Universitat Politècnica de Catalunya

Computational Solid Mechanics and Dynamics

Master's Degree in Numerical Methods in Engineering

Axisymmetric Shells

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1 Axisymmetric shells under arbitrary loading

The shell formulation that has been studied for symmetric loads can be extended for analysis of axisymmetric shells under arbitrary loading. The expressions for the displacements, generalized strains and resultant stresses are equal to those for flat shells and rectangular plates. The basic expressions are derived starting from Reissner-Mindlin troncoconical shell theory, and now the displacements are expanded in Fourier series along the circumferential direction, and to account for arbitrary loading, the displacement field is split in symmetric and anti-symmetric components with respect to the meridional plane at $\beta = 0$, where the angle β defines the position of the points over the circular arch along the circular prismatic direction [1].

The loads are expanded in Fourier series using the same harmonic functions as for the displacements, and the resulting expression is:

$$\mathbf{t} = \sum_{l=0}^{m} \mathbf{S^l t^l} + \overline{\mathbf{S^l t^l}} \tag{1}$$

Where the over-lined terms $\mathbf{S^l}$, $\mathbf{t^l}$refer to anti-symmetric loads and load amplitudes, respectively. The anti-symmetric field can be obtained by considering these terms [1].

Then, as for the constitutive matrix $\mathbf{D}$, and the local stiffness matrix $\mathbf{K_{ij}}$, relating the local resultant stresses and the generalized strains, are modified accordingly for the different harmonic terms previously calculated [1].

It is known that the stiffness matrix and the equivalent nodal nodal force amplitude vector for uniformly distributed loading for the 2-noded element and the l_{th} harmonic term are a function of

$$[\mathbf{K_{il}^{ll}}]^{\mathbf{e}} = f(\alpha/2, \mathbf{B}, \mathbf{D}, \mathbf{r}) \tag{2}$$

For the anti-symmetric loading case, thus, matrix $\mathbf{B^l}$ is obtained with the corresponding Fourier harmonic term l for the symmetric and $-l$ for the anti-axisymmetric cases. The parameter $\alpha/2$ is also substituted by C, where C is 2π for $l = 0$ and π for $l \neq 0$ [1].

2 The shape of the B matrix and the integration rule

When using thin beams formulation, the Kirchoff assumption with which the normals to the generatrix remain straight and orthogonal to the generatrix after deformation has been used. This way, the normal rotation coincides with the slope of the generatrix [1].

This way, since the element is straight, $Rs = \infty$ and Cs = 1 which implies $\partial/\partial s = \partial/\partial x$. For simplicity we will also be assumed that $C_\alpha = 1$. This way, the generalized local strain vector is found. The generalized local strain vector includes the axial stretching, the (pseudo) curvature and the shear, as for a plane frame or an arch. It also incorporates the circumferential stretching and the circumferential curvature. Eventually, the matrix B has the shape of:

$$
\mathbf{B}'_i = \left\{ \begin{array}{c} \mathbf{B}'_{m_i} \\ \hdashline \mathbf{B}'_{b_i} \\ \hdashline \mathbf{B}'_{s_i} \end{array} \right\} = \left[\begin{array}{ccc} \dfrac{\partial N_i}{\partial s} & 0 & 0 \\ \dfrac{N_i \text{cos}\phi}{r} & -\dfrac{N_i \text{sen}\phi}{r} & 0 \\ \hdashline 0 & 0 & \dfrac{\partial N_i}{\partial s} \\ 0 & 0 & \dfrac{N_i \text{cos}\phi}{r} \\ \hdashline 0 & \dfrac{\partial N_i}{\partial s} & -N_i \end{array} \right]
$$

Figure 1: Extracted from [1].

where N_i are the shape functions of 1D Lagrange elements [1].

Axisymmetric strip elements based on Reissner-Mindlin theory require reduced/selective quadratures to prevent locking. The linear, quadratic and cubic strips use certain full, selective and reduced quadratures [1].

Shear locking in the linear (2-noded) Reissner-Mindlin troncoconical element can be avoided by using a one point reduced quadrature. The 2-noded Reissner-Mindlin troncoconical element with a single integrating point is the simplest and most popular axisymmetric shell element. The one-point quadrature for the all the stiffness terms preserves the correct rank in the overall element stiffness matrix, whereas the 3-noded (quadratic) Reissner-Mindlin troncoconical element requires a reduced two-point quadrature to avoid shear locking [1].

References

[1] E. Oñate *Structural Analysis with the Finite Element Method. Linear Statics. Volume 2. Beams, Plates and Shells.*, 2013 ISBN : 978-1-4020-8742-4