Master's Degree Numerical Methods in Engineering

Universitat Politècnica de Catalunya
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Computational Structural Mechanics and Dynamics

Assignment 9: Dynamics

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1) The equation used for the model is:

$$m\ddot{u} + ku = F$$

The displacement is build from two solutions, one solved for the model with F = 0 and one constant value.

$$u = C_1 sin(\omega t) + C_2 cos(\omega t) + \frac{F}{k}$$

Using an arbitrary initial condition of u_0 and v_0 we will find the constants as:

$$t = 0 \rightarrow u = u_0 \rightarrow u_0 = C_2 + \frac{F}{k} \rightarrow C_2 = u_0 - \frac{F}{k}$$

$$t = 0 \rightarrow \dot{u} = v_0 \rightarrow v_0 = C_1 \omega cos(\omega t) \rightarrow C_1 = \frac{v_0}{\omega}$$

$$u = \frac{v_0}{\omega} sin(\omega t) + (u_0 - \frac{F}{k}) cos(\omega t) + \frac{F}{k}$$

The F does not effect the natural frequency of the system and as before its calculated as $\sqrt{\frac{k}{m}}$. The effect of $\boldsymbol{F}$ on the model as seen on the displacement is a constant added and an amplitude changer.

2) The natural frequency of the system is calculated as $\sqrt{\frac{k}{m}}$ where the m is the mass of the system and the k for the clamped beam at both ends can be calculated as :

$$k = \frac{192EI}{L^3} as I = \frac{1}{12} A^2 \rightarrow k = \frac{16EA^2}{L^3}$$

So the natural frequency can be calculated as:

$$\omega = \sqrt{\frac{16EA^2}{mL^3}}$$

3) The m matrix is calculated as:

$$m = \int \boldsymbol{N^T N} N dV$$

Where :

$$N = \begin{bmatrix} \frac{L-x}{L} & \frac{x}{L} \end{bmatrix} \rightarrow \boldsymbol{N^T N} = \begin{bmatrix} (\frac{L-x}{L})^2 & \frac{x(L-x)}{L^2} \\ \frac{x(L-x)}{L^2} & (\frac{x}{L})^2 \end{bmatrix}$$

Integrating and changing dV to $\rho A dx$ and integrating over x we will have:

$$m = \rho A // L^2 \begin{bmatrix} \frac{L^3}{3} & \frac{L^3}{6} \\ \frac{L^3}{6} & \frac{L^3}{3} \end{bmatrix}$$

4) We first have to find the formula for the change of the area with respect to the element length.

$$A(x) = A_1 + \frac{x}{L}(A_2 - A_1)$$

Using the formulas in the third problem we again integrate in order to obtain the m matrix.

$$m = \rho \int (A_1 + \frac{x}{L}(A_2 - A_1)) \begin{bmatrix} (\frac{L-x}{L})^2 & \frac{x(L-x)}{L^2} \\ \frac{x(L-x)}{L^2} & (\frac{x}{L})^2 \end{bmatrix} dx$$

$$m = \rho(A_1/L^2 \begin{bmatrix} \frac{L^3}{3} & \frac{L^3}{6} \\ \frac{L^3}{6} & \frac{L^3}{3} \end{bmatrix} + (A_2 - A_1)/L^2 \begin{bmatrix} \frac{L^3}{12} & \frac{L^3}{12} \\ \frac{L^3}{12} & \frac{L^3}{4} \end{bmatrix}$$

$$m = \rho L/12 \begin{bmatrix} 3A_1 + A_2 & 2A_1 + A_2 \\ 2A_1 + A_2 & A_1 + A_2 \end{bmatrix}$$

5) The m matrix of thetwo-noded bar element in the 3D space should have three d.o.fs for each node so it should be a 6 by 6 matrix or a diagonlized matrix of size 6. The weight of the element is ρAL so the m matrix will be as:

$$m_e = \rho AL[111111]_{diag} = \rho AL\boldsymbol{I_6}$$