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BARCELONATECH

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Assignment 9- SHELL

1. *Describe in extension how can be applied a non-symmetric load on this formulation.*

In the case of a non-symmetrical load applied on an asymmetrical shell, the stress that will be created in the shell will be both symmetrical and asymmetrical. Using the Fourier series the problem can be formulated, describing the harmonic behavior of the structure with respect to the circumferential direction, thus treating it in a single dimension. Thanks to the Fourier series we can divide the force into the symmetric and anti-symmetric components and obtain two stiffness matrices K_a and K_b. The loads and displacements are described in polar coordinates, therefore the Fourier series will also receive the loads described in this formulation:

$$K_a = \pi \int_A B_a^T D B_a dA$$
$$K_b = \pi \int_A B_b^T D B_b dA$$

$$f(\theta) = \frac{a_0}{2} + \sum \left(\frac{cosn\theta}{\pi} \int_{-\infty}^{+\infty} f(\theta) cosn\theta d\theta \right) + \frac{sinn\theta}{\pi} \int_{-\infty}^{+\infty} f(\theta) sinn\theta d\theta$$

And the strain vector:

$$\epsilon = \begin{bmatrix} \epsilon_a \\ \epsilon_\theta \\ \gamma_{a\theta} \\ \lambda_a \\ \lambda_\theta \\ \lambda_{a\theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial a} \\ \frac{1}{r}\frac{\partial v}{\partial \theta} + (ucos\theta - \frac{1}{r} wsin\theta) \\ \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial a} - \frac{1}{r} vcos\theta \\ -\frac{\partial^2 w}{\partial a^2} \\ -\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} - \frac{1}{r}\frac{\partial w}{\partial a} cos\theta + \frac{1}{r}\frac{\partial v}{\partial \theta} sin\theta \\ \frac{2}{r}(-\frac{\partial^2 w}{\partial a \partial \theta} + \frac{1}{r}\frac{\partial w}{\partial \theta} + \frac{\partial v}{\partial a} - \frac{1}{r} vsin\theta cos\theta) \end{bmatrix}$$

2. Using thin beams formulation, describe the shape of the B^e matrix and comment the integration rule.

Matrix B will be composed only of the deformation matrix of the membrane and of the bending one, because according to Kirchoff's assumption for thin beams, the shear stresses are negligible. Therefore the general deformation B^e matrix will be:

$$B^e = \begin{bmatrix} B_m^e \\ B_b^e \end{bmatrix} = \begin{bmatrix} \frac{\partial N_u^e}{\partial a} & 0 & 0 \\ \frac{N_u^e cos\theta}{r} & -\frac{N_w^e sin\theta}{r} & -\frac{\mathbf{N}_w^e sin\theta}{r} \\ 0 & \frac{\partial^2 N_w^e}{\partial a^2} & \frac{\partial^2 \mathbf{N}_w^e}{\partial a^2} \\ 0 & \frac{cos\theta}{r}\frac{\partial N_w^e}{\partial a} & \frac{cos\theta}{r}\frac{\partial \mathbf{N}_w^e}{\partial a} \end{bmatrix}$$

We can observed that for $r \to 0$ there wil be some problems because of B depend on 1/r. We can solve these problems using the gauss quadrature method