UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

Universitat Politècnica de Catalunya

Escola Tècnica Superior d'Enginyers de Camins, Canals i Ports

MASTER EN INGENIERÍA ESTRUCTURAL Y DE LA CONSTRUCCIÓN

Course:

COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS

Assignment 8

On "Axisymmetric Shells and Arches"

By

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Assignment 9:

a) Describe in extension how can be applied a non-symmetric load on this formulation.

b) Using thin beams formulation, describe the shape of the B(e) matrix and comment the integration rule.

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a) On this formulation, in a solid of revolution can be applied a non – symmetric load using a Fourier decomposition method. The non – symmetric load has to be expressed in Fourier series form in the circumferential direction. The expression is

$$f(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\theta + b_n \sin n\theta)$$

With

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\theta) \cos n\theta \, d\theta \qquad n = 0,1,2, \dots, \infty$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\theta) \sin n\theta \, d\theta \qquad n = 1,2, \dots, \infty$$

After that, the response of the structure to each harmonic term retained in the series is calculated, and adding the results, the global structure response is obtained. This superposition technique, is limited to linear problems.

As done with the load, the displacement field also has to be expressed in the series form.

$$u = \sum_{n=1}^{\infty} (u_{a_n} \cos n\theta + u_{b_n} \sin n\theta)$$

$$w = \sum_{n=1}^{\infty} (w_{a_n} \cos n\theta + w_{b_n} \sin n\theta)$$

$$\theta = \sum_{n=1}^{\infty} (\theta_{a_n} \cos n\theta + \theta_{b_n} \sin n\theta)$$

Now, the strain vector is redefined and the stiffness matrix for $n = 0$ term and $n = 1,2, \dots, \infty$ are re-defined. Also the loads vectors.

b) With thin beam formulation the $B_{(e)}$ matrix has the form

$$B_i = \begin{bmatrix} \frac{\partial N_i}{\partial s} cos\phi & \frac{\partial N_i}{\partial s} sin\phi & 0 \\ \frac{N_i}{x} & 0 & 0 \\ 0 & 0 & -\frac{\partial N_i}{\partial s} \\ 0 & 0 & -\frac{N_i cos\phi}{x} \\ -\frac{\partial N_i}{\partial s} sin\phi & \frac{\partial N_i}{\partial s} cos\phi & -N_i \end{bmatrix}$$

The first 2 rows represent the membrane behaviour of the element, the 3rd and 4th rows the beam behaviour and the last one the shear behaviour.

If is used Lobatto integration rule in elements with a node in the axis of revolution are not going to be able to compute stiffness matrix, due the $\frac{1}{x}$ members in B matrix (that would make terms of B_i matrix tending to ∞). Using Gauss integration rule, with integration points inside the element, this problem is avoided. Using linear shape functions, to avoid the shear locking effect, it is used a single gauss integration point in the middle of the element. If we increase the order of the shape function, we are able to increase the number of integration gauss points in the element, but to avoid locking effect must to be used a sub integration using less number of points for the shear behaviour.