & \frac{EA_3}{2L_2} \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix}$$

Formation of the Master stiffness matrix through Equilibrium Rule:-

$$f = f^{(1)} + f^{(2)} + f^{(3)} = (K^{(1)} + K^{(2)} + K^{(3)})u = Ku$$

$$K_{bar} = K^{(1)} + K^{(2)} + K^{(3)}$$

$$= \begin{bmatrix} EA_1 & 0 & -EA_1 & 0 & 0 & 0 \\ 0 & EA_1 & 0 & 0 & 0 & -EA_1 \\ -EA_1 & 0 & EA_1 + \frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} \\ 0 & 0 & -\frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} \\ 0 & 0 & -\frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} \\ 0 & -EA_1 & \frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} & EA_1 + \frac{EA_3}{2\sqrt{2}} \end{bmatrix}$$

while

$$K_{tri} = \begin{bmatrix} 0.75E & 0.25E & -0.5E & -0.25E & -0.25E & 0 \\ 0.25E & 0.75E & 0 & -0.25E & -0.25E & -0.5E \\ -0.5E & 0 & 0.5E & 0 & 0 & 0 \\ -0.25E & -0.25E & 0 & 0.25E & 0.25E & 0 \\ -0.25E & -0.25E & 0 & 0.25E & 0.25E & 0 \\ 0 & -0.5E & 0 & 0 & 0 & 0.5E \end{bmatrix}$$

$A_3 = 0$
$A_1 = 0.5$

② NO, There are not any set of values for the cross sections $A_1 = A_2$ and A_3 to make both the stiffness matrix equivalent $K^{bar} = K^{tri}$.

If we take

Set (i) :- $\frac{EA_3}{2\sqrt{2}} = 0.25E \Rightarrow A_3 = \frac{1}{\sqrt{2}}$;

$EA_1 = 0.75E \Rightarrow A_1 = 3/4$; $A_1 = A_2 = 3/4$

We can make K_{bar} and K_{tri} more similar.

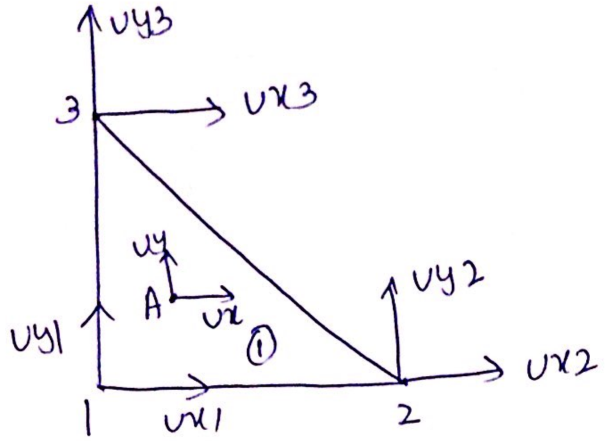
Also there exist another set of values for A_1 and A_2.

we can equate

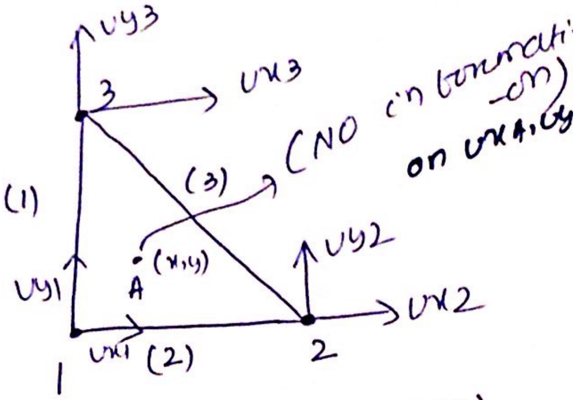
Set (ii) :- $EA_1 = 0.5E \Rightarrow A_1 = 1/2$; $A_3 = 1/\sqrt{2}$;

$(A_1 = A_2 = 1/2;\ A_3 = 1/\sqrt{2})$

(3)



(Turner's triangle)



(3 bar elements)
(NO information about UXA, UYA)

In the case of a planar linear triangular Element (Turner's triangle); the displacement at any point in the Element or in the triangular domain is interpolated as

$$U_x = U_{x1} N_1 + U_{x2} N_2 + U_{x3} N_3$$
$$U_y = U_{y1} N_1 + U_{y2} N_2 + U_{y3} N_3$$

The kinematic equations for the element considers this relationship inorder to evaluate the strain and stress inside the element. Stresses and strains are related by constitutive relations. The element stiffness matrix is evaluated from the variation of the total potential energy considering the above mentioned relationship of displacement at any point. It provides information about the transverse stresses and strains in the element for the plane strain and plane stress cases respectively when $v \neq 0$.

→ But the element stiffness matrix obtained using three bars does not provide any information about the displacements of the interior points which lie inside the triangular domain. We only can recover the axial stress and strain inside the bar elements. It is not possible to get any information about the transverse strains and stresses for the plane stress and plane strain problems when $\nu \neq 0$.

That's why two obtained element stiffness matrices K_{bar} and K_{tri} are not similar.

(4)

When $\nu \neq 0$; For the plane stress problems $\epsilon_{zz} = \frac{-\nu}{E}(\sigma_{xx} + \sigma_{yy})$. The transverse strain will be non-zero because of the poisson's ratio effect.

Using a plane linear triangular element (Turner's triangle), we can find the stress matrix over the Element using the constitutive relationship.

$$\underline{\underline{\sigma}} = \underline{\underline{\tilde{E}}}\,\underline{\underline{e}} = \underline{\underline{\tilde{E}}}\,\underline{\underline{B}}\,\underline{u}^e$$

From the stress matrix we can determine ϵ_{zz}.

But using the Bar elements we only can recover the axial stresses in the bar elements. They do not provide any information regarding the transverse strain due to poisson's effect.

Similarly for the plane-strain cases using bar elements we can't determine $\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$, the transverse stress over the element.

The stiffness matrix for the plane-stress case ($\nu \neq 0$) has been described.

When $\nu \neq 0$

$(x_1, y_1) = (0,0)$

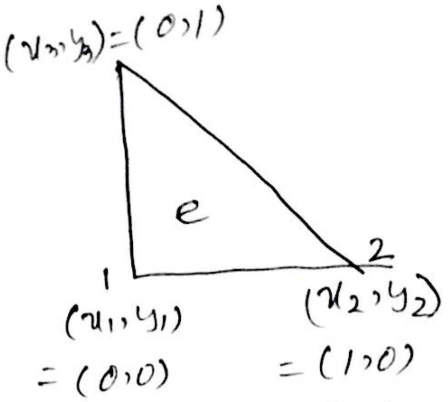
$(x_2, y_2) = (1,0)$

$(x_3, y_3) = (0,1)$

$h = 1$ (Given)

$x_{ij} = x_i - x_j$

$y_{ij} = y_i - y_j$



$x_{21} = 1$

$$K^{tri} = \frac{Eh}{2(1-\nu^2)} \begin{bmatrix} \frac{y_3}{x_{21}} & \frac{\nu x_2}{x_{21}} & \frac{-y_3}{x_{21}} & \frac{-\nu x_1}{x_{21}} & 0 & -\nu \\ \frac{\nu x_2}{x_{21}} & \frac{x_2}{y_3 x_{21}} & \frac{-\nu x_2}{x_{21}} & \frac{-x_1 x_2}{y_3 x_{21}} & 0 & \frac{-x_2}{y_3} \\ \frac{-y_3}{x_{21}} & \frac{-\nu x_2}{x_{21}} & \frac{y_3}{x_{21}} & \frac{\nu x_1}{x_{21}} & 0 & \nu \\ \frac{-\nu x_1}{x_{21}} & \frac{-x_1 x_2}{y_3 x_{21}} & \frac{\nu x_1}{x_{21}} & \frac{x_1^2}{y_3 x_{21}} & 0 & \frac{x_1}{y_3} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\nu & -\frac{x_2}{y_3} & \nu & \frac{x_1}{y_3} & 0 & \frac{x_{21}}{y_3} \end{bmatrix}$$

$$+ \frac{Gh}{2} \begin{bmatrix} \frac{x_2^2}{y_3 x_{21}} & \frac{x_2}{x_{21}} & \frac{-x_1 x_2}{y_3 x_{21}} & \frac{-x_2}{x_{21}} & -\frac{x_2}{y_3} & 0 \\ \frac{x_2}{x_{21}} & \frac{y_3}{x_{21}} & \frac{-x_1}{x_{21}} & \frac{-y_3}{x_{21}} & -1 & 0 \\ \frac{-x_1 x_2}{y_3 x_{21}} & \frac{-x_1}{x_{21}} & \frac{x_1^2}{y_3 x_{21}} & \frac{x_1}{x_{21}} & \frac{x_1}{y_3} & 0 \\ \frac{-x_2}{x_{21}} & \frac{-y_3}{x_{21}} & \frac{x_1}{x_{21}} & \frac{y_3}{x_{21}} & 1 & 0 \\ -\frac{x_2}{y_3} & -1 & \frac{x_1}{y_3} & 1 & \frac{x_{21}}{y_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$K^{tri} = \frac{E}{2(1-\nu^2)} \begin{bmatrix} 1 & \nu & -1 & 0 & 0 & -\nu \\ \nu & 1 & -\nu & 0 & 0 & -1 \\ -1 & -\nu & 1 & 0 & 0 & \nu \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\nu & -1 & \nu & 0 & 0 & 1 \end{bmatrix}$$

$$+ \frac{G}{2} \begin{bmatrix} 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 & 1 & 0 \\ -1 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(when $\nu = 0$
$E = 2G(1+\nu)$
$\Rightarrow G = E/2$]

(AM)