Assignment 10. Dynamics

Student: Diego Roldán Ukrlep

1. $r(t) \Rightarrow$ constant force F

Effect of F on time-dependent displacement $u(t)$ and natural frequency of vibration?

This problem is described as:

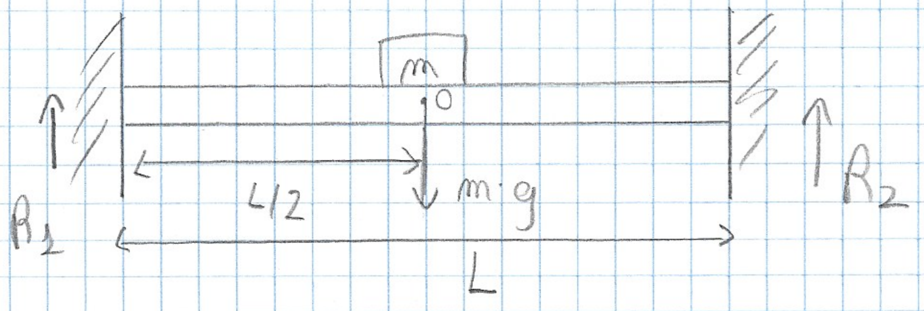
$$m\ddot{u} + Ku = F$$

Therefore, this problem is considered as a forced undamped vibration system. As there is an external force $F = r(t)$ applied to the system and it does not have any damping effect attached to the system, the displacements will be sinusoidal, so the displacements oscillates.
However, in real systems the energy dissipates because the system will be damped. Then, if the system is forced to vibrate at its natural frequency, there will be resonance and it will be observed big amplitude oscillations.
Finally, the natural frequency is defined as:

$$\omega = \sqrt{\frac{k}{m}}$$

Therefore, the applied force will not have any effect on the natural frequency of vibration. It will have an effect to the amplitude of vibration and the range of movement.

2. Estimate the natural frequency



$\Sigma F_y = 0$; $\uparrow \oplus$ $\quad R_1 + R_2 = m g$ (Equilibrium of forces)

$\Sigma M_O = 0$, $\oplus$ $\quad -R_2 \cdot L/2 + R_1 \cdot L/2 = 0$

$$\boxed{R_1 = R_2}$$

Then,

$$2R_1 = m g; \quad \boxed{R_1 = \frac{mg}{2}}$$

* Displacement at the centre, by definition:

$$\delta(x = L/2) = \frac{m \cdot g \, L^3}{192 EI}$$

* Effective stiffness

$$k = \frac{m g}{\delta} = \frac{\cancel{mg}\, 192 EI}{\cancel{mg}\, L^3} = \frac{192 EI}{L^3}$$

* Natural frequency

$$\boxed{w = \sqrt{\frac{k}{m}} = \sqrt{\frac{192 EI}{m L^3}}}$$

Considering the beam with a square section:

$$I = \frac{b\,h^3}{12} = \frac{A^2}{12}$$

Natural frequency:

$$\omega = \sqrt{\frac{16\,E A^2}{m L^3}} = \frac{4A}{L}\sqrt{\frac{E}{m \cdot L}}$$

3.

The consistent element mass matrix is calculated as:

$$m = \int_{\Omega^e} \underline{N}^T \underline{N} \rho \, dV$$

* Using an isoparametric representation of the two-node bar element:

$$m = \rho A \int_{-1}^{1} \begin{bmatrix} 1/2\,(1-\xi) \\ 1/2\,(1+\xi) \end{bmatrix} \begin{bmatrix} \frac{1}{2}(1-\xi) & \frac{1}{2}(1+\xi) \end{bmatrix} |J| \, d\xi =$$

$$= \rho \frac{AL}{8} \int_{-1}^{1} \begin{bmatrix} (1-\xi)^2 & (1-\xi)(1+\xi) \\ (1-\xi)(1+\xi) & (1+\xi)^2 \end{bmatrix} d\xi = \left(J = \frac{dx}{d\xi} = \frac{L}{2} \right)$$

$$= \rho \frac{AL}{8} \int_{-1}^{1} \begin{bmatrix} 1-2\xi+\xi^2 & 1-\xi^2 \\ 1-\xi^2 & 1+2\xi+\xi^2 \end{bmatrix} d\xi =$$

$$= \rho \frac{AL}{8} \begin{bmatrix} \xi-\xi^2+\frac{\xi^3}{3} & \xi-\frac{\xi^3}{3} \\ \xi-\frac{\xi^3}{3} & \xi+\xi^2+\frac{\xi^3}{3} \end{bmatrix} \Rightarrow$$

$$\Rightarrow m = \begin{bmatrix} \rho \frac{AL}{3} & \rho \frac{AL}{6} \\ \rho \frac{AL}{6} & \rho \frac{AL}{3} \end{bmatrix}$$

4.

Cross-sectional area that varies from A_1 to A_2.

+ Variation of area as:

$$A(\xi) = \sum_{i=1}^{2} N_i(\xi) A_i = \frac{A_1}{2}(1-\xi) + \frac{A_2}{2}(1+\xi)$$

+ Mass matrix:

$$m = \int_{-1}^{1} \underline{N}^T \underline{N} \rho A |J| d\xi = \rho \frac{L}{2} \int_{-1}^{1} \begin{bmatrix} \frac{1}{2}(1-\xi) \\ \frac{1}{2}(1+\xi) \end{bmatrix} \left[\frac{1}{2}(1-\xi) \ \frac{1}{2}(1+\xi) \right] \cdot$$

$$\left(\frac{A_1}{2}(1-\xi) + \frac{A_2}{2}(1+\xi) \right) d\xi = \text{"Steps done apart}$$

$$= \rho \frac{L}{16} \left[A_1 \begin{bmatrix} \xi - \frac{3\xi^2}{2} + \xi^3 - \frac{\xi^4}{4} & \xi - \frac{\xi^2}{2} - \frac{\xi^3}{3} + \frac{\xi^4}{4} \\ \xi - \frac{\xi^2}{2} - \frac{\xi^3}{3} + \frac{\xi^4}{4} & \xi + \frac{\xi^2}{2} - \frac{\xi^3}{3} - \frac{\xi^4}{4} \end{bmatrix} \right.$$

$$\left. + A_2 \begin{bmatrix} \xi - \frac{\xi^2}{2} - \frac{\xi^3}{3} + \frac{\xi^4}{4} & \xi + \frac{\xi^2}{2} - \frac{\xi^3}{3} - \frac{\xi^4}{4} \\ \xi + \frac{\xi^2}{2} - \frac{\xi^3}{3} - \frac{\xi^4}{4} & \xi + \frac{3\xi^2}{2} + \xi^3 + \frac{\xi^4}{4} \end{bmatrix} \right]_{-1}^{1}$$

$$m = \begin{bmatrix} \rho L \frac{3A_1 + A_2}{12} & \rho L \frac{A_1 + A_2}{12} \\ \rho L \frac{A_1 + A_2}{12} & \rho L \frac{A_1 + 3A_2}{12} \end{bmatrix}$$

5. Diagonal mass matrix

Lumped matrix of the 3D 2-node bar, assigning half of the mass of the bar at every node:

$$\boxed{m = \frac{\rho A L}{2} \underline{I_6}}$$

$\underline{I_6}$: identity matrix

The element has six degrees of freedom corresponding to the three displacements of each node.