# Assignment 9, Computational Structural Mechanics and Dynamics

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## Problem a

- Describe in extension how can be applied a nonsymmetric load on this formulation.

When addressing external forces in axisymmetric shells, the formulation can be extended for an analysis under an arbitrary loading pattern. The analysis of axisymmetric solid structures with non-symmetric load distribution has to be expressed by representation of structural configuration using two- dimensional model and the non-symmetric load to be expressed in the **Fourier series** form. The main idea is to take advantage of the fourier series to obtain a numerically solution of the **f**-vector. $f(\theta)$, the fourier series representing the load vector is displayed as:

$$f(\theta) = \frac{a_0}{2} + \sum (a_n cos(n\theta) + b_n sin(n\theta))$$

where $a_n$ and $b_n$ is obtained by the expressions below:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)cos(n\theta)d\theta, \quad n = 1, 2, 3, 4...$$
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)sin(n\theta)d\theta, \quad n = 1, 2, 3, 4...$$

where $\theta$ is the coordinate along the circumferential direction.

The load is now expressed in the form of Fourier series, but the displacement field with in the body also has to be expressed in a similar way. Therefore the displacement field within the body is expressed as follows:

$$u = \sum_{n=1}^{\infty}(u_{a_n} cos(n\theta) + u_{b_n} sin(n\theta))$$
$$v = \sum_{n=1}^{\infty}(v_{a_n} cos(n\theta) + v_{b_n} sin(n\theta))$$
$$u = \sum_{n=1}^{\infty}(w_{a_n} cos(n\theta) + w_{b_n} sin(n\theta))$$

where u, v and w express the displacement in axial, radial and circumferential directions. When both the displacements and the loading is expressed by Fourier series, the problem can be solved numerically.

# 1 Problem b

- Using thin beams formulation, describe the shape of the $B^{(e)}$ matrix and comment the integration rule.

Thin beam formulations means to use the Kirchoff Plate theory for thin plates and shells. This theory uses the assumption of negligible shear energy. Therefore, the transvere shear strain term is neglected, and the total strain only contains bending and membrane strains

$$\varepsilon = \begin{bmatrix} \varepsilon_{bending} \\ \varepsilon_{membrane} \end{bmatrix}, \;\; \varepsilon_{shear} = 0$$

Therefore will there also be a lack of the transverse shear term in the $B^{(e)}$ matrix, and this results in the term

$$B^{(e)} = \begin{bmatrix} B_b^{(e)} \\ B_m^{(e)} \end{bmatrix}$$

The main issue that occurs with the Kirchoff thin shell elements is the possibility of membrane locking. An usual way to prevent this problem is using the Selectively Reduced Integration scheme for the membrane terms. This erase the possibility of membrane locking, and will therefore result in a value that is more precise based on the analytic solution.