Assignment 9.1

Describe in extension how can be applied a non-symmetric load on this formulation

In this case where non symmetric loads are applied to an axisymmetric structure, both symmetric and non-symmetric displacements (tangential displacements different than zero) need to be accounted for in the shell formulation.

According to Zienkiewicz and Taylor, the problem of an axisymmetric solid subjected to non-symmetric loads may be solved by expressing displacements, as well as force components, as Fourier series expansions, splitting them into symmetric and antisymmetric components to obtain two stiffness matrices defined as following:

$$K_a = \pi \int_A B_a^T D B_a dA$$
$$K_b = \pi \int_A B_b^T D B_b dA$$

It let us describe the harmonic behaviour of the structure with respect to the circumferential direction, allowing us to describe it in one dimension.

Being the loads and displacements described in polar coordinates and the loads from the Fourier series as:

$$f(\theta) = \frac{a_0}{2} + \sum(\frac{cosn\theta}{\pi}\int_{-\infty}^{+\infty} f(\theta)cosn\theta d\theta) + \frac{sinn\theta}{\pi}\int_{-\infty}^{+\infty} f(\theta)sinn\theta d\theta$$

For the case of axisymmetric thin shells (Euler-Bernoulli formulation), the definition of strains is also modified to take into account the displacements and force in all three directions. The strain vector then becomes:

$$\epsilon = \begin{bmatrix} \epsilon_a \\ \epsilon_\theta \\ \gamma_{a\theta} \\ \lambda_a \\ \lambda_\theta \\ \lambda_{a\theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial a} \\ \frac{1}{r}\frac{\partial v}{\partial \theta} + (ucos\theta - \frac{1}{r}wsin\theta) \\ \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial a} - \frac{1}{r}vcos\theta \\ -\frac{\partial^2 w}{\partial a^2} \\ -\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} - \frac{1}{r}\frac{\partial w}{\partial a}cos\theta + \frac{1}{r}\frac{\partial v}{\partial \theta}sin\theta \\ \frac{2}{r}(-\frac{\partial^2 w}{\partial a \partial \theta} + \frac{1}{r}\frac{\partial w}{\partial \theta} + \frac{\partial v}{\partial a} - \frac{1}{r}vsin\theta cos\theta) \end{bmatrix}$$

Now, three membrane and three bending effects are now present in the stress vector.

$$\sigma = \begin{bmatrix} N_s & N_\theta & N_{s\theta} & M_s & M_\theta & M_{s\theta} \end{bmatrix}'$$

Assignment 9.2

Using thin beams formulation, describe the shape of the B(e) matrix and component the integration rule.

For the Kirchhoff formulation of axisymmetric shells, shear effects are neglected and the B matrix will be composed only by the membrane strain and bending strain.

So the general deformation matrix is defined as following:

$$B^e = \begin{bmatrix} B_m^e \\ B_b^e \end{bmatrix} = \begin{bmatrix} \frac{\partial N_a^e}{\partial a} & 0 & 0 \\ \frac{N_u^e cos\theta}{r} & -\frac{N_w^e sin\theta}{r} & -\frac{\bar{N}_w^e sin\theta}{r} \\ 0 & \frac{\partial^2 N_w^e}{\partial a^2} & \frac{\partial^2 \bar{N}_w^e}{\partial a^2} \\ 0 & \frac{cos\theta}{r}\frac{\partial^2 N_w^e}{\partial a^2} & \frac{cos\theta}{r}\frac{\partial^2 \bar{N}_w^e}{\partial a^2} \end{bmatrix}$$

Regarding the integration rule, it may be observed that for r→0 the integration method will present problems. This because some terms of B depends on 1/r by r. This could mean that an integration method for which it is necessary to evaluate the integrals at the nodes would present problems at nodes with r = 0.

The inconvenient is however solved by using a numerical integration scheme such as Gauss quadratures, where the function need not be evaluated at the nodes.