Assignmet 2.1: on FEM Modeling

1. Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated:

a) a circular disk under two diametrically opposite point forces (the famous "Brazilian test" for concrete)

b) the same disk under two diametrically opposite force pairs

c) a clamped semiannulus under a force pair oriented as shown

d) a stretched rectangular plate with a central circular hole.

e) and f) are half-planes under concentrated loads.

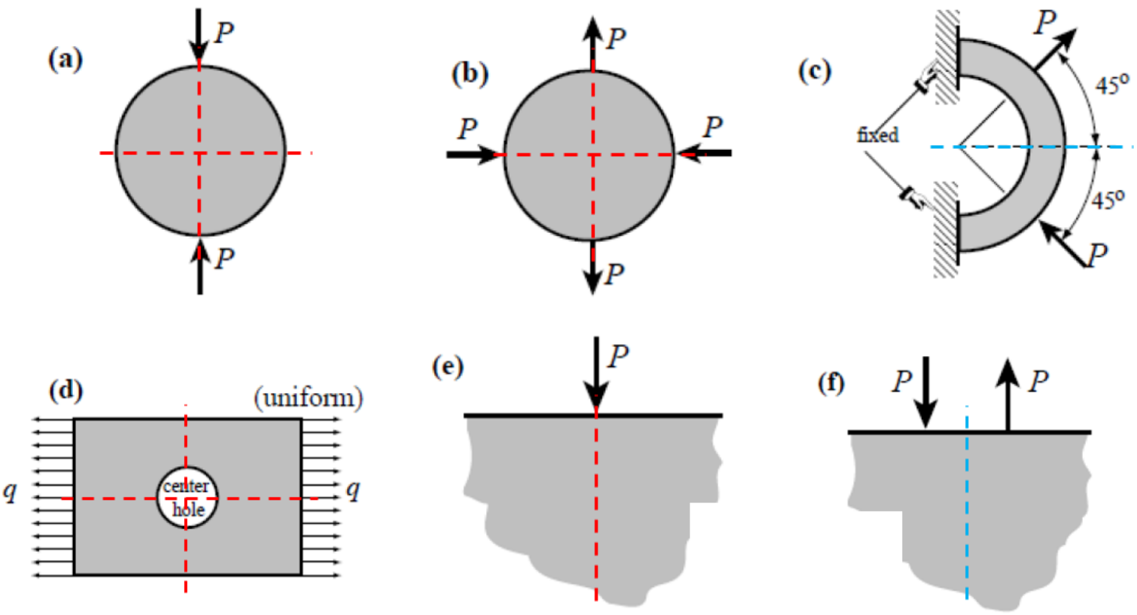

Figure 1: Symmetry (in red) and antisymmetry (in blue) axis of figures.

2. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.:

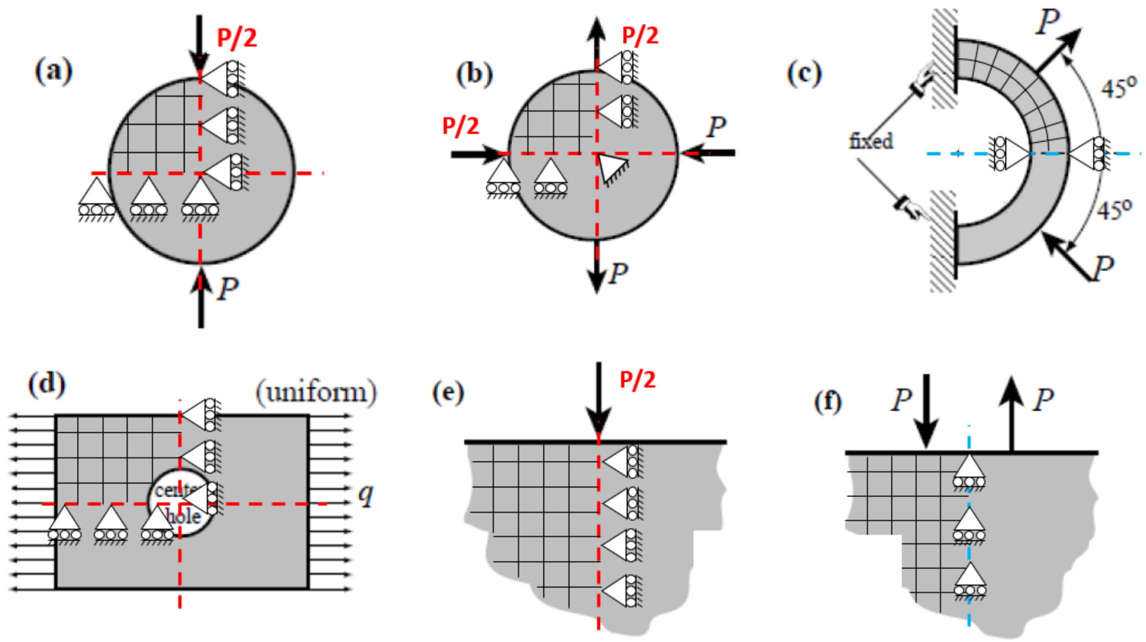

Figure 2: Minimum part with its BC's to be computed that represents the whole cases.

Assignmet 2.2:

Explain difference between "*verification*" and "*validation*":

Below is shown an scheme for physical FEM system:

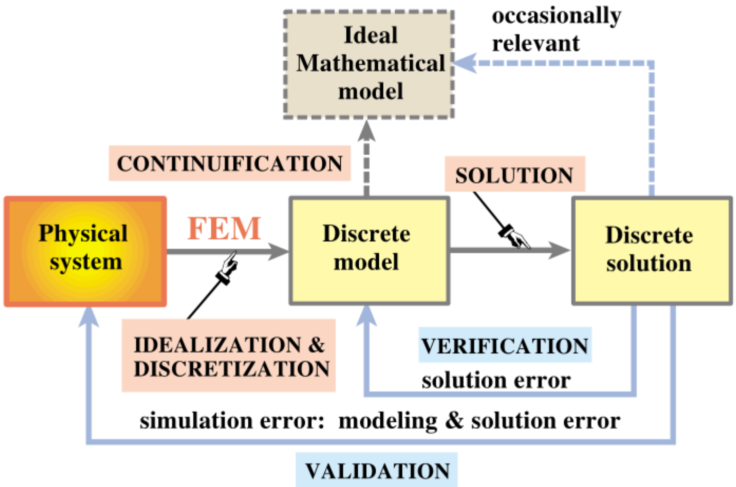

Figure 3: The Physical FEM system -Introduction to Finite Element Methods, C.A.Fellipa, 2004.

As it can be seen in the figure:

1. *Verification* consists on quantify the solution error which can be obtained by replacing the obtained results on the original discrete model. "Solution error" in computation should be very small and often is unimportant in comparison to the "Idealization Discretization error" (also called "Modeling error"). It just quantify the error of the solving method.

2. *Validation* consists on quantify the overall simulation error (Modeling Error + Solution Error), nevertheless the more important error is the modeling error (Idealization + discretization). The validation process is use to quantify if the modeled problem represents sufficiently accurate the behavior of the physical real problem.

Assignmet 2.3: Variational Formulation

A tapered bar element of length l and areas A i and A j with A interpolated as $A = A_i(1-\xi) + A_j\xi$ and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i , the centrifugal axial force is $q = \rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$. Find the consistent node forces as functions of ρ, A_i , A_j , ω and l, and specialize the result to the prismatic bar .

$$x = \xi l$$
$$N = \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix}$$
$$q = \rho(A_i(1-\xi) + A_j\xi)\omega^2 x$$

Replacing the previous relations in the terms of force equation:

$$f = \int_0^1 N q J^{-1} d\xi$$
$$f = \int_0^1 \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} \cdot [\rho(A_i(1-\xi) + A_j\xi) \cdot \omega^2 \cdot \xi l] \cdot [l] \cdot d\xi$$

After integrating force results as:

$$f = \omega^2 \rho l^2 \begin{bmatrix} A_i \frac{1}{12} + A_j \frac{1}{12} \\ A_i \frac{1}{12} + A_j \frac{1}{4} \end{bmatrix}$$

For a prismatic bar where $A = A_i = A_j$, the resulting force is:

$$f = A\omega^2 \rho l^2 \begin{bmatrix} \frac{1}{6} \\ \frac{1}{3} \end{bmatrix}$$