UPC - BARCELONA TECH
MSc Computational Mechanics
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Computational Structual Mechanics and Dynamics

Homework 9: Axissymmetric Shells

Prasad ADHAV

1 Describe in extension how can be applied a non-symmetric load on this formulation?

When using any type of revolution formulation, it assumed that the loads, supports and by extension geometry is axissymmetric. But we can still use this formulation for non-symmetric loads, by using a modified version though. The way to approach this problem is by modeling the complete structural configuration using 3-Dimensional model and then apply the non-symmetric loads are required position.

In a practical way, the angle θ around the symmetric axis is discretized by linear finite elements. Thus the formulation now has new dimension, like a FEM solution in time for example where t is the new dimension. So, instead of applying 2 the PVW integral, the angle θ is integrated element by element in the tangential direction from zero to the circumferential length L. In order to apply discontinuous loads along the circumferential domain, the use of Fourier series is proposed. In this approach for keeping the consistency, rotations and displacements are also discritized using a Fourier series.

2 Using thin beams formulation, describe the shape of the B(e) matrix and comment the integration rule.

The thin beam formulation for element can be derived by introducing normal orthogonally condition in the kinetic field, i.e neglecting the effect of transverse shear strain in the analysis). This formulation is applicable to the Thin Shell problems only.

$$\hat{\varepsilon} = 0; \qquad \theta_s = \frac{\partial w_0^{'}}{r\partial \beta}; \qquad \theta_t = \frac{1}{r}\frac{\partial w_0^{'}}{\partial \beta} + \frac{v_0^{'}}{r}s$$

Taking into account the above terms, strain matrix and B matrix are

$$\hat{\boldsymbol{\varepsilon}}' = \begin{Bmatrix} \hat{\boldsymbol{\varepsilon}}'_m \\ \hat{\boldsymbol{\varepsilon}}'_b \end{Bmatrix} \quad ; \quad \begin{cases} \hat{\boldsymbol{\varepsilon}}'_m = \begin{Bmatrix} \dfrac{\partial u'_0}{\partial s} \\ \dfrac{1}{r}\dfrac{\partial v'_0}{\partial \beta} + \dfrac{u'_0}{r}C - \dfrac{w'_0}{r}S \\ \dfrac{\partial v'_0}{\partial s} + \dfrac{1}{r}\dfrac{\partial u'_0}{\partial \beta} - \dfrac{v'_0}{r}C \end{Bmatrix} \\ \hat{\boldsymbol{\varepsilon}}'_b = \begin{Bmatrix} \dfrac{\partial^2 w'_0}{\partial s^2} \\ \dfrac{1}{r^2}\dfrac{\partial^2 w'_0}{\partial \beta^2} + \dfrac{S}{r^2}\dfrac{\partial v'_0}{\partial \beta} + \dfrac{C}{r}\dfrac{\partial w'_0}{\partial s} \\ \dfrac{2}{r}\dfrac{\partial^2 w'_0}{\partial s \partial \beta} - \dfrac{2CS}{r^2}v'_0 - \dfrac{2C}{r^2}\dfrac{\partial w'_0}{\partial \beta} + \dfrac{S}{r}\dfrac{\partial v'_0}{\partial s} \end{Bmatrix} \end{cases}$$

$$\mathbf{B}_i'^l = \begin{Bmatrix} \mathbf{B}_{m_i}'^l \\ \mathbf{B}_{b_i}'^l \end{Bmatrix}; \begin{cases} \mathbf{B}_{m_i}'^l = \begin{bmatrix} \dfrac{\partial N_i}{\partial s} & 0 & 0 & 0 \\ \dfrac{N_i}{r}C & -\dfrac{N_i}{r}\gamma & -\dfrac{H'_i}{r}S & -\dfrac{\bar{H}_i}{r}S \\ \dfrac{N_i}{r}\gamma & \left(\dfrac{\partial N_i}{\partial s} - \dfrac{N_i}{r}C\right) & 0 & 0 \end{bmatrix} \\ \mathbf{B}_{b_i}'^l = \begin{bmatrix} 0 & 0 & \dfrac{\partial^2 H_i}{\partial s^2} & \dfrac{\partial^2 \bar{H}_i}{\partial s^2} \\ 0 & \dfrac{N_i}{r^2}S\gamma & \left[\dfrac{C}{r}\dfrac{\partial H_i}{\partial s} - \left(\dfrac{\gamma}{r}\right)^2 H_i\right] & \left[\dfrac{C}{r}\dfrac{\partial \bar{H}_i}{\partial s} - \left(\dfrac{\gamma}{r}\right)^2 \bar{H}_i\right] \\ 0 & \left(\dfrac{S}{r}\dfrac{\partial N_i}{\partial s} - \dfrac{2N_i}{r^2}CS\right) & \left(\dfrac{2\gamma}{r}\dfrac{\partial H_i}{\partial s} - \dfrac{2H_i}{r^2}C\gamma\right) & \left(\dfrac{2\gamma}{r}\dfrac{\partial \bar{H}_i}{\partial s} - \dfrac{2\bar{H}_i}{r^2}C\gamma\right) \end{bmatrix} \end{cases}$$

Where N_i is Langrangian Shape Function, $H_i; \bar{H}_t$ is Hermite Shape Function.
The integration rule recommended is two-point quadrature for computing the given integrals. We can also obtain results using one-point quadrature.