Course
Computational Structural Mechanics and Dynamics

Assignment 9
on
Revolution Shells

by

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Exercise: Revolution Shells

Ques 1. Describe in extension how can be applied a non-symmetric load on this formulation?

Solution: Under any arbitrary loading, in axisymmetric shells the length of the structure is considered as a whole circumference (i.e. 2π is replace in place of angle α). Along the circumferential direction, the displacements are expanded in Fourier series. As a result the displacement field gets split in symmetric and non-symmetric components.
The displacement vector (u') is given by the formula:

$$\mathbf{U}' = \sum_{l=0}^{m}\sum_{i=1}^{n} \mathbf{N}_i\{(\bar{s}\,\overline{a_i'^l}) + (\bar{\bar{s}}\overline{\overline{a_i'^l}})\}$$

Where, symmetric component represented by single bar and non-symmetric component represented by double bar respectively.

U', W' and θ_s which are the displacement components, contained in the symmetry plane are zero for an anti-symmetric loading. This zero harmonic term corresponds to its deformation where β is constant which have same displacement components.

The loads are expanded in *Fourier series using harmonic functions for displacements*,

$$\boldsymbol{t} = \sum_{l=0}^{m}\{(\bar{s}\bar{t}') + (\bar{\bar{s}}\bar{\bar{t}}')\}$$

t' indicates load amplitude.

For an axisymmetric strip element local stiffness matrix is

$$[\mathbf{K}'^{ll}_{ij}]^{(e)} = C\int_{a^{(e)}} [\mathbf{B}'^{l}_{i}]^T \hat{\mathbf{D}}' \mathbf{B}'^{l}_{j} r ds$$

Where D is constitute matrix, $C = \begin{cases} 2\pi\ for\ \ l = 0 \\ \pi\ \ for\ l \neq 0 \end{cases}$, r is radius of revolution shell.

$$\hat{\boldsymbol{\varepsilon}}' = \sum_{l=1}^{m}\sum_{i=1}^{n} \hat{\mathbf{S}}^l\, \mathbf{B}'^l_i\, \mathbf{a}^l_i$$

$$\hat{\boldsymbol{\varepsilon}}' = \begin{Bmatrix} \hat{\boldsymbol{\varepsilon}}'_m \\ \hat{\boldsymbol{\varepsilon}}'_b \\ \hat{\boldsymbol{\varepsilon}}'_s \end{Bmatrix} ; \quad \left\{ \begin{array}{l} \hat{\boldsymbol{\varepsilon}}'_m = \begin{Bmatrix} \dfrac{\partial u'_0}{\partial x'} \\ \dfrac{\partial v'_0}{\partial y'} \\ \dfrac{\partial u'_0}{\partial y'} + \dfrac{\partial v'_0}{\partial x'} \end{Bmatrix} \\ \hat{\boldsymbol{\varepsilon}}'_f = \begin{Bmatrix} \dfrac{\partial \theta x'}{\partial x'} \\ \dfrac{\partial \theta y'}{\partial y'} \\ \left(\dfrac{\partial \theta x'}{\partial y'} + \dfrac{\partial \theta y'}{\partial x'}\right) \end{Bmatrix} \\ \hat{\boldsymbol{\varepsilon}}'_s = \begin{Bmatrix} \dfrac{\partial w'_0}{\partial x'} - \theta x' \\ \dfrac{\partial w'_0}{\partial y'} - \theta y' \end{Bmatrix} \end{array} \right.$$

$$\mathbf{B}'^l_i = \begin{Bmatrix} \mathbf{B}'^l_{m_i} \\ \mathbf{B}'^l_{b_i} \\ \mathbf{B}'^l_{s_i} \end{Bmatrix} ; \quad \left\{ \begin{array}{l} \mathbf{B}'^l_{m_i} = \begin{bmatrix} \dfrac{\partial N_i}{\partial x'} & 0 & 0\,0\,0 \\ 0 & -N_i\gamma & 0\,0\,0 \\ N_i\gamma & \dfrac{\partial N_i}{\partial x'} & 0\,0\,0 \end{bmatrix} \\ \mathbf{B}'^l_{b_i} = \begin{bmatrix} 0\,0\,0 & \dfrac{\partial N_i}{\partial x'} & 0 \\ 0\,0\,0 & 0 & N_i\gamma \\ 0\,0\,0 & N_i\gamma & \dfrac{\partial N_i}{\partial x'} \end{bmatrix} \\ \mathbf{B}'^l_{s_i} = \begin{bmatrix} 0\,0 & \dfrac{\partial N_i}{\partial x'} & -N_i & 0 \\ 0\,0 & N_i\gamma & 0 & -N_i \end{bmatrix} \end{array} \right.$$

for an axisymmetric strip element the global stiffness matrix is

$$[\mathbf{K}_{ij}^{ll}]^{(e)} = C \int_{a^{(e)}} [\mathbf{B}_i^l]^T \hat{\mathbf{D}}' \mathbf{B}_j^l r ds$$

Hence , a revolution shell formulation can be expanded on the nonsymmetric loading using Fourier Series.

Ques 2. Using thin beams formulation, describe the shape of the B(e) matrix and comment the integration rule.

Solution:

a. **The B(e) matrix shape :**

For an element, the thin Beam formulation (Kirchhoff theory) can be derived by introducing normal orthogonality condition in the kinetic field (i.e., neglecting the effect of transverse shear strain in the analysis). It can only be applied to the Thin Shell problems.

$$\hat{\varepsilon}_s = 0; \quad \theta_s = \frac{\partial w_0'}{\partial s}; \quad \theta_t = \frac{1}{r}\frac{\partial w_0'}{\partial \beta} + \frac{v_0'}{r} s$$

Taking into account above terms, strain matrix and B matrix are

$$\hat{\boldsymbol{\varepsilon}}' = \begin{Bmatrix} \hat{\boldsymbol{\varepsilon}}_m' \\ \hat{\boldsymbol{\varepsilon}}_b' \end{Bmatrix} \quad ; \quad \left\{ \begin{array}{l} \hat{\boldsymbol{\varepsilon}}_m' = \begin{Bmatrix} \dfrac{\partial u_0'}{\partial s} \\ \dfrac{1}{r}\dfrac{\partial v_0'}{\partial \beta} + \dfrac{u_0'}{r}C - \dfrac{w_0'}{r}S \\ \dfrac{\partial v_0'}{\partial s} + \dfrac{1}{r}\dfrac{\partial u_0'}{\partial \beta} - \dfrac{v_0'}{r}C \end{Bmatrix} \\ \hat{\boldsymbol{\varepsilon}}_b' = \begin{Bmatrix} \dfrac{\partial^2 w_0'}{\partial s^2} \\ \dfrac{1}{r^2}\dfrac{\partial^2 w_0'}{\partial \beta^2} + \dfrac{S}{r^2}\dfrac{\partial v_0'}{\partial \beta} + \dfrac{C}{r}\dfrac{\partial w_0'}{\partial s} \\ \dfrac{2}{r}\dfrac{\partial^2 w_0'}{\partial s \partial \beta} - \dfrac{2CS}{r^2}v_0' - \dfrac{2C}{r^2}\dfrac{\partial w_0'}{\partial \beta} + \dfrac{S}{r}\dfrac{\partial v_0'}{\partial s} \end{Bmatrix} \end{array} \right.$$

$$\mathbf{B}_i'^l = \begin{Bmatrix} \mathbf{B}_{m_i}'^l \\ \mathbf{B}_{b_i}'^l \end{Bmatrix}; \left\{ \begin{array}{l} \mathbf{B}_{m_i}'^l = \begin{bmatrix} \dfrac{\partial N_i}{\partial s} & 0 & 0 & 0 \\ \dfrac{N_i}{r}C & -\dfrac{N_i}{r}\gamma & -\dfrac{H_i'}{r}S & -\dfrac{\bar{H}_i}{r}S \\ \dfrac{N_i}{r}\gamma & \left(\dfrac{\partial N_i}{\partial s} - \dfrac{N_i}{r}C\right) & 0 & 0 \end{bmatrix} \\ \mathbf{B}_{b_i}'^l = \begin{bmatrix} 0 & 0 & \dfrac{\partial^2 H_i}{\partial s^2} & \dfrac{\partial^2 \bar{H}_i}{\partial s^2} \\ 0 & \dfrac{N_i}{r^2}S\gamma & \left[\dfrac{C}{r}\dfrac{\partial H_i}{\partial s} - \left(\dfrac{\gamma}{r}\right)^2 H_i\right] & \left[\dfrac{C}{r}\dfrac{\partial \bar{H}_i}{\partial s} - \left(\dfrac{\gamma}{r}\right)^2 \bar{H}_i\right] \\ 0 & \left(\dfrac{S}{r}\dfrac{\partial N_i}{\partial s} - \dfrac{2N_i}{r^2}CS\right) & \left(\dfrac{2\gamma}{r}\dfrac{\partial H_i}{\partial s} - \dfrac{2H_i}{r^2}C\gamma\right) & \left(\dfrac{2\gamma}{r}\dfrac{\partial \bar{H}_i}{\partial s} - \dfrac{2\bar{H}_i}{r^2}C\gamma\right) \end{bmatrix} \end{array} \right.$$

Where, N_i - Lagrange Shape Function, H_i ; $\overline{H_i}$ – Hermite Shape Function.

b. Integration Rule: For computing the previous integrals, two-point quadrature is highly recommended. But also results can be obtained using simplest reduced one-point quadrature.