Computational Structural Mechanics and Dynamics

Master's Degree in Numerical Methods in Engineering

Assignment 9: Rev. Shells

Authors:
Diego Roldán

Supervisor:
Prof. S. Francisco Zárate

Academic Year 2019-2020

a) Describe in extension how can be applied a non-symmetric load on this formulation.

When a non-symmetric load is applied to a revolution-shell structure, called a non-axisymmetric load, the formulation of the problem is more difficult than a straightforward axisymmetric load. The most complex part is the load vector in the right-hand side of the system of equations.

In this case, it is when Fourier series play an important role to the formulation of this problem to divide the load into a sum of cosines and sinuses depending on the circumferential direction (θ), working out the behaviour of the specific structure to each harmonic term of the series and overlapping the results. It is required to calculate the non-symmetric and symmetric terms and combine them into a formulation of the type homogenous harmonic.

It is necessary to mention that the displacements and forces of the structure must be totally harmonic (as an addition of the sum of cosines and sinuses).

Then, the loads and displacements are decomposed into a series of Fourier functions.

The displacements (axial, radial and circumferential, respectively) are described as:

$$u = \sum_{i=1}^{\infty}(u_{a_n} cosn\theta + u_{b_n} sinn\theta)$$

$$v = \sum_{i=1}^{\infty}(v_{a_n} cosn\theta + v_{b_n} sinn\theta)$$

$$w = \sum_{i=1}^{\infty}(w_{a_n} cosn\theta + w_{b_n} sinn\theta)$$

And the loads as:

$$f(\theta) = \frac{a_0}{2} + \sum_{i=1}^{\infty}(\frac{cosn\theta}{\pi}\int_{-\infty}^{\infty} f(\theta)cosn\theta d\theta + \frac{sinn\theta}{\pi}\int_{-\infty}^{\infty} f(\theta)sinn\theta d\theta)$$

Therefore, the strain matrices depend on θ (circumferential variable). Due to the fact that Fourier series generate non-symmetric and symmetric solutions, the functions which are approximated divide into non-symmetric and symmetric. The stiffness matrices are (b for non-symmetric n=1,3… and a for symmetric n=2,4…).

$$\mathbf{k}_{an} = \pi \int_A \mathbf{B}_{ai}^T \mathbf{D} \mathbf{B}_{ai} dA$$
$$\mathbf{k}_{bn} = \pi \int_A \mathbf{B}_{bi}^T \mathbf{D} \mathbf{B}_{bi} dA$$

Where,

D is the constitutive matrix,

dA=r * ds, where r is the radius of the revolution shell

B is the strain matrix with the form of, varying for non-symmetric and symmetric.

$$\left[B_{a_n}^i\right] = \begin{bmatrix} \frac{\partial N_i}{\partial r} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial z} & 0 \\ \frac{N_i}{r} & 0 & +\frac{nN_i}{r} \\ \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial r} & 0 \\ -\frac{nN_i}{r} & 0 & \left(\frac{\partial N_i}{\partial r} - \frac{N_i}{r}\right) \\ 0 & -\frac{nN_i}{r} & \frac{\partial N_i}{\partial z} \end{bmatrix} \qquad \left[B_{b_n}^i\right] = \begin{bmatrix} \frac{\partial N_i}{\partial r} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial z} & 0 \\ \frac{N_i}{r} & 0 & -\frac{nN_i}{r} \\ \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial r} & 0 \\ \frac{nN_i}{r} & 0 & \left(\frac{\partial N_i}{\partial r} - \frac{N_i}{r}\right) \\ 0 & \frac{nN_i}{r} & \frac{\partial N_i}{\partial z} \end{bmatrix}$$

The load vector, f(θ), is discretized as it is mentioned before with shape functions.

The solution is also a function of θ.

b) Using thin beams formulation, describe the shape of the B(e) matrix and comment the integration rule.

It is possible to use two different formulations for thin beams, based on the principal deformation produced because of bending.

Firstly, the Euler-Bernoulli formulation where the B(e) matrix has the bending deformation as the principal problem.

Secondly, the Timoshenko formulation, where the shear locking effect plays an important role in the formulation that produces the need to reduce the integration in the B(e) matrix for the shear part.

The global revolution-shell stiffness matrix, for the membrane and bending problem, is calculated using full integration rule (2 Gauss points inside the element). It is possible to reduce the shear locking effect using only 1 Gauss point that makes the shear value of the stiffness underestimated.

Coming back to the Euler-Bernoulli elements, it is required to use 2 Gauss points for both bending and membrane parts. Usually, in the formulation for revolution-shell structures, the rotation is around the z-axis; therefore, in many cases, it is not possible to make use of the Lobatto integration rule because it presents problematic location for the integration points. In this case, the formulation presents a numerical block because some values of the B(e) matrix goes to infinite.

The shape of B(e) matrix for thin beams is divided into three B matrices (membrane, bending and shear).

.

$$\mathbf{B}_i = \begin{bmatrix} \mathbf{B}_m \\ \mathbf{B}_b \\ \mathbf{B}_s \end{bmatrix}$$